Percentage Rents and Stand-Alone Property:  
Share Contracting as a Barrier to Entry¹

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Abstract

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¹ This paper has benefitted from the input of numerous individuals and seminar attendees. We wish to single out three individuals for their special contributions to the paper. First, we are grateful to Rob Bond of the Bond Companies for suggesting this idea to us. Second, Riddiough thanks Bill Wheaton and Joe Williams for the many hours spent discussing various aspects of retail contracting theory and practices. We do not, however, wish to implicate anyone – all contrivances, errors and shortcomings are the authors and the authors alone.
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Abstract

Although share (percentage rent) lease contracts have been rationalized when inter-store externalities exist, the case of stand-alone property has yet to be addressed. To consider this case, we develop a model of a local trade area with a retail tenant that makes non-contractable specific investment at the time of initial contracting and a monopolist landlord that controls the timing of follow-on entry. A two-part share contract is analyzed, in which a positive fraction of sales revenues is passed from the incumbent retail tenant to the landlord. This contract is shown to be optimal as well as robust to constrained renegotiation incentives. The percentage rent contract is, however, dominated by an enhanced contract that includes a lump-sum payment made by the landlord to the incumbent tenant at the time of competitive entry. Implementation of the welfare maximizing contract is also analyzed and policy implications are discussed.

I. Introduction and Summary of Findings

Brueckner’s (1993) seminal paper on inter-store externalities has generated a great deal of research interest on the topic of retail lease contracts. In his paper, after considering optimal space allocation decisions, Brueckner puzzles over the fact that standard principal-agent theory with effort provision predicts that negative percentage rent contracts should be observed rather than the share contract seen in practice in which the percentage rents flow from the tenant to the landlord.

Two more recent papers provide rationales for the percentage rent contract that depend on agency and moral hazard. Wheaton (2000) rationalizes share contracts in a shopping center setting as a credible commitment device. In his model with long-term lease contracts, percentage rents paid by the tenants to the landlord prevent the landlord from deviating from an optimal tenant mix should an individual tenant prematurely disappear (e.g., go bankrupt, exercise a cancellation option) after the initial set of tenants is determined. The model addresses a particular type of hold-up problem, with the unique twist of incorporating complementarities in agent
production to characterize the optimal contracting problem. Williams (2013) stresses a different type of moral hazard, which is the landlord’s incentive to skimp on property maintenance and reinvestment once a set of long-term leases have been signed by a cluster of retail tenants. In his model percentage rent contracts create a virtuous cycle, supported by both the landlord and tenants in equilibrium. Through the optimal percentage rent contract, tenants share revenues with the landlord in order to provide resources to maintain and reinvest in the property as a whole, where maintenance/reinvestment serves to increase tenant productivity. This in turn creates greater revenues for both the tenants and the landlord, some of which is used to fund the next round of maintenance and investment.\(^2\)

To date, the retail lease contracting literature has focused almost exclusively on inter-store externalities that exist within shopping centers or clusters of retail stores. Yet there exist percentage rent contracts in retail and franchise settings where there are no obvious complementarities between a subject site and surrounding sites. As evidence that share contracting exists with stand-alone retail property, we have obtained data on stand-alone Walgreen convenience store leases for which the property owner (landlord) debt-financed it’s leasehold position through the commercial mortgage-backed securities market.\(^3\) The data are from 1998 through 2006. A review of the data produces the following stylized empirical facts. Based on a sample of 149 Walgreen leased-fee properties, 79 executed a percentage rent contract that contained a fixed rent component and 70 executed a lease with a fixed rent component only. Of the 79 properties with a percentage rent contract and for which the detailed data exist, all paid either 1.0 or 2.0 percent of sales to the landlord. Lease maturity on the percentage rent contracts ranged from 240 to 900 months, with an average lease maturity of 763 months (over 60 years).

Industry practice thus indicates that sharing contracts are frequently utilized with stand-alone urban retail property, where sharing percentages are relatively low-powered and

\(^2\) There are numerous other papers that provide models to rationalize share contracting with retail property. For example, Miceli and Sirmans (1995) consider risk sharing motives, and Cho and Shilling (2007) analyze shopping center leases with sales externalities in a real options context with tenant default. A number of empirical papers analyze shopping center leases, including Benjamin, Boyle and Sirmans (1990), Eppli and Shilling (1995) and Gould, Pashigian and Prendergast (2005). For recent work on issues of shopping center concentration, contracting effects and space allocation outcomes, see Des Rosiers et al. (2009), Liu and Liu (2013) and You and Lizieri (2013). See in particular Williams (2013) for a comprehensive and up-to-date review of the relevant retail and shopping center contracting literature.

\(^3\) These data were obtained from a commercial mortgage-backed securities purchaser with whom one of the authors on this paper had known for some time. We note that there is no information on the landlord in the data set. The data reported above are available from the authors by request.
lease maturities are long term. The objective of this paper is to rationalize and characterize this share contract phenomenon in a setting where local land ownership is concentrated and incumbent tenants are concerned about the adverse effects of nearby competitive entry on retail sales.4

Importantly, retail property is unique among commercial property types as it relates to incumbent tenant effects. Additional competitive supply typically benefits incumbent tenants of non-retail property, since new supply puts downward pressure on rents and does not negatively affect tenant productivity (think of typical office, apartment or hotel tenants). In contrast, the new local supply of competitive retail space will, in many circumstances, negatively affect sales productivity of the incumbent tenant. It is precisely the negative implications of competitive entry into a retail trade area that creates incentives for both the landlord and the incumbent retail tenant to implement a contract in which retail sales revenues are shared, and shared in such a way that they create a barrier for new entry into the market.

In an attempt to capture these salient features, in this paper we construct a model with a double moral hazard. One moral hazard relates to the controlling landlord’s decision of when to allow follow-on competitive entry to occur into a local trade area. The landlord’s decision will depend on the costs and benefits of facilitating entry, where net benefits depend on the realization of new lease flows relative to the costs of building out the new space. A sharing contract with the incumbent tenant imposes an additional cost on the landlord, however, in that entry will decrease incumbent tenant sales to reduce lease revenue flow, thus providing an incentive for the landlord to delay entry. The incumbent tenant of course prefers delayed entry while not necessarily preferring to share revenues with the landlord. This creates the second moral hazard by affecting the tenant’s decision of how much effort to exert as it impacts sales. Addressing this tension between the timing of the landlord’s entry decision and the incumbent tenant’s effort decision as it depends on anticipated future actions of the landlord creates the basis for the optimal share contract.5

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4 There is a long-standing and quite prominent literature, going back to Marshall, on agricultural share contracting that spurred a general share contracting literature. For classic sharecropping applications, see Cheung (1968) and Stiglitz (1974). None of the share contracting literature to our knowledge has focused on it as a barrier to entry into markets.

5 Entry games have been long been a focus in the real estate economics literature as way to characterize supply response in local markets. See, for example, Grenadier (1996, 1999), Childs, Ott and Riddiough (2002) and Wang and Zhou (2006). In these models there are no long-term contracts, nor is contracting used to create entry barriers. In particular, in contrast to this cited literature, our focus is primarily on explaining share contracts as a means to an
In our model there are three agents: a monopoly upstream supplier (the landlord) that controls the timing of follow-on entry into the downstream (retail) market, an incumbent downstream producer (incumbent retail tenant, say a Walgreen convenience store) and a potential downstream entrant (second or entering retail tenant, say a CVS convenience store). Time is continuous over an infinite horizon, where final product demand is subject to a series of exogenous shocks. Production cost functions of the incumbent and the potential entering tenant are identical in our model. Specific investment by each tenant is endogenously determined at the time of initial contracting, with the costly investment paying dividends over time in the form of increased sales. Entry is accommodative in our model, in the sense that the incumbent tenant shares the retail product market with an entrant, sustaining an incremental loss in revenue at the time of entry but not a permanent loss of its entire revenue stream.

As an initial step in our analysis, we show that the optimal contract executed between the landlord and the follow-on entrant tenant (which enters sometime after the initial incumbent tenant lease is signed) is a fixed payment lease contract. The fixed payment contract is optimal because it causes an efficient level of specific investment by the entrant. In contrast, the optimal contract between the landlord and the incumbent tenant is shown to result in revenue sharing. Optimal specific investment by the incumbent tenant is, however, shown to be decreasing in the share percentage. Consequently, a delicate tradeoff is involved in structuring the optimal long-term contract: an increase in the share percentage delays entry to increase incumbent effort production while, conditional on the entry threshold value, an increase in the share percentage decreases effort production. The benefit of delayed entry exceeds the marginal cost of effort at the fixed payment-only margin to result in a (relatively small or low-powered) positive share percentage.

The share contracts we consider are unable to produce privately efficient (first-best) outcomes in a double moral hazard setting. This is because effort (controlled by the incumbent tenant) is too low and entry timing (controlled by the landlord) is too soon relative to first-best—where the two-part share contract can only delay entry at the expense of a reduction in incumbent effort. As an extension to our benchmark model, in consideration of retail contracts commonly observed in practice, we analyze a share contract that also includes a percentage rent breakpoint and a starting retail sales value that is below the breakpoint. This contract is shown to end, where the end is slowing new supply.
be more efficient than the benchmark contract (yet does not produce first-best outcomes), as the breakpoint provides an additional instrument with which to manage incentives of the tenant and the landlord. Other extensions to the benchmark model are considered, including the possibility of tenant default.

The introduction of a lump-sum transfer made by the landlord to the incumbent tenant at the time of entry is capable of generating an efficient contracting outcome. Furthermore, because a fixed payment-only lease contract induces efficient effort production by both the incumbent and the entrant tenants, revenue sharing is not required. Given that such a “simple” fixed lease contract is capable of dominating the standard percentage rent contract, we ask why we don’t see it in practice. We believe there are two reasons. First, it is likely that there are incumbent tenant concerns over possible breach and subsequent enforcement of the lump-sum payoff owed by the landlord at the time of entry, and second, the contractually specified lump-sum payoff may be a bit too obvious in terms of creating entry barriers into markets. A percentage rent contract, although less effective than the first-best contract, is much less obvious to regulators as a barrier to entry.6

Private contracting outcomes do not account for the consumers’ surplus. Welfare analysis suggests that effort production from a social perspective is too low relative to private contracting outcomes, and that entry occurs either too early or too late depending on the size of consumer surplus generated upon entry. In consideration of these effects, and taking the monopolistic upstream market structure as given, we analyze the optimal contract from a social perspective.

We show that this contract cannot be implemented using either the simple share contract or the augmented privately efficient contract. Instead, the socially optimal contract requires an additional flow subsidy that creates a differential share percentage between the landlord and the incumbent tenant. The simplest such contract involves: i) fixed payments made by the incumbent tenant to the landlord, ii) a percentage of sales subsidy paid by local government to the landlord, and iii) a lump-sum transfer subsidy or tax is imposed on the landlord at the time of entry that depends on whether consumer surplus is relatively large (subsidy) or small (tax). We note that certain incentive schemes seen in practice—e.g., those executed by local government and real estate developers that work with tenants to revitalize blighted neighborhoods—match well with

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6 Concern over regulator scrutiny explains why landlord and incumbent tenants do not enter into contracts that explicitly bar competitive entry.
the contracting incentive scheme that falls out from our model.

The main body of the paper is organized as follows. In section II the benchmark model is developed and the optimal (second-best) sharing contract is derived. We also derive the first-best (privately efficient) levels of tenant effort and landlord entry timing. Graphical relations and extensions to the benchmark model are also considered in section II. Section III examines the privately efficient contract that produces first-best levels of incumbent effort and landlord entry timing. Finally in section IV the socially efficient contract is examined and policy implications are discussed. The paper concludes in the final section.

II. The Benchmark Model

II.A. Model Set-Up

Envision an urban setting with a commercial property owner (which we refer to as the landlord) that monopolizes local land use. More specifically, we will consider two and only two properties that are owned by the landlord, both of which are identical in their physical and locational amenities when constructed. The highest and best use for the properties is also identical. Think of some sort of retail or franchise operation such as convenience stores, gas stations, or fast food outlets, which require physical space as an input for production. For exogenous reasons market participants value a separation of property ownership and day-to-day control of the space. We will refer to the space user as the tenant. This value to separation of ownership and control is fixed and therefore sunk, and will be addressed later in the paper in our analysis of first-best.

Separation of ownership and control raises the issue of executing a formal contractual agreement on space usage for a specified period of time. Recalling the long Walgreen lease terms noted in the introduction, we assume that for exogenous control and transaction cost reasons a long-term contract is optimal. This allows us to focus our attention on the optimality of other contractual terms and conditions. Again, as with the own-versus-rent issue, we further address the contract maturity issue in our analysis of first-best and a discussion of renegotiation incentives.

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7 Local land use and zoning restrictions often impose size and design standards that are binding constraints for developers and their tenants. In terms of locational attributes, envision two identical plots of land on opposing corner lots at a busy intersection with accessible parking.
The formal model is as follows. Specific inputs provided by the landlord are uniform, in the sense that the only difference between tenants is the timing of entry into the market and the tenants’ effort-investment decision. Landlord inputs are required by the tenant to generate a baseline quantity of sales into a retail market. The tenant provides management expertise and related investment that affect continuous sales activity. Resulting gross instantaneous sales of tenant \(i\) is:

\[
S_i = xe_i \pi(Q)
\]

where \(i=1\) denotes the incumbent tenant and \(i=2\) denotes the follow-on entrant tenant. Control rights with respect to follow-on entry rest with the monopolist landlord.

Sales are seen to have three components. The first component, \(x\), expresses exogenously determined baseline stochastic product demand that evolves continuously according to geometric Brownian motion, with \(\mu\) and \(\sigma\) denoting drift and volatility parameters of the process.

The second component, \(e_i\), is non-contractable effort production of tenant \(i\). Effort is supplied at time of entry, where effort effects are long-lived in the sense that they permanently adjust the flow of sales through time. One way to rationalize tenant effort is as an initial investment in brand or reputation-based marketing or advertising that persistently affects sales.8

For simplicity we will assume that effort production is quadratic in cost, as follows:

\[
c(e_i) = \frac{1}{2} \alpha e_i^2
\]

where \(\alpha>0\) is a cost parameter.9 Note that this cost is proportional to the stochastic sales parameter, \(x\), that is realized at the time of entry. This implies that production costs are indexed to, or are perfectly correlated with, stochastic product demand.

The third component is \(\pi(Q)\). \(Q\) is equal to 1 if only the incumbent tenant is operating in the market and equals 2 if both tenants are operating in the market. To abstract from micro-level tenant production issues, we assume that one constant-quality product is made and marketed to consumers. Retail product demand is imperfectly elastic, implying that \(\pi(2) < \pi(1)\). Note that

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8 This approach is general in that it would also apply to effort production that occurred through time, determined at contract execution. An alternative approach would be to consider the continuous supply of effort through time as an optimal control problem. We comment on this alternative later in the paper as an extension to the benchmark model. The extension also addresses renegotiation incentives on the part of the tenant as they relate to sunk investments and hold-up.

9 It would be straightforward to allow for different production cost parameters, but for simplicity of presentation we impose an equivalent cost parameter to both tenants.
homogeneous product demand here is standard in the sense that the schedule slopes downward, which is in contrast to a shopping center setting in which heterogeneous products offered by clustered stores are typically characterized as strategic complements.

Thus, summarizing, final product demand is stochastic and is expected to grow exponentially over time. Entry capital is immortal in the sense that it is infinitely durable (Eaton and Lipsey (1981)). Entry capital is also indivisible in the sense that capacity is exogenous and firms are identical in (unit) scale. Both the capital immortality and indivisibility assumptions provide good approximations for improved commercial property that is subject to zoning restrictions. But, the productivity of this capital is permanently altered by perfectly divisible specific investment made at the time of entry/initial contracting. This seems reasonable in the context of brand-based advertising and the like.

A long-term legally enforceable (lease) contract is structured to compensate the landlord for complementary product inputs and to motivate specific investment by the tenants. The particular form of the contract is as follows. A simple sharing (or, equivalently, percentage rent) contract is available that is linear in sales and completely characterized by two parameters: a fixed payment parameter \( a_i \) and sales percentage parameter \( p_i \). The instantaneous contract payment made by tenant \( i \) to the landlord is:

\[
R_i = a_i + p_i S_i \tag{3}
\]

Let \( \gamma \) denote the tenant’s profit margin as a fraction of sales, \( 0 < \gamma < 1 \), implying that the net operating profit of the tenant is \( \gamma S_i \) when current gross sales are \( S_i \).\(^{10}\) This net flow is available to fund the contractually specified payments made to the landlord, where net income to tenant \( i \) after operating expenses and the contract payment is \( \gamma S_i - R_i \).\(^{11}\) Finally, assume that all agents are risk neutral with an identical discount rate \( r \), where \( r > \mu \) to ensure convergence in value solutions.

After the incumbent tenant has signed a contract and has made its effort choice, the landlord decides when to allow follow-on entry into the market. Entry occurs at such time that the stochastic sales variable \( x \) equals or exceeds the endogenously determined threshold value, \( x^*_2 \). The landlord incurs a fixed cost, \( K \geq 0 \), at the time of follow-on entry, which reflects the fixed

\(^{10}\) As with the production cost parameter, for simplicity we impose equivalent profit margins across the two tenants. Again, this restriction is not required, as all results carry through with non-equivalent profit margin parameters.

\(^{11}\) In this benchmark model we will not consider bankruptcy issues or renegotiation incentives that accompany a decline in sales, implying that the tenant has access to outside resources to fund contractually specified payments when those payments exceed the current net operating profit. As an extension to the benchmark model, we will consider the effects of tenant contract payment default on the optimal contracting-entry timing problem.
marginal costs of supplying inputs for the entrant. In a commercial property context, infrastructure and build-out costs are examples of fixed costs incurred by the landlord to accommodate entry. Once the follow-on entry decision is made and the entry cost is incurred, a long-term contract is signed between the landlord and the entering tenant. Immediately thereafter, effort is chosen by the entering tenant. No additional entry occurs after this point. Figure 1 summarizes timing and contract action choices of the market participants.

II.B. The Optimal Contracting Problem

The optimal contracting problem is solved using backward induction. First consider the infinite-term percentage rent contract executed between the second entering tenant and the monopolist landlord. After that contract is executed, the effort production decision is made by the entering tenant. Conditional on the current level of baseline sales, as indexed by $x$, and the contracting variables, $a_2$ and $p_2$, the entrant’s value function is given by

$$G_2(x) = \int_0^\infty e^{-\gamma t} \left[ (\gamma - p_2) x(t) \pi(2) e - a_2 \right] dt \tag{4}$$

With this value function, and conditional on $x=x_2$ at the point of entry, the entrant chooses optimal effort by solving:

$$e_2^* = \arg \max_{e \geq 0} \left\{ G_2(x_2) - \frac{1}{2} x_2 \alpha e_2^2 \right\} \tag{5}$$

subject to participation, implying that $e_2 > 0$. Given that the participation constraint is satisfied, the optimal level of effort is:

$$e_2^* = \frac{(\gamma - p_2) \pi(2)}{\alpha (r - \mu)} \tag{6}$$

The comparative static relations evident in (6) are straightforward and intuitive. Specifically note that effort production is independent of the entry threshold value, $x_2$, as well as the fixed payment variable, $a_2$, and that it moves inversely with $p_2$, the percentage-of-sales contract variable.

Now consider the optimal contract. Assume the monopolistic landlord is in a position to offer the contract on a take-it or leave-it basis to the tenants, with the objective of maximizing the expected value of the incoming contract payments subject to participation by the tenant.
With no future entry, a fixed payment contract with no percentage-of-sales component is optimal. This result is formalized with the following proposition.

**Proposition 1:** Upon allowing entry to occur, the landlord offers a contract to the entering tenant that includes a fixed payment component only, implying that \( p_2^* = 0 \) and \( a_2^* > 0 \). Furthermore, \( a_2^* = \frac{r \gamma^2 \pi^2 (2) x_2}{2 \alpha (r - \mu)^2} \).

**Proof:** See Appendix 1.

With full bargaining power, the landlord extracts all of the surplus from the entering tenant vis-à-vis the participation constraint. Because no further entry can occur, the landlord’s objective is to maximize total net sales revenue generated by the entrant after accounting for effort costs. A fixed contract is optimal, since it does not distort effort production. Also note that, conditional on the observable demand variable, \( x_2 \), the optimal contract does not depend on any variables specific to the incumbent tenant—in particular, its contract terms.

Conditional on contract terms specified in the incumbent’s contract and its effort choice, \( \{a_1, p_1, e_1\} \), the landlord’s value function after follow-on entry has occurred can be written as:

\[
V_2(x) = \int_0^x e^{-\gamma t} (a_1 + a_2^* + p_1 x(t) \pi(2) e_1) dt
= \frac{p_1 e_1 \pi(2) x}{r - \mu} + \frac{a_1}{r} + \frac{\gamma^2 \pi^2 (2) x_2}{2 \alpha (r - \mu)^2}
\]  

(7)

Given \( V_2(x) \), the landlord's value function prior to follow-on entry can be determined. Let \( V_1(x) \) denote this value function, which depends explicitly on the baseline stochastic sales variable, \( x \), conditional on fixed contract parameter values \( a_1 \) and \( p_1 \), and the level of effort, \( e_1 \). \( V_1(x) \) satisfies a standard ordinary differential equation (ODE), subject to the boundary condition that \( V_1(x) = a_1/r \) if \( x = 0 \). This ODE has a solution of the form:

\[
V_1(x) = Ax^\beta_1 + \frac{a_1}{r} + \frac{p_1 e_1 \pi(1) x}{r - \mu}
\]  

(8)

where \( \beta_1 = \frac{-((\mu - \sigma^2) + \sqrt{((\mu - \sigma^2)^2 + 2 \sigma^2 r})}{\sigma^2} \) and \( A \) is a constant to be determined.

The landlord’s decision of when to allow follow-on entry into the market is an optimal stopping problem. As depicted in Figure 1, this problem can be characterized by determination of a threshold value, \( x_2^* \), with corresponding stopping time, \( t_2^* \), such that
At $x = x^*_2$, incentive compatibility (value matching and smooth pasting) conditions must be satisfied, which, when stated and solved, deliver the following results:

$$x^*_2 = \left[ \frac{\pi \left( \frac{\pi(2) - \pi(1)}{r - \mu} \right)}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \right]^{-1} \frac{\beta_1}{\beta_1 - 1} K$$  \hspace{1cm} (9)

and

$$A = \left[ \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} + \frac{p \pi \left( \frac{\pi(2) - \pi(1)}{r - \mu} \right)}{r - \mu} \right] \frac{x^* - \beta_1}{\beta_1} \hspace{1cm} (10)$$

Demand must be sufficiently elastic for entry to occur; thus, $2\pi(2) > \pi(1)$ is required as a parameter restriction to ensure that follow-on entry will occur for some finite $x^*$.\(^{12}\)

The share contract’s entry deterrence effect can be seen in equation (9). The critical follow-on entry value, $x^*_2$, is increasing in the percentage-of-sales parameter, $p_1$, as a result of $\pi(2) > \pi(1)$. Thus, all else equal, greater revenue sharing causes an increase in the expected time to entry. This benefits the incumbent tenant, which otherwise suffers a decline in profits as a result of entry.

Certain other comparative static relations are worth highlighting. The entry threshold is increasing in entry cost, $K$, implying that a higher entry cost substitutes for revenue sharing as a barrier to entry. The entry threshold is also increasing in the baseline sales volatility parameter, $\sigma$. Higher baseline sales volatility therefore reduces the incentive to revenue-share as a barrier to entry. This predicted relation is contrary to predictions that follow from risk-sharing rationales for revenue sharing.

The critical entry value increases in the entrant’s effort cost parameter, $\alpha$, and decreases in the profit margin parameter, $\gamma$. A higher cost of effort decreases tenant investment to decrease sales. This in turn causes a delay in entry, since a lower level of sales decreases payments flowing from the entering tenant to the landlord. An increase in profit margin increases payments made by the entering tenant to the landlord, which decreases the entry boundary.

\(^{12}\) This parameter restriction is standard with this type of dynamic Cournot entry game, implying greater total profits accruing to the landlord with two downstream producers operating in the market relative to just one. To see why this is reasonable, recall the market structure in which stochastic demand is exogenous and tenants are identical in scale (in their marginal production costs as well as in their product offering). Entry doubles the quantity of goods available to the market, so there is simply movement down the product demand curve (in which stochastic shifts in demand occur over time). The landlord takes these demand factors into account when considering whether or not to incur the fixed costs of entry, $K$, where entry accommodation doubles the supply of retail offerings. There are no other strategic considerations, which, with a different market structure, might result in greater profits in a monopoly as compared to an imperfectly competitive downstream market.
Now, let $F_i(x)$ denote the incumbent tenant's value function prior to follow-on entry. To ensure that $x$ is less than the entry threshold for arbitrary revenue share variable, $p_1$, we will restrict the initial value, $x_0$, so that $x_0 < \frac{2\alpha(r - \mu)^2}{\gamma\pi^2(2)} \frac{\beta_1}{\beta_1 - 1} K$. With this restriction, $F_i(x)$ initially satisfies a standard ODE with the boundary condition that, when sales equal zero, the incumbent tenant is obligated to pay the fixed component only (i.e., $F(x) = -a_1/r$ when $x=0$). As a consequence, the solution has the form:

$$F_i(x) = Bx^{\beta_i} + \left(\frac{\gamma - p_1}{r - \mu}\right)\frac{\pi(1)x}{r} - \frac{a_1}{r}$$  \hspace{1cm} (11)

where $\beta_i$ is as defined previously and $B$ is a constant to be determined.

The solution also satisfies the value matching condition at $x_2^*$, the point of entry by the follow-on tenant. This allows us to isolate and explicitly solve for $B$:

$$B = \left(\frac{\gamma - p_1}{r - \mu}\right)\frac{\pi(2) - \pi(1)}{x_2^{\nu - \beta_i}}$$  \hspace{1cm} (12)

Recall that the landlord’s entry timing decision depends on the effort supplied by the incumbent tenant, since sales affect landlord profits. Given this relation and an initial value of $x = x_0 < x_2^*$, the incumbent tenant chooses its effort production according to:

$$e_i^* = \arg \max_{e_i} \left\{ F_i(x_0) - \frac{1}{2} x_0 \alpha e_i^2 \right\}$$  \hspace{1cm} (13)

An interior solution is assured when $x_0 < \frac{4\alpha K \pi(1)(r - \mu)^2}{\gamma\pi^2(2)(\pi(1) - \pi(2))}$, which is a parameter restriction that is imposed. We note that this restriction is automatically satisfied for most plausible parameter value constellations given that $x_0 < \frac{2\alpha(r - \mu)^2}{\gamma\pi^2(2)} \frac{\beta_1}{\beta_1 - 1} K$ as previously required. This includes the special case of $\beta_i = 2$ that is considered in detail below.

The first-order condition that follows from (13) is non-linear in $e_i$, so a closed-form solution is elusive in general. However, in the special case of $\beta_i = 2$, which can result from plausible parameter value combinations of $r$, $\mu$ and $\sigma$, an explicit expression for $e_i^*$ can be stated.

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13 This least upper bound follows from $p_1 = 0$ in equation (10), since the entry threshold is increasing in $p_1$. It presumes that $p_1 \geq 0$. A negative $p_1$, should it be feasible in equilibrium, would require an even stronger restriction on $x$. We later demonstrate that $p_1 > 0$ in equilibrium.
Proposition 2: If $\beta = 2$, then: 1) $e_i^*$ exists and is unique, 2) $e_i^* = \frac{4K\alpha (r - \mu)^2 (\gamma - p_i)\pi(1) - \pi^2(2)[\pi(1) - \pi(2)]\gamma^2(\gamma - p_i)x_0}{4K\alpha^2(r - \mu)^3 - 4\alpha[\pi(1) - \pi(2)]^2(r - \mu)p_i(\gamma - p_i)x_0} > 0$ and 3) $\frac{\partial e_i^*}{\partial p_1} < 0$.

Proof: See Appendix 1

The comparative static relation stated in part 3 of the proposition reveals that an increase in the percent-of-sales component, $p_1$, unambiguously decreases effort supplied by the incumbent. Intuition suggests this to be the case, but, as previously shown, an increase in $p_1$ also increases the entry threshold to increase the expected profits of the incumbent. This in turn provides an incentive to increase effort. Since the delayed entry effect is incorporated into the optimal effort choice, the comparative static relation stated in Proposition 2 shows that the negative effort incentive effect associated greater revenue sharing dominates the positive effect associated with delayed entry.

The optimal contract offered to the incumbent tenant is solved by considering the landlord’s valuation problem, as stated in equation (8), subject to participation by the incumbent tenant. That is, the monopolist landlord solves:

$$
\max_{(a_i, p_i)} \left\{ A x_0^\beta + \frac{a_i}{r} + p_i e_i^* \pi(1)x_0 \right\}, \quad \text{s.t. } F_i(x_0) - \frac{1}{2} x_0 \alpha e_i^{2*} \geq 0
$$

Because the participation constraint in (14) will be binding, the optimal contract is obtained by solving the problem as a two-part tariff (see, e.g., Mas-Colell (1995)). First, conditional on $p_1$, the landlord calculates a fixed payment amount, $a_i^*$, that extracts all surplus from the incumbent tenant. Then, given the optimal $a_i^*$, the landlord sets $p_i^*$ to maximize the incumbent tenant’s value function.

The following proposition summarizes the result, along with the crucial property that $p_1$ is strictly positive in the case of $\beta = 2$.

Proposition 3: The landlord chooses $a_i^*$ such that $a_i^* = r \left( B x_0^\beta + \frac{(\gamma - p_i)e_i^* \pi(1)x_0}{r - \mu} - \frac{1}{2} x_0 \alpha e_i^{2*} \right)$, and then solves the following unconstrained problem:

$$
\max_{p_i} \left\{ (A + B)x_0^\beta + \frac{e_i^* \pi(1)x_0}{r - \mu} - \frac{1}{2} x_0 \alpha e_i^{2*} \right\}.
$$
In the case of $\beta_1=2$, $p_1^* > 0$ and a solution for $p_1^*$ exists with $0 < p_1^* < \gamma$

**Proof:** See Appendix 1.

If non-contractable effort supplied by the incumbent tenant was independent of entry by the second tenant, the optimal contract would require a fixed payment component only. But, since the landlord’s entry decision depends endogenously on the effort level of the first tenant, and because entry by the second tenant dilutes profits of the incumbent tenant, costs and benefits associated with effort and entry outcomes are managed by the landlord through the optimal contract. And, crucially, the revenue share parameter, $p_1$, is shown to be positive.

Prior to articulating more specific tradeoffs associated with the share contract, for comparative purposes the following corollary identifies characteristics of the **fixed payment-only contract**.

**Corollary 1:** Consider long-term contracting between a landlord and two tenants, where a fixed payment-only contract is executed (i.e., $p_1$ is restricted to equal zero). The value function of the landlord prior to entry by the follow-on tenant can be expressed as:

$$V_{1,RC}(x) = \frac{a_{1,RC}}{r} + \left( \frac{x}{x_{2,RC}^*} \right)^{\beta_r} \left[ \frac{\gamma^2 \pi^2 (2) x_{2,RC}^* - K}{2 \alpha (r - \mu)^2} \right]$$

where $x_{2,RC}^*$ is the entry threshold. This value is given by:

$$x_{2,RC}^* = \frac{2 \alpha (r - \mu) \beta_1}{\gamma^2 \pi^2 (2) \beta_1 - 1} K$$

The optimal fixed payment required of the incumbent tenant is:

$$a_{1,RC}^* = \frac{r \gamma x_0}{2 \alpha (r - \mu)^2} \left[ \pi(1) - \left( \frac{x_0}{x_{2,RC}^*} \right)^{\beta_1 - 1} \left( \pi(1) - \pi(2) \right) \right]^2$$

where the optimal effort choice of the incumbent is:

$$e_1^* = \frac{\gamma}{\alpha (r - \mu)} \left[ \pi(1) - \left( \frac{x_0}{x_{2,RC}^*} \right)^{\beta_1 - 1} \left( \pi(1) - \pi(2) \right) \right]$$

If $\beta_1=2$, it can be explicitly shown that $e_1^* < e_{1,RC}^*$, $x_2^* > x_{2,RC}^*$, and $V_1 > V_{1,RC}$.

**Proof:** See Appendix 1.

This corollary demonstrates that, relative to the enhanced share contract in which $p_1$ is available as an instrument to affect tenant effort production decisions, the fixed payment-only contract results in greater incumbent tenant effort, a lower entry threshold value and a lower total value to the landlord.
With this corollary, we can now describe tradeoffs embedded in the optimal share contract. A positive percent-of-sales component simultaneously reduces incumbent tenant effort and delays entry of the second tenant. Shared revenues paid to the landlord combined with delayed entry have the effect of increasing landlord profits relative to those obtained from a fixed payment-only contract. Consequently, incumbent tenant effort, while reduced relative to the fixed payment-only contract, is not reduced as much as would be the case if entry were not delayed—an effect which increases total profits available to the landlord.

II.C. First-Best

Non-contractability of effort production and its dependence on the timing of follow-on entry causes distortions relative to first-best. In this section we solve the first-best problem by eliminating the coordination (effort-entry timing) problem that exists between agents. In a subsequent section we take the endogenously determined effort level and entry timing solutions obtained in this section and apply them to the question of how to implement a privately optimal contract to produce fully incentive compatible first-best outcomes.

To solve the first-best problem, suppose that the landlord controls effort production and collects all the profits. Working backwards we first solve for the optimal effort choice associated with the entering tenant, which produces:

$$\tilde{\varepsilon}_2 = \frac{\gamma \pi(2)}{\alpha (r - \mu)}$$

Total post-entry value associated with the second tenant is thus

$$\tilde{\varepsilon}_2^* = \frac{\gamma \pi(2)}{\alpha (r - \mu)}.$$  

Note that a fixed payment contract produces an effort level by the entering tenant that is equal to first-best, as previously shown.

Denote the landlord’s value function prior to entry as $V(x)$. This function satisfies an appropriately specified equation of motion (ODE). Given the boundary condition that $V(x)=0$ when $x=0$, the ODE has the general solution of the form:

$$V(x) = Cx^{\beta_1} + \frac{\gamma \rho(1)x}{r - \mu}$$

where $\beta_1$ is as previously defined and $C$ is a constant to be determined.

The optimal stopping problem is characterized by the critical entry threshold value, $x^*$.
with corresponding stopping time, $t^*$, such that $t^* = \inf\{t \geq 0 : x(t) \geq x^*\}$. By satisfying the appropriate incentive compatibility conditions and solving for $x^*$ and $C$, we find that:

$$x^* = \frac{\gamma \varepsilon_1 (\pi(2) - \pi(1))}{r - \mu} - \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \left( \frac{b_1}{b_1 - 1} \right) K$$  \hspace{1cm} (17)

and

$$C = \left[ \frac{\gamma \varepsilon_1 (\pi(2) - \pi(1))}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \right] \frac{x^{1-b_1}}{b_1}$$  \hspace{1cm} (18)

With this, we can now state the major results of this section.

**Lemma 1:** If $\varepsilon_1 > e_1$, then $x^* > x_2^*$.  

**Proof:** Follows directly from the fact that $p_1 < \gamma$ in equilibrium (see Proposition 3).

The result is intuitive. With first-best, increased effort leads to increased landlord value in proportion to the profit margin, $\gamma$, whereas increased effort in the benchmark contracting case leads to increased landlord value in proportion to $p_1$. Greater marginal returns to effort consequently causes delayed entry.

Effort production associated with the first asset, $\varepsilon_1$, can be found by solving the following problem:

$$\varepsilon_1^* = \arg\max_{\varepsilon_1} \left\{ C x_0^{\beta_1} + \frac{\gamma \varepsilon_1 \pi(1) x_0}{r - \mu} - \frac{1}{2} x_0 \alpha \varepsilon_1^2 \right\}$$  \hspace{1cm} (19)

where $x_0 < x^*$. With this and results from Lemma 1, the following proposition can be stated.

**Proposition 4:** In the case of $\beta_1 = 2$, $\varepsilon_1 > e_1$. Consequently, from Lemma 1, $x^* > x_2^*$.  

**Proof:** See Appendix 1.

The proposition states that the first-best level of effort production associated with the incumbent tenant exceeds that produced in a second-best world with a percent-of-sales contract. This follows because gains to effort in the first-best case are not distorted by an incumbent tenant that does not want to expend greater effort given the decline in profitability realized upon entry by the second tenant. This also explains why $p_1 < \gamma$ with the optimal (second-best) contract: effort
production by the incumbent would not increase to the level required at \( p_1 = \gamma \) to justify such an investment.

II.D. Graphical Relations

In this section the endogenously determined values of \( e_1^* \), \( p_1^* \) and \( e_t^* \) are obtained by numerical procedure. The following parameter values are adopted to generate base case solutions: \( \mu = .03, \sigma = .10, \alpha = .10, \pi(1) = 1.3, \pi(2) = 1.0, r = .06, \gamma = .20, x_0 = .01, K = 3 \). With these values, from equations (6) and (15) we see that \( e_t^* = e_1^* = 66.67 \). The numerically generated solutions for incumbent effort and percent-of-sales are \( e_1^* = 74.45 \) and \( p_1^* = .011 \), respectively. Other analytically determined values in the benchmark model are \( a_2^* = .43 \) (see Proposition 1); \( x_2^* = .032 \) (see equation (9)); and \( a_1^* = .17 \) (see Proposition 3). For the first-best case, we find that \( e_t^* = .83.84 \) and \( x_2^* = .126 \) (see equation (19)). These numerical results immediately reveal that, as previously noted, the incumbent tenant underinvests in effort production and landlord-controlled entry occurs early relative to first-best.

For comparison purposes we also consider the case in which the share percentage is restricted to be zero to result in a fixed payment-only contract. In this case we find that \( x_2^*_{RC} = .031; \ a_1^*_{RC} = .184; \) and \( e_1^*_{RC} = .78.31 \) (see Corollary 1). Thus, relative to the benchmark model, we see that fixed payments are higher, effort is higher, and entry is earlier when \( p_1 \) is restricted to equal zero.

Figure 2 highlights the main insights associated with our model of share contracting as a barrier to entry. In the figure we plot the share percentage, \( p_1 \), versus optimal incumbent effort, \( e_t^* \), and the optimal entry threshold, \( x_2^* \). We also indicate the \( p_1^* \) that results from base case parameter values as well as a “first-best” \( p_1 \) that would induce efficient effort. The figure shows that effort decreases and the entry threshold increases as a function of the share percentage in the range around \( p_1^* \). Thus, a positive optimal share percentage, while decreasing effort to reduce sales, does increase the time to entry to increase total value. The figure also illustrates how this simple share contract is incapable of simultaneously inducing both privately efficient effort production and entry timing.

To see how potential entry affects the optimal contract, in Figure 3 we plot the
equilibrium fixed payment and share percentage contract terms against the entry cost, \( K \). The share percentage is seen to decrease with the entry cost and asymptotically approach zero. This result is intuitive, since, as \( K \) increases, entry is delayed to lessen the need to use shared revenues as a barrier to entry.

Figure 4 displays the expected time to entry against the entry cost for three cases. The first case is the expected time to entry in the benchmark model in which \( a_1 \) and \( p_1 \) are optimally determined, the second is the expected time to entry with a contract that contains a fixed payment component only, and the third is the expected time to entry in the first-best model. The expected entry time in the benchmark model is seen to be longer than the model with fixed payment only, demonstrating that the share contract delays entry. Note that the expected entry time is longest in the first-best model. This is because, in the first-best case, the landlord internalizes all of the net profits from the first asset, \( \gamma S \), without having to contract with an agent on effort. In comparison, the landlord receives marginal cash flows of \( p_1^*S \) in the benchmark model and no marginal flows as a function of sales from a fixed payment only contract. Because \( p_1^* < \gamma_1 \), the first-best case provides the landlord greater incentives to delay entry, since it benefits the most from the monopoly profit received from the incumbent asset. And, because \( p_1^* \) is relatively small in equilibrium (\( p_1^* = .011 \)), delay is only slightly longer in the benchmark model as compared to the fixed payment only model. Note that \( p_1^* \) in this case is in the empirically observed range of 1.0 to 2.0 percent, as discussed in the introductory section of this paper.

Figure 5 compares the landlord’s value function at time zero, i.e., \( x=x_0 \), for the three models. Entry cost has two opposing effects on the landlord's initial value. The first effect results from higher entry cost reducing value. The second effect is more subtle, in that higher entry cost makes future entry less likely. As a result, the incumbent tenant exerts more effort in anticipation of delayed future entry, which increases value. In the first-best case, the first effect dominates the second (there is a significant level of effort to begin with), where landlord value is seen to decrease in entry cost. But, in the benchmark model and the model with fixed payment only, in which effort is relatively low to begin with, marginal returns to effort are higher, and the effort effect dominates the cost effect.

Figure 6 compares the incumbent tenant's effort choice for the three models. For all
models, tenant effort production is increasing in the entry cost, since higher landlord entry cost implies delayed entry, thus increasing returns to tenant effort. As previously shown in the $\beta_i=2$ case, effort production is highest in the first-best model while it is lowest in the benchmark model. This figure provides another perspective on the opposing effects of a share contract. On the one hand, the share contract pushes the follow-on entry time toward first-best; on the other hand, the percent-of-sales component pulls optimal effort production away from first best. As a result, the landlord through the share contract trades off efficient effort production by the tenant and its own optimal entry time.

Figure 7 shows the effect of sales revenue volatility on the optimal contract, in which the share percentage is seen to be decreasing in the volatility. This relation follows because higher volatility causes delayed entry by the second tenant, thereby substituting for revenue sharing as a barrier to entry. As noted earlier, this empirical implication is contrary to predictions from the risk-sharing approach. With that approach, higher volatility increases incentives for the less risk averse landlord to share risk with the more risk averse tenant.

**II.E. Extensions of the Benchmark Model**

In this sub-section we present and discuss three extensions to the benchmark model in order to assess its robustness. The extensions specifically consider: i) the inclusion of a percentage breakpoint provision in the lease contract, ii) continuous effort production by each tenant throughout the contract period, and iii) the risk of contract payment default by the incumbent tenant as a result of low sales outcomes. The first case is of practical importance, as retail leasing contracts often contain breakpoints in which sharing occurs only when revenues exceed a specified level (see, e.g., most recently Williams (2013) on the use of breakpoints in retail percentage rent contracting). The latter two cases address the issue of contract renegotiation incentives of the incumbent tenant as they depend on effort production and low versus high realized sales production.

Our focus in this section is to briefly summarize the main results, as a full analysis of these three cases is rather extensive. Details are available from the authors upon request.

**Percentage Rent Breakpoint Contract**

In the benchmark model we consider a simple linear percentage lease contract that also
includes a fixed lease payment. This contract is often seen in practice, but, given only one free parameter \( p_1 \), the landlord is unable to manage both effort production and entry timing in a double moral hazard setting. In this extension we retain all the assumptions made in the benchmark model, with the exception that the baseline lease contract is augmented to include an additional parameter that is often referred in practice as a breakpoint. The basic idea is that a fixed rent only is paid when sales are below the breakpoint and that both the fixed rent and a percentage-of-sales are paid when sales are above the breakpoint.

The breakpoint thus shifts the weighting between the fixed and percentage-of-sales lease components, where a higher breakpoint implies less weight to the percentage lease component in a present value sense. The model is complicated by the fact that lease payments can, depending on the level of sales, dynamically switch between fixed payments only and fixed payments with a percentage rent component.

Our major findings as compared to the benchmark and first-best models are that: 1) The incumbent tenant supplies greater effort than in the benchmark model. This happens because the relatively high initial breakpoint eliminates percentage rents in the near term, which increases tenant effort; 2) The entry threshold increases relative to the benchmark model. Greater effort production increases incumbent tenant sales to reduce landlord incentives to allow entry to occur; 3) Both the fixed rent and percentage rent components increase relative to the benchmark model case. Because the percentage rent component is initially out-of-the-money at the time of lease execution, percentage rents are a less obvious mechanism to deter entry, which may help explain their common usage as an indirect barrier to entry; and 4) The breakpoint contract is significantly more efficient than the benchmark percentage rent contract when entry costs are low, but there is little difference in the contracts at higher entry cost levels.

**Continuous Effort Production**

In this model, the endogenous effort production decision by each tenant is made continuously and instantaneously, rather than discretely at the time of contract execution. Thus, in lieu of renegotiating contract terms as a result of entry, the incumbent tenant has the opportunity to adjust its effort production over time as a function of potential entry in response to changing market conditions.
Investment in the model is, as a consequence, no longer sunk after time zero. Instead, conditional on share contract terms, the incumbent tenant can modify investment in response to entry or anticipated entry. The main results of this extension are that the entry threshold in the extended model exceeds that in the benchmark model and that the percentage rent component is positive and bounded from above by $\gamma / 2$. The first result follows because the incumbent supplies a greater amount of effort prior to entry in the continuous effort case, which causes the landlord to delay entry. The second result shows that a positive share percentage in the optimal second-best contract is robust to continuous effort provision.

**Incumbent Contract Payment Default**

In the benchmark model, tenant default does not occur regardless of the level of retail sales. We will now relax this assumption by assuming that the incumbent tenant defaults as soon as the net profit from retail operations is insufficient to cover the fixed contract payment (i.e., liquidity payment default risk is posited). This extension thus partially addresses the issue of contract enforcement and renegotiation, where incumbent default substitutes for renegotiation as a result of low sales outcomes.\(^{14}\) Related literature includes Cho and Shilling (2007) and Williams (2013), who consider the effects of possible tenant default on the optimal contract in a shopping center setting.

The main results of this model extension as they compare to the benchmark model are as follows. First, conditional on no-default by the incumbent tenant, the entry threshold is higher than in the benchmark model. A higher entry threshold is realized prior to default because a high percentage rent delays entry as long as the incumbent has not yet defaulted. Second, effort supplied by the incumbent is low relative to the benchmark model. This happens because the returns to effort production, in which costs are incurred up-front with benefits flowing in through time, decrease due to the possibility of default. Third, a share contract offered to the incumbent is optimal, in which share percentages are higher and fixed payments are lower as compared to those in the benchmark model. Incumbent default risk enhances the role of revenue sharing as a mechanism to increase landlord profits and create a barrier to entry. Larger share percentages

\(^{14}\) This raises the issue of why the landlord would allow default to occur in the first place, given the opportunity to initiate renegotiation. One rationale, which is outside the scope of the model, is that low sales outcomes may be the result of poor tenant management, where default is a mechanism used to replace the incumbent with a more productive tenant.
allocate greater profits to the landlord when retail sales are high, since the landlord bears the costs of default when sales are low.

As a final note, it can be shown that, even when no further entry occurs, a share contract offered to the incumbent is optimal when default risk exists. Thus, incumbent tenant contract payment default risk is sufficient to explain the existence of sharing contracts with stand-alone property, implying that the empiricist should separately identify effects of default risk from competitive entry. Our analysis suggests that, when both effects are relevant, they are complementary and therefore enhance the role of revenue sharing in managing the incumbent’s non-contractable effort production decision.

III. Implementing The Privately Efficient Contract

With dual agency the share contract analyzed in the benchmark model is incapable of implementing first-best, as it cannot successfully coordinate both the effort and the entry timing decisions. In fact, it can be shown that no contract exists that is capable of implementing first-best that does not include a contingent payoff made on or after the point of entry by the second tenant.

In this section we will show how to implement a first-best contract that is a simple extension of the benchmark share contract. This augmented contract includes a fixed lump-sum transfer made between the landlord and the incumbent tenant at the time of entry. It is forward-looking in nature, in that the amount of the transfer is agreed upon at the time of contract execution and is funded at the time of entry. The resulting contract is fully incentive compatible. After deriving the privately efficient contract, we will discuss practical challenges to contract implemenation.

To start, we observe that no modifications must be made to the first-best contract offered to the (follow-on) entering tenant, since it is unaffected by the incumbent’s contract or entry timing.

The original share contract offered to the incumbent tenant and the lump-sum transfer amount are instruments used to jointly coordinate efficient effort and entry decisions. Four basic steps are required to implement this contract.

*Step 1:* Modify the benchmark model to incorporate ex post incentive compatibility constraints.

Model modification requires that: i) the entry threshold, $x^*_z$, as seen in equation (9),
changes from an entry cost of $K$ to an entry cost of $K + \chi$, where $\chi$ is a lump-sum transfer made by the landlord to the incumbent tenant at the time of entry; and ii) that $B$, which is a constant in the incumbent tenant’s value function as seen in equation (12), now includes an additional term of $\chi(x_2)$. That is, using the superscript $FB$ to indicate first-best contracting outcomes, we find that:

$$x_2^{FB} = \left[ \frac{p_1 e_i (\pi(2) - \pi(1))}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \right]^{-1} \frac{\beta_1}{\beta_1 - 1} (K + \chi)$$  \hspace{1cm} (20)$$

and

$$B^{FB} = \frac{(\gamma - p_1) e_i (\pi(2) - \pi(1)) (x_2^{FB})^{-\beta_1} + \chi(x_2^{FB})^{-\beta_1}}{r - \mu}$$  \hspace{1cm} (21)$$

Otherwise, the structure of the benchmark model remains as previously specified.

**Step 2**: Choose a percent-of-sales parameter, $p_1^{FB}$, to induce a first-best level of effort production by the incumbent tenant.

The first-best level of effort production, $e_1^*$, follows from the maximization stated in equation (19). Given this quantity, the landlord chooses $p_1^{FB}$ to induce effort production by the incumbent such that the maximization problem stated in equation (13) produces $e_1^{FB} = e_1^*$. This procedure is fully incentive compatible. Because $e_1^*$ exceeds $e_1^*$ from the benchmark model, and because $\frac{\partial e}{\partial p} < 0$, $p_1^{FB}$ is less than $p_1^*$.

**Step 3**: The landlord identifies $\chi$ to induce entry at $x_2^{FB} = x^*$, where $x^*$ is as specified in equation (17).

This procedure is fully ex post incentive compatible, thus guaranteeing first-best entry timing. Algebraic manipulation produces the following lump-sum transfer amount:

$$\chi = K \left[ \frac{2\alpha(r - \mu) e_i^* (\pi(2) - \pi(1)) (p_1^{FB} - \gamma)}{2\alpha(r - \mu)^2 \gamma e_i^* (\pi(2) - \pi(1)) + \gamma^2 \pi^2(2)} \right] > 0$$  \hspace{1cm} (22)$$

**Step 4**: Given $p_1^{FB}$, $B^{FB}$, $e_1^{FB} = e_1^*$, $x_2^{FB} = x^*$ and $\chi$, the landlord chooses the fixed payment, $d_1^{FB}$, according to Proposition 3.

\[15\text{ Note that the incumbent tenant’s value function changes per step 1.}\]
Because the incumbent tenant anticipates the lump-sum transfer made at the time of entry, the fixed payment is higher than it would be without the transfer. Moreover, given that: i) \( p_1^{FB} < p_1^* \), ii) the first-best contract creates greater total value than in the benchmark model, and iii) the incumbent’s participation constraint is binding, the fixed payment is such that \( a_1^{FB} > a_1^* \).

Specifically,

\[
a_1^{FB} = r \left( B^{FB} x_0 \beta_0 + \frac{(\gamma - p_1^{FB}) e_1^* \pi(1) x_0}{r - \mu} - \frac{1}{2} x_0 \alpha e_1^{*2} \right)
\]

(23)

Keeping in mind that efficiency is achieved by taking the production market structure as given, there are relevant policy implications associated with implementing the first-best contract. To start, it is clear that the landlord delays follow-on entry in order to continue to harvest monopoly rents from the incumbent tenant. However, as the benchmark model plainly shows, effort supplied by the incumbent tenant is well below that preferred by the landlord, which decreases contract flows to the landlord and therefore causes entry to occur earlier than it otherwise would. Implementation of the first-best contract addresses the effort underinvestment problem, resulting in delayed entry.

With this program, we can now state the major result of the section. This result generalizes a previous result already noted in the analysis of Figure 2, which is that efficient effort production is achieved when \( p_1 = 0 \).

**Proposition 5:** For any given \( e_1^* \) and \( x^* \), \( p_1^{FB} = 0 \) and

\[
\chi = K \left[ \frac{\gamma e_1^2}{r^2} + \frac{\gamma^2 e_1^2 (r-\mu)^2}{2 \alpha (r-\mu)^2} \right]^{-1} - \left[ \frac{\gamma^2 e_1^2}{2 \alpha (r-\mu)^2} \right]^{-1} > 0.
\]

**Proof:** See Appendix A

The result is intuitive and quite general. As previously shown, a fixed payment-only contract causes an efficient level of effort production, implying \( p_1^{FB} = 0 \). The lump sum payment, \( \chi \), is adjusted to induce entry at the appropriate point in time.

The privately efficient contract offered to the incumbent tenant is simple, with only fixed lease payments and a lump sum paid by the landlord to the tenant at the time of follow-on entry. Yet we are not aware of this contract being implemented in practice, where instead the more
“complex” percentage rent contracts are often implemented with stand-alone retail and franchise tenants.

Why? We can offer two rationales. First, it may well be that the incumbent tenant is worried about a breach by the landlord at the time of follow-on entry. After all, the landlord will have incentives to hold up the incumbent tenant, and contract enforceability is costly and may be imperfect. Second, writing a contract with such an explicit payoff by the landlord to an incumbent tenant at the time of entry may catch the attention of regulators, interpreted as a restraint of trade. A percentage rent contract, in contrast, is less obvious in terms of creating a barrier to entry.

IV. Implementing a Socially Efficient Contract

Entry creates consumer surplus that is ignored by the landlord and tenant in their optimal contracting problem. In this section we recognize and incorporate consumer surplus into the model and extract some policy implications. We further suggest a contracting scheme that achieves a fully incentive compatible welfare maximizing outcome.

Throughout the paper we have focused on the strategic interactions of a landlord and tenants in a well-defined market, and have paid less attention to the details underlying consumer behavior. In particular, retail consumer demand has been defined and summarized by the sales parameters $\pi(1)$ and $\pi(2)$. To keep the welfare analysis as simple as possible, but to also focus attention on consumer surplus associated with entry, we will assume consumer surplus exists only on or after the point of entry by the second tenant, and only for those customers that previously bought and continue to buy from the incumbent tenant. These consumers consequently enjoy the benefit of price declines associated with competitive entry.

We will bound consumer surplus between zero and the decline in gross sales experienced by the incumbent tenant at the point of entry by the second tenant. That is, we will assume that consumer surplus is zero prior to entry and that it equals $e(x_2(\pi(1) - \pi(2)))$, $0 \leq \xi \leq 1$, on and after the point of entry by the second tenant.

Welfare analysis proceeds as follows. Because welfare gains are experienced only by incumbent tenant customers, the welfare maximizing amount of effort produced by the second tenant upon entry, for reasons articulated previously, equals effort production provided in the
benchmark and first-best cases. Similar arguments result in a fixed rental contract being the socially optimal contract for the entering tenant.

As a next step, prior to entry, it is straightforward to modify the landlord’s value function to form the social value function. The resulting socially optimal entry threshold, $x^*_w$ is:

$$x^*_w = \left[ \frac{e^{w^*}\left[ (\gamma - \xi)(\pi(2) - \pi(1)) \right]}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \right]^{-1} \frac{\beta_1}{\beta_1 - 1} K$$

where $e^{w^*}$ indicates socially optimal effort production. Note also that $C$ as expressed in equation (18) is similarly modified to incorporate the valuation effects of consumer surplus.

A comparison of the socially optimal entry threshold, $x^*_w$, with the first-best entry threshold, $x^*$, as seen in equation (17), reveals that $x^*_w < x^*$ when consumer surplus is positive and socially optimal effort production exceeds first-best effort production. Socially inefficient delay occurs with privately efficient contracting because the monopolist landlord does not internalize consumer surplus that is realized upon entry.

A comparison of $x^*_w$ with the entry threshold from the private contracting benchmark model, $x^*_2$, as seen in equation (9), reveals that the socially optimal entry threshold may be higher or lower than the threshold that occurs in a private contract setting. The following proposition states the result:

**Lemma 2:** Given that $\beta_1 = 2$, there exists a unique critical value, $\xi^* = \left( \frac{\gamma e^{w^*} - \pi_1 e^*}{e^{w^*}} \right)$, $0 < \xi^* < 1$, such that $x^*_w < (>) x^*_2$ when $\xi^* > (>) \xi^*$.

**Proof:** See Appendix A

The socially optimal entry threshold equals first-best when consumer surplus is zero at the point of entry. In this case, as was previously shown in proposition 4, the first-best entry threshold exceeds the entry threshold in the benchmark case. More generally, the socially optimal entry threshold $x^*_w$ exceeds $x^*_2$ when consumer surplus is “small”. On the other hand, when consumer surplus is sufficiently “large”, the socially optimal entry threshold is less than the entry threshold in the benchmark case.

Socially optimal effort production follows the program stated in equation (19), given
appropriate modifications to the entry threshold value and value function. Because $C^W$ in the socially optimal value function (see equation (18), as modified to account for consumer surplus) is increasing in $\xi$, socially optimal effort production exceeds first-best, which in turn exceeds effort production realized in the benchmark case. The reason too little effort is produced in the first-best case is that the landlord does not internalize permanent benefits associated with effort production that accrue to consumers in the form of surplus when entry occurs. Underinvestment in effort in the benchmark case is even more pronounced for reasons cited previously.

Thus, summarizing, it is clear that, from a social planning (SP) perspective, effort production is too low in a world with private contracting, and entry timing will, in general, occur either too early or too late depending whether consumer surplus is “small” or “large”. If the privately optimal contract is implementable, entry in that case is always too late relative to the social optimum. Consequently, it is desirable to consider whether an implementable scheme can be designed by SP that induces socially optimal effort production and entry timing outcomes, while also satisfying incentive compatibility constraints of private market participants.

To pursue this question, we will start by observing that SP in the form of local government often uses its power to regulate, tax and subsidize in order to provide incentives to private market participants. But, at the same time, there could be credibility issues associated with SP delivering on promises through time and potential problems with private market participants engaging in side agreements, if it is in their interests, where the side agreements effectively undermine local government objectives.

With these caveats in mind, consider the following scheme that induces socially optimal effort production and entry outcomes As a first step, because SP has within its powers to impose a subsidy or tax on the landlord at the time of entry, *ex post* incentive compatibility constraints associated with entry can be introduced to allow for a lump sum transfer to occur at the time of entry (which is similar to modifications required when implementing the first-best contract).

In contrast to the first-best contract implementation case, a percentage rent paid by the tenant to the landlord and a lump sum transfer at the time of entry are not sufficient to induce both socially optimal effort production and entry timing. This follows because, with SP’s desire to account for consumer surplus in the optimal contracting problem, interests of three agents must be satisfied, and those two instruments alone cannot simultaneously satisfy tenant and landlord incentive compatibility constraints.
In addition to percentage rent and a lump sum transfer paid at the point of entry, an appropriate percentage-of-sales subsidy paid by SP directly to the landlord is sufficient to induce socially optimal effort production and entry timing. With this scheme, implementation can be simplified by restricting the standard tenant-landlord percentage rent payment to equal zero so that the incumbent tenant pays only a fixed rent.

The following proposition states the major result of this section, in which a unique set of transfer payments made between SP and the landlord are capable of inducing socially optimal effort production and entry timing.

**Proposition 6:** Assume a contracting scheme in which local government mandates a fixed rental payment made by the tenant to the landlord. Local government can then implement a socially optimal contract, in which consumer surplus is summarized by the parameter $\xi$, $0 \leq \xi \leq 1$, with two kinds of transfers made between local government and the landlord: i) a positive flow subsidy from local government to the landlord that is a percentage of total sales, and ii) a lump-sum transfer that is made at the time of entry. These two transfers, denoted as $p^W$ and $\chi^W$, respectively, are such that:

\[
p^W = \frac{\gamma^2 \pi^2 (2)}{2 \epsilon^W \alpha (r - \mu) \left( \pi (2) - \pi (1) \right) \left( \gamma (1 - \beta_i) - \xi \right)} > 0
\]

and

\[
\chi^W = K \left[ \frac{\gamma^2 \pi^2 (2)}{2 \alpha (r - \mu)^2} \left( \frac{\gamma^2 \pi^2 (2)}{2 \alpha (r - \mu)^2} \right) ^{-1} \left( \frac{\gamma^2 \pi^2 (2)}{2 \alpha (r - \mu)^2} \right) ^{-1} \right]
\]

The lump-sum subsidy is positive (negative) when $p^W > (<) \gamma - \xi$.

**Proof:** See Appendix A

Note that the fixed rent charged to the incumbent tenant is

\[
a^*_W = r \left( \frac{\gamma^2 \pi^2 (2)}{2 \alpha (r - \mu)^2} \left( \frac{\gamma^2 \pi^2 (2)}{2 \alpha (r - \mu)^2} \right) ^{-1} \left( \frac{\gamma^2 \pi^2 (2)}{2 \alpha (r - \mu)^2} \right) ^{-1} \right)
\]

If needed, this rent, which can be split and combined with taxes on the relevant consumer surplus, can be used offset SP subsidies and a possible lump-sum transfer by the landlord to SP. This rent is sufficient to satisfy participation constraints of SP and the landlord. That is, participation constraints of landlord and SP are satisfied (if needed) through, first, distribution of residual tenant profits and, second, through taxes on consumer surplus.

This is the only two-dimensional contractual instrument combination that is capable of producing a socially optimal result. That is, for a non-zero percentage rental payment made
between the incumbent tenant and landlord, a lump-sum subsidy equal to zero cannot induce first-best; nor can $p_{W}=0$. With a fixed rent-only contract offered to the incumbent tenant, entry must occur at the appropriate time for the incumbent to produce the required amount of effort. A lump-sum payoff alone is unable to induce the landlord to enter at the socially optimal time, since it alone cannot satisfy incentive compatibility. However, with the addition of a positive flow transfer from SP to the landlord, incentive compatibility can be achieved. A positive flow transfer might seem unintuitive, since it suggests delay in entry when entry occurs sooner in the welfare case as compared to the first-best case. But the lump-sum transfer combines with the flow subsidy to achieve the social optimum that is also incentive compatible.

As stated earlier, implementation of this kind of scheme is not trivial. But these conditions match closely with actual methods employed by local government to address local development concerns, where exactions, subsidies (such as tax incremental financing on the flow side), contractual fine-tuning and development restrictions (or relaxation in the case of reducing landlord entry costs) are simultaneously negotiated and imposed by local government to satisfy socially acceptable (if not optimal) objectives.

V. Concluding Comments

We have studied incentive contracting in a stand-alone retail leasing context in which competitive entry can occur. Tenants make non-contractable specific investments that affect retail sales, where entry timing is endogenously determined by the landlord. A percentage rent contract is considered and is implemented in equilibrium, where revenue sharing imposes a cost on the landlord when follow-on entry occurs. Effort production and entry deterrence through revenue sharing are jointly determined by the optimal contract, causing the incumbent tenant to underinvest in effort production and the landlord to dynamically overinvest in total market capacity. Percent-of-sales lease contracting is shown to be robust to extensions that include a percentage breakpoint contract, continuous effort production and possible payment default by the incumbent tenant.

Privately efficient contract design is examined. We show that the privately efficient contract can be implemented in two parts: a fixed payment component that is combined with a lump-sum transfer paid by the landlord to the incumbent tenant at the time of follow-on entry. This contract is not often observed in practice, likely due to incumbent tenant concerns over
landlord breach with respect to payment of the lump sum amount upon entry and because the lump-sum transfer payment is more obviously associated with conspiring to limit competition.

We have also considered the socially optimal contract by accounting for consumers’ surplus. The socially optimal contract is relatively complex, involving a flow subsidy paid by government to the landlord and lump-sum payment that, depending on the magnitude of consumer surplus generated upon entry, is either a subsidy or a tax imposed on the landlord at the time of entry. Interestingly, these contracting features match closely with actual methods employed by local governments to address local real estate and economic development concerns. For example, exactions, subsidies (such as tax incremental financing on the flow side), contractual fine-tuning and development restrictions (or relaxation in the case of reducing entry costs) are simultaneously negotiated and imposed by local governments to arrive at socially acceptable (if not optimal) outcomes.

Empirical implications that follow from our model include the existence of percentage rent contracts with a cluster of retail tenants under common management that have no obvious complementarities, relatively low-powered share percentages in the percentage rent contract, sequential entry in which incumbent tenants are more likely to pay percentage rents than follow-on tenants, a reduction in percentage rents as a function of local land use restrictions that make entry into a local trade area more difficult, a concomitant increase in tenant advertising and other brand-enhancing investments in high barrier markets, and decreasing contract share percentages as a function of the volatility of retail sales. Many of these predictions are unique to our model, requiring only lease contracting and market trade area data for empirical model specification and testing.
Figures

Figure 1: Time Line of the Benchmark Model

Figure 2: Optimal Incumbent Effort Production and Follow-on Entry Threshold as a Function of the Share Percentage
Figure 3: Optimal Contract Variables as a Function of Entry Cost

![Graph showing Fixed payment and Share percentage as functions of Entry cost.]

Figure 4: Expected Follow-On Entry Time as a Function of Entry Cost

![Graph showing Expected follow-on entry time as a function of Entry cost.]

Figure 5: Landlord Value as a Function of Entry Cost

![Graph showing Landlord value as a function of Entry cost.]

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Figure 6: Incumbent Tenant Effort Production as a Function of Entry Cost

Figure 7: Optimal Contract Variables as a Function of Sales Revenue Volatility


Williams, Joseph T., 2013, Percentage rents with agency, forthcoming, *Real Estate Economics*

Appendix A

Proofs of Lemmas, Propositions, and Corollaries

Prior to stating the proofs, it will be useful to restate parameter restrictions that were noted in the text of the paper. First, unless otherwise stated, all parameters are assumed to be positive. Other particularly relevant parameter restrictions are:

1) $0 < \gamma < 1$
2) $r > \mu$
3) $\pi(1) > \pi(2)$
4) $2\pi(2) > \pi(1)$
5) $x_0 < \frac{2\alpha(r - \mu)^2}{\gamma^2 \pi^2(2)} \frac{\beta_i}{\beta_i - 1} K$

Parameter restriction 2 ensures convergence in valuation solutions, parameter restriction 4 ensures that, for some finite value of $x$, entry by the second follow-on tenant will occur, and parameter restriction 5 ensures that $x$ at $t=0$ is sufficiently low so that follow-on entry does not occur immediately. Given these restrictions, the following preliminary result can be stated:

**Lemma A1**: If $\beta_i=2$, $x_0 < \frac{4\alpha K \pi(1)(r - \mu)^2}{\gamma^2 \pi^2(2)(\pi(1) - \pi(2))}$.

**Proof**: Given previously stated parameter restrictions, all that is required is to show that

$$\frac{4\alpha K \pi(1)(r - \mu)^2}{\gamma^2 \pi^2(2)(\pi(1) - \pi(2))} > \frac{2\alpha(r - \mu)^2}{\gamma^2 \pi^2(2)} \frac{\beta_i}{\beta_i - 1} K$$

where the right-hand side of the inequality is seen in restriction 5. The result follows immediately when $\beta_i=2$.

This lemma will be used below. When $\beta_i \neq 2$, for reasonable parameter values it is also true that $x_0 < \frac{4\alpha K \pi(1)(r - \mu)^2}{\gamma^2 \pi^2(2)(\pi(1) - \pi(2))}$ when restriction 5 is applied.
Proof of Proposition 1

Assume the entrant is offered a share contract, \( \{ a_2, p_1, p_2 \} \). The landlord’s problem is:

\[
\begin{align*}
\max_{\{ a_2, p_2 \}} \quad & \frac{p_2 e^*_2 x_2 \pi(2)}{r - \mu} + \frac{a_2}{r}, \\
\text{s.t.} \quad & \frac{(\gamma - p_2) x_2 \pi(2) e^*_2}{r - \mu} - \frac{a_2}{r} - \frac{1}{2} \alpha_2 x_2 e^*_2 r^2 \geq 0.
\end{align*}
\]

Substitute \( e^*_2 \) from equation (6) into the above problem, which can now be written as

\[
\begin{align*}
\max_{\{ a_2, p_2 \}} \quad & \frac{p_2 (\gamma_2 - p_2) x_2 \pi^2(2)}{\alpha_2 (r - \mu)^2} + \frac{a_2}{r} + \lambda \left\{ \frac{(\gamma_2 - p_2)^2 x_2 \pi^2(2)}{2 \alpha_2 (r - \mu)^2} - \frac{a_2}{r} \right\}, \\
\text{s.t.} \quad & -\frac{2 (\gamma_2 - p_2)^2 x_2 \pi^2(2)}{2 \alpha_2 (r - \mu)^2} \geq 0.
\end{align*}
\]

The first order conditions are given as

\[
\begin{align*}
\frac{\partial L}{\partial p_2} &= \frac{(\gamma_2 - p_2)^2 x_2 \pi^2(2)}{\alpha_2 (r - \mu)^2} + \lambda \left\{ -\frac{2 (\gamma_2 - p_2) x_2 \pi^2(2)}{2 \alpha_2 (r - \mu)^2} \right\} = 0; \\
\frac{\partial L}{\partial \lambda} &= \frac{(\gamma_2 - p_2)^2 x_2 \pi^2(2)}{2 \alpha_2 (r - \mu)^2} + \frac{a_2}{r} = 0; \\
\frac{\partial L}{\partial a_2} &= -\frac{1}{r - \lambda} = 0.
\end{align*}
\]

Solving these equations produces:

\[
\lambda = 1; \quad p_2 = 0; \quad a_2 = \frac{r \gamma_2^2 - p_2 x_2 \pi^2(2)}{2 \alpha_2 (r - \mu)^2}.
\]

Proof of Proposition 2

With \( \beta_1 = 2 \), from equations (10) and (14) we see that

\[
x^*_2 = 2 \left[ \frac{\gamma^2 \pi^2(2)}{2 \alpha (r - \mu)^2} + \frac{p_1 e^*_1 \pi(2)}{r - \mu} \right] K
\]

and

\[
B = \frac{\gamma^2_2 (\gamma - p_1) e^*_1 \pi^2(2) \pi(2)}{4K \alpha (r - \mu)^2} + \frac{p_1 (\gamma - p_1) e^*_1 \pi^2(2) \pi(2)}{2K (r - \mu)^3}.
\]

These quantities can be used to help determine optimal effort production of the incumbent, as follows:

\[
e^*_1 = \arg \max_{e^*_1} \left[ \frac{p_1 (\gamma - p_1) [\pi(2) - \pi(1)] x_0^b}{2K (r - \mu)^3} - \frac{1}{2} \alpha \gamma_0 \right] e^*_1 + \left[ \frac{\gamma^2_2 (\gamma - p_1) e^*_1 \pi^2(2) \pi(2)}{4K \alpha (r - \mu)^3} + \frac{p_1 (\gamma - p_1) \pi(1) x_0}{r - \mu} \right] e^*_1 - \frac{a_1}{r}.
\]

The first order condition produces

\[
e^*_1 = \frac{4K \alpha (r - \mu)^2 (\gamma - p_1) \pi(1) - \gamma^2_2 (\gamma - p_1) e^*_1 \pi^2(2) \pi(1) - \pi(2)) x_0}{4K \alpha^2 (r - \mu)^3 - 4\alpha (r - \mu) p_1 (\gamma - p_1) [\pi(1) - \pi(2)]^2 x_0}, \text{ while uniqueness requires}
\]

\[
e^*_1 = \frac{4K \alpha (r - \mu)^2 (\gamma - p_1) \pi(1) - \gamma^2_2 (\gamma - p_1) e^*_1 \pi^2(2) \pi(1) - \pi(2)) x_0}{4K \alpha^2 (r - \mu)^3 - 4\alpha (r - \mu) p_1 (\gamma - p_1) [\pi(1) - \pi(2)]^2 x_0}.
\]
that \( aK(r - \mu)^2 > p_i(\gamma - p_i)(\pi(1) - \pi(2))^2 \). Now recall that restriction 4 and 5, which in
the \( \beta_i = 2 \) case are: i) \( \pi(1) < 2\pi(2) \) and ii) \( x_0 < \frac{4\alpha(r - \mu)^2 K}{\gamma^2 \pi^2(2)} \). These restrictions imply that the
second-order condition holds. Thus \( e^*_1 \) exists and is unique, with the exact solution that follows
from the first-order condition. Lemma A1 ensures that \( e^*_1 \) is positive. To study the comparative
statics of \( e^*_1 \) with respect to \( p_1 \), rewrite equation the first-order condition as:
\[
e^*_1 = \frac{4K\alpha(r - \mu)^2 \pi(1) - \gamma^2 \pi^2(2)(\pi(1) - \pi(2))x_0}{4K\alpha^2(r - \mu)^3 / (\gamma - p_1) - 4\alpha(\pi(1) - \pi(2))^2 (r - \mu) p_1 x_0}.
\]
With this, it follows that
\[
\frac{\partial e^*_1}{\partial p_1} = \left[ \pi^2(2)\gamma^2(\pi(2) - \pi(1))x_0 + 4\alpha K(r - \mu)^2 \pi(1) \right] \left[ (r - \mu)(\pi(1) - \pi(2))^2 x_0 - \alpha K(r - \mu)^3 (\gamma - p_1)^2 \right] \quad \frac{4\alpha [\alpha(r - \mu)^3 K - (r - \mu) p_1 (\gamma - p_1)(\pi(1) - \pi(2))^2 x_0]^2}{\gamma^2 \pi^2(2)}.
\]
Examination reduces the comparative statics relation to identifying
\[
\text{sign} \left( \frac{\partial e_1}{\partial p_1} \right) = -\text{sign} \left[ 4\alpha K(r - \mu)^2 - 4[\pi(1) - \pi(2)]^2 x_0(\gamma - p_1)^2 \right].
\]
Notice that we have \( x_0 < \frac{4\alpha(r - \mu)^2 K}{\gamma^2 \pi^2(2)} < \frac{4\alpha(r - \mu)^2 K}{\gamma^2(\pi(1) - \pi(2))^2} < \frac{4\alpha(r - \mu)^2 K}{(\gamma - p_1)^2(\pi(1) - \pi(2))^2} \), where the
first inequality follows directly from restriction 5. The second inequality holds because
\( \pi(1) < 2\pi(2) \), implying that \( (\pi(1) - \pi(2))^2 < \pi^2(2) \). The third inequality holds because
\( (\gamma - p_1)^2 < \gamma^2 \). It then follows that \( 4\alpha K(r - \mu)^2 - 4[\pi(1) - \pi(2)]^2 x_0(\gamma - p_1)^2 > 0 \), which implies
that \( \frac{\partial e_1^*}{\partial p_1} < 0 \).

**Proof of Proposition 3**

Determination of \( a^*_1 \) and \( p^*_1 \) follow directly from the paragraph description just prior to the
the statement of the proposition. For the case of \( \beta_i = 2 \), the landlord's problem is to choose \( p_1 \) to
maximize its value at time 0, as follows:
\[
\max_{p_1} \left\{ \frac{\gamma \pi(1) e_1^* x_0}{r - \mu} + \left( x_0 \frac{\pi^2(2) x_2^*}{2\alpha(r - \mu)^2} + \frac{\pi e_1^* (\pi(2) - \pi(1)) x_2^*}{r - \mu} - K \right) - \frac{1}{2} \alpha e_1^* \right\}, \text{ where}
\]
\[ e_1^* = \frac{4K\alpha(r - \mu)^2(\gamma - p_1)\pi(1) - \pi^2(2)[\pi(1) - \pi(2)]}{4K\alpha^2(r - \mu)^3 - 4\alpha[\pi(1) - \pi(2)]^2(r - \mu)p_1(\gamma - p_1)x_0} \] and

\[ x_2^* = \left[ \frac{\pi^2(2)\gamma^2}{2\alpha(r - \mu)^2} + \frac{p_1e_1^*[\pi(2) - \pi(1)]}{r - \mu} \right]^{-1} 2K. \] The associated first order condition is:

\[ \frac{(\gamma - p_1)e_1^*}{(r - \mu)} \left( \pi(1) - \pi(2) \right)^2 \]

\[ \frac{\gamma\pi(1)}{r - \mu} - \alpha e_1^* + \frac{x_0}{2K} \left[ \frac{\pi^2(2)\gamma^2(\pi(2) - \pi(1))}{2\alpha(r - \mu)^3} + \frac{p_1e_1^*(2\gamma - p_1)(\pi(1) - \pi(2))^2}{2(r - \mu)^2} \right] \]

Now define the following:

\[ g(p_1) \equiv \frac{(\gamma - p_1)e_1^*}{(r - \mu)} \left( \pi(1) - \pi(2) \right)^2 + \]

\[ \frac{\gamma\pi(1)}{r - \mu} - \alpha e_1^* + \frac{x_0}{2K} \left[ \frac{\pi^2(2)\gamma^2(\pi(2) - \pi(1))}{2\alpha(r - \mu)^3} + \frac{p_1e_1^*(2\gamma - p_1)(\pi(1) - \pi(2))^2}{2(r - \mu)^2} \right] \]

We then have the following properties of the function \( g(p_1) \):

\[ g(0) = \frac{(\gamma - p_1)e_1^*}{(r - \mu)} \left( \pi(1) - \pi(2) \right)^2 > 0 \] and

\[ g(\gamma) = \frac{\gamma}{4K\alpha(r - \mu)^3} [4K\alpha\pi(1)(r - \mu)^2 - x_0\pi^2(2)\gamma^2(\pi(1) - \pi(2))] \frac{\partial e_1^*}{\partial p_1} \]

Given the previous restrictions and lemmas A1, the term in the square bracket is positive, and according to proposition 2, \( \frac{\partial e_1^*}{\partial p_1} < 0 \). Therefore, \( g(\gamma) < 0 \). The above two properties of \( g(p_1) \), combined with the continuity of this function, imply that on solution exists in the interval \( (0, \gamma) \) that satisfies the first-order condition. We also observe that \( g(p_1) < 0 \) for any \( p_1 \leq 0 \). With these relations we conclude that \( p_1^* \) is positive and that a solution for the optimal percentage \( p_1^* \) exists with \( 0 < p_1^* < \gamma \). ■
Proof of Corollary 1

We solve the model for the case when the percent-of-sales component is restricted to equal zero. Working backwards in time, it follows that the optimal effort supplied by the second tenant is equal to that shown in equation (6). It then follows that the fixed payment made by the second tenant is

\[ a_{2, RC}^* = \frac{\gamma^2 \pi^2 (2) x_2^*}{2 \alpha (r - \mu)^2} \]

since, as shown in Proposition 1, a fixed payment contract is optimal for the entrant in the more general contracting case. We can now determine optimal effort production of the incumbent and characterize the optimal contract between the landlord and the incumbent tenant. Determination of the landlord value function and entry threshold is straightforward. With these quantities, the value function of the incumbent prior to the entry of the second tenant is given as:

\[
F_{1, RC}(x) = \frac{\gamma e_{1,RC} \pi(1)x}{r - \mu} - \left( \frac{x}{x_{2,RC}^*} \right)^{\beta} \frac{\gamma e_{1,RC} \pi(1) - \pi(2)x_{2,RC}^*}{r - \mu} - \frac{a_{1,RC}}{r}.
\]

Optimal effort production of the incumbent is solved by the following optimization problem:

\[
e_{1, RC}^* = \arg \max_{e_1} \left\{ F_{1, RC}(x) - \frac{1}{2} \alpha x e_{1,RC}^2 \right\}.
\]

Solving the first order condition, we find that:

\[
e_{1, RC}^* = \frac{\gamma}{\alpha (r - \mu)} \left[ \pi(1) - \left( \frac{x_0}{x_{2,RC}^*} \right)^{\beta-1} (\pi(1) - \pi(2)) \right].
\]

The participation constraint of the incumbent implies that the optimal \( a_{1, RC}^* \) should be such that \( F_{1, RC}(x_0) = 0 \). The resulting \( a_{1, RC}^* \) is given by

\[
a_{1, RC}^* = \frac{r \gamma x_0}{2 \alpha (r - \mu)^2} \left[ \pi(1) - \left( \frac{x_0}{x_{2,RC}^*} \right)^{\beta-1} (\pi(1) - \pi(2)) \right]^2.
\]

In the case of \( \beta = 2 \), according to Proposition 2 we know that optimal effort production is decreasing in the share percentage \( p_1 \) and that the restricted contract is one in which \( p_1 \) is required to equal zero. According to Proposition 3, we know that the optimal share percentage is positive when \( p_1 \) is unrestricted. Combining Propositions 2 and 3, we have that \( e_{1, RC}^* < e_{1, RC}^* \). \( x_2^* > x_{2, RC}^* \) follows directly given that \( p_1^* > 0 \). The fact that the optimal share percentage is non-zero implies that \( V_1 > V_{1, RC} \). ■
**Proof of Proposition 4**

When $\beta_i = 2$ in the first-best problem, it follows that

$$x^* = \left[\frac{\gamma \pi(2) - \pi(1)}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2}\right]^{-1} 2K$$

and

$$C = \frac{1}{4K} \left[\frac{\gamma \pi(2) - \pi(1)}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2}\right]^2.$$  

The first order condition for $\varepsilon_1$ is

$$\frac{1}{2K} \left[\frac{\gamma \pi(2) - \pi(1)}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2}\right] \gamma \pi(2) - \pi(1) x_0 + \frac{\gamma \pi(1)}{r - \mu} - \alpha \varepsilon_1 = 0,$$

which implies that

$$\varepsilon_1^* = \frac{\gamma^3 \pi^3(2)(\pi(2) - \pi(1)) x_0 + 4K \gamma \pi(1)(r - \mu)^2}{4K \alpha(2)(r - \mu)^3 - 2\alpha^2 \gamma^2 \pi(2) - \pi(1))^2(r - \mu) x_0}.$$  

We can now compare $\varepsilon_1^*$ with $\varepsilon_1^*$:

$$\varepsilon_1^* = \frac{4K \gamma \pi(2) - \pi(1) x_0 + 4K \gamma \pi(1)(r - \mu)^2}{4K \alpha(2)(r - \mu)^3 - 2\alpha^2 \gamma^2 \pi(2) - \pi(1))^2(r - \mu) x_0}$$

The first inequality holds because $p_1^* > 0$; the second inequality holds because $p_1^* < \gamma^1$, which implies that $p_1^*(\gamma - p_1^*) \leq \gamma^1 / 4$; and the third inequality holds because

$$\alpha[\pi(1) - \pi(2)]^2(r - \mu) \gamma^2 x_0 < 2\alpha[\pi(1) - \pi(2)]^2(r - \mu) \gamma^2 x_0.$$  

**Proof of Proposition 5**

We need to show that under $p_1^{FB} = 0$, the first order condition for the incumbent's effort choice is the same as in the first best case, and together with $\chi$, $p_1^{FB} = 0$ also induces the first-best entry threshold. After some algebraic manipulation, the first-order condition for the first-best case can be written as

$$\frac{\gamma(\pi(2) - \pi(1))}{r - \mu} x_0^\alpha x^{\alpha - \beta} + \frac{\gamma \pi(1) x_0}{r - \mu} - \alpha \varepsilon_1 = 0,$$  

where $x^*$ is a function of $\varepsilon_1$ and $\varepsilon_1^*$ satisfies this first-order condition. Assuming that $p_1^{FB} = 0$, from equation (18) we
see that \( x_2^{FB^*} = \left[ \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \right]^{-1} \frac{\beta_1}{\beta_1 - 1} (K + \chi) \), and from equation (19) \( B^{FB} = \frac{\gamma e_1'(\pi(2) - \pi(1)) (x_2^{FB^*})^{\beta_1}}{r - \mu} + \chi (x_2^{FB^*})^{\beta_1} \). As a result, the first order condition for \( e_1 \) in this case can be written explicitly and simplifies to \( \frac{\gamma'(\pi(2) - \pi(1))}{r - \mu} x_0^\beta (\chi^{FB*})^{\beta_1} + \frac{\gamma \pi(1)x_0}{r - \mu} - \alpha x_0 e_1 = 0 \). Observe that when \( e_1^* = e_1^* \), in order to satisfy the above first order relation, the relation is satisfied as long as \( x_2^{FB^*} = x^* \), which requires that

\[
\chi = K \left[ \frac{p_0(\pi(2) - \pi(1))}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \right]^{-1} \left[ \frac{\gamma \pi(2)}{2\alpha(r - \mu)^2} \right]^{-1}
\]

Inspection reveals that this quantity is positive. 

**Proof of Proposition 6**

Recall that consumer surplus equals \( e_1 x_0^\xi (\pi(1) - \pi(2)) \), \( 0 \leq \xi \leq 1 \), on and after the point of entry by the second tenant, and is zero otherwise. With this, it was previously shown that the welfare maximizing entry threshold, \( x_W^* \), is as stated in equation (24). The first-order condition required to achieve socially optimal effort production can, after some algebraic manipulation, be stated as \( \frac{(\gamma - \xi)(\pi(2) - \pi(1))x_W^*}{r - \mu} \left( \frac{x_0}{x_W^*} \right)^{\beta_1} + \frac{\gamma \pi(1)x_0}{r - \mu} - \alpha x_0 e_1^W = 0 \), where \( x_W^* \) also depends on effort.

Tenant effort production depends on both the lease contract terms and entry timing. Given the restriction that the tenant’s percentage lease payment equal zero, IC effort production must be achieved through determination of the entry threshold. Moreover, it must be done in a way such that the landlord has proper incentives to enter at the socially optimal entry threshold. Let \( \chi^W \) denote a lump-sum paid by SP to the landlord at the time of entry, where \( \chi^W \) can be positive or negative. The resulting IC entry threshold for the landlord is

\[
x_2^W = \left[ \frac{e_1^W p^W (\pi(2) - \pi(1))}{r - \mu} + \frac{\gamma^2 \pi^2(2)}{2\alpha(r - \mu)^2} \right]^{-1} \left( \frac{\beta_1}{\beta_1 - 1} (K - \chi^W) \right), \text{ where } p^W \text{ is a flow subsidy paid by SP to the landlord as a percentage of total sales. With this entry threshold, and the restriction}
\]
that a fixed rental contract is imposed on the incumbent tenant, the first-order condition for tenant effort production can be stated as

\[
\frac{\gamma(\pi(2) - \pi(1))x_2^w}{r - \mu} \left( \frac{x_0}{x_2^w} \right)^\beta \left[ 1 - e_1^w(1 - \beta_1) \left( \frac{p_w^w(\pi(2) - \pi(1))}{r - \mu} \right) \right] + \frac{\gamma \pi(1)x_0}{r - \mu} - \alpha_0e_1^w = 0
\]

Comparison of the above two first-order conditions, together with the requirements that 
\(e_1^w = e_1^{w*}\) and \(x_2^w = x_2^w\), results in \(p_w^w > 0\) as stated in the proposition. With \(p_w^w\), the above two entry thresholds can be equated and \(\chi^w\) can be solved for with the result as stated in the proposition. Inspection of the result reveals the relation that \(\chi^w\) is positive (negative) when \(p_w^w > (<) \gamma - \xi\).