Prospect Theory and Loss Aversion in the Housing Market

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Abstract

A stylized fact of the housing market is the strong positive correlation between prices and trading volume. Loss aversion from the sellers is one of the most often suggested explanations for this phenomenon, through an increase in sellers’ reservation value. This article demonstrates that on the contrary the effect of loss aversion is to decrease the reservation value and not to increase it. We suggest alternative behavioral explanations for the observed stylized fact.

Keywords: Loss aversion, prospect theory, housing market, behavioral economics.

1 Introduction

It has been known for a while that in the housing market, there are consistently more transactions when the market is up than when it is down (Stein (1995)). Two explanations have been provided in the literature for this phenomenon: first, liquidity constraints which are binding when the resale price is lower than the buying price (Stein (1995), Genesove and Mayer (1997), Chan (2001)); second, loss aversion from sellers (Genesove and Mayer (2001), Engelhardt (2003), Einiö, Kaustia, and Puttonen (2008), Bokhari and Geltner (2011)). Leung, Lau, and Leong (2002) and Shi, Young, and Hargreaves (2010) use econometric methods to determine the direction of causality between volume and price; they find that in most markets the causality goes from volume to prices, which would suggest that search frictions cause this relationship, but they cannot rule out liquidity constraints or loss aversion in several markets.

The liquidity constraint explanation relies on the fact that house buyers are highly leveraged in the United States, with down-payments of 10% being usual. Consider an agent who bought a $100,000 house with a $10,000 down-payment. If the house value increases the next day to $110,000, after selling the house and paying back the mortgage, the agent will have a $20,000 equity
position with which she can buy a $200,000 house with a down-payment of 10%, assuming that there are no binding income or debt ratio constraints, and ignoring for the sake of simplicity taxes and real estate agent commissions. Conversely, if the house value decreases the next day to $90,000 and the agent wants to sell it, after selling the house and paying back the mortgage, the agent would have zero equity, and she would be unable to buy a new house if she has no other savings. Therefore, it is rational for an agent to set an asking price not too far below her previous buying price (even if the probability of selling the house is low) instead of completely adjusting her asking price to the market conditions, as long as the waiting or carrying costs are lower than the expected disutility of not being able to buy a new house.

The loss aversion explanation relies on the fact, first demonstrated by psychologists, that a loss is more painful than a gain is enjoyable, which makes people reluctant to realize a nominal loss (Kahneman and Tversky (1979)).

Both explanations may justify sellers’ reluctance to sell at a lower price than their previous buying price. To disentangle the relative weights of these two explanations, the articles mentioned above use the fact that the buying price and the Loan-to-Value (LTV), i.e. the agent’s level of equity, are not perfectly correlated. When an agent has a high previous buying price but she has no liquidity constraint, her reluctance to sell at a loss may be ascribed to loss aversion. However, the previous buying price and the LTV share common determinants (such as the characteristics of the house and of the agent), and they are quite strongly correlated, which makes it practically difficult to determine their relative weights. A further limitation of this approach is that it relies on econometric models designed for estimation, without modeling theoretically the relationship between loss aversion and asking prices in a search model.

In this article, we develop a model of the decision to sell a house when the seller is subject to loss aversion. We show that the theoretical effect of loss aversion goes in the direction opposite to the one formerly hypothesized: loss
aversion tends to decrease the reservation value of a seller. The strength of
the effect depends on the time horizon we select: the longer the time horizon,
the weaker the effect. For a model in infinite time, loss aversion no longer
has an effect on the reservation value.

The plan of the article is as follows: first we build a two-period model,
which allows for clearer intuition. Then we show that the negative impact
of loss aversion on the reservation value extends to any model in finite time.
Finally, we show that loss aversion does not have any impact at all in infinite
time.

2 Two-period model

The housing market is a bilateral search and matching market: buyers and
sellers are both heterogeneous, and the final transaction prices result from
bilateral bargaining between buyers and sellers. But here we are interested
mainly in the sellers' behavior, so we will take the behavior of buyers as fixed.
In this case, the sellers’ problem boils down to an optimal stopping problem:
they receive successive offers from buyers and they have to decide when to
accept an offer. We will first consider the problem of a seller with classical
(linear) preferences, then the problem of a seller subject to loss aversion.

As we will see below, the optimal behavior of a loss-averse seller is very
different in the finite-horizon case compared to the infinite-horizon case. This
justifies considering first a two-period model, the simplest finite-horizon pos-
sibility, where the seller’s problem is to choose between the offer she has at
hand today or wait for another offer tomorrow, which we assume she will
have to accept (e.g. because she has to move to some other place). This
restrictive assumption is made here only for the ease of exposition and it
is relaxed in the infinite time model, which consider the general case where
the agent is never forced to sell. The reservation value is then basically the
certainty equivalent to the search “lottery,” which allow us to give the result
an interpretation in line with the literature on loss aversion in lotteries.
Formally, we consider an agent (a seller) who draws an offer from a given probability distribution, with cumulative density function \( F(x) \) and probability density function \( f(x) \) on an interval \( I = [x_{\min}, x_{\max}] \). If the agent refuses the first offer, she can draw another offer (noted \( y \)) that will be final. The second draw is independent from the first. Any second-period offer is discounted relative to first-period offers at a discount rate \( \beta \) and utility is linear: \( u(x) = x \). The discount rate \( \beta \) is taken to embody the time value of money, i.e. the time preference of the seller, as well as any holding cost proportional to the value of the house. It would also be possible to model a fixed holding cost \( c \) by subtracting it from the selling price \( (u(x) = x - c) \); but it seems more realistic to consider that the holding cost is proportional to the value of the house.

We model the choice of accepting the first offer or drawing a second one with a recursive value function, building on the theoretical framework of Lucas, Stokey, and Prescott (1989) and Ljungqvist and Sargent (2004). The use of a recursive value function is justified by the fact that a recursive value function is much more tractable for models in infinite time, but can also be used in finite time models. In this framework, the uncertainty in the first period is neglected, because the first draw is essentially “free,” as usual in models of search; the choice of the agent boils down to choosing between accepting the first offer or drawing a new one at a discount on the next period.

The value function for having received the first (random) offer \( x \) is

\[
V(x) = \max \left\{ u(x), \beta \int_I u(y)f(y)dy \right\}
\]

\[
= \max \left\{ x, \beta \int_I yf(y)dy \right\}
\]

\[
= \max \{ x, \beta E(y) \}
\]

\(^1\)The interval \( I \) could be extended to infinity for the sake of generality, with \( F(x) = 0 \) for any \( x \) above a certain threshold, but we think that finite boundaries make the modeling clearer, without loss of generality.
A standard result in search theory is that the solution strategy takes the form of a reservation value: in the first period, the agent accepts any offer higher than $\bar{x} = \beta E(y)$ and rejects any offer lower than $\bar{x}$. This means that the expected value for the agent before drawing the first offer is

$$EV = \int V(x)f(x)dx = \int_{x_{\min}}^{\bar{x}} \beta \int u(y)f(y)dydx + \int_{\bar{x}}^{x_{\max}} u(x)f(x)dx$$

$$= \beta F(\bar{x}) \int y f(y)dy + \int_{\bar{x}}^{x_{\max}} xf(x)dx$$

$$= \beta F(\bar{x})E(y) + E(x|x \geq \bar{x})$$

(2)

Therefore, it would be possible to use a non-recursive model in the two-period case, and directly solve for the optimal $\bar{x}$. However, that would not be possible for infinite time, so for consistency we use a recursive set-up everywhere.

We then consider an agent with loss aversion (Kahneman and Tversky (1979)). In the original formulation, Prospect Theory includes three features:

- Loss aversion, i.e. the slope of the utility function is higher below the reference point (losses) than above it (gains);
- Diminishing sensitivity to gains and losses, i.e. the utility function is concave for gain and convex for losses;
- Probability deformation, i.e. the fact that low probabilities are overestimated and high probabilities are underestimated.

It is often customary in the literature to consider loss aversion alone, without taking into account the last two features. It is what Genesove and Mayer (2001) do and we will do likewise here.

In that case the utility function for a reference point $r$ is

$$u(x, r) = x - r \quad \text{if } x \geq r$$

$$= \lambda(x - r) \quad \text{if } x < r$$

(3)
In this equation, $\lambda \geq 1$ is the parameter representing the strength of loss aversion: $\lambda = 1$ means no loss aversion and for $\lambda > 1$, the higher $\lambda$ the stronger loss aversion. In the seminal work of Kahneman and Tversky, the empirically determined value for $\lambda$ was 2.25.

An important question in the general literature on loss aversion is to determine the relevant reference point (for a discussion of this problem and a proposition of solution, cf. Köszegi and Rabin (2006)). A common assumption with loss aversion in the housing market is that the relevant reference point is the previous buying price, which is consistent with anecdotal evidence and interviews of sellers. Nonetheless, Seiler, Seiler, Traub, and Harrison (2008) show the importance of a “false reference point,” namely the highest potential selling price reached by the property since the owner has bought it. Seiler, Collins, and Fefferman (2013) present an example of the outstanding loan balance as a false reference point in the context of a social network. This distinction is immaterial for the present article, which focuses on the theoretical effect of any reference point to the reservation value, and the model is not calibrated with empirical values. We therefore consider $r$ as the reference point, whatever its origin.

We have then two different cases to examine, depending on whether $r \notin \text{Int}(I)$ or $r \in \text{Int}(I)$, i.e., the reference point is accessible or not. When $r \notin \text{Int}(I)$, the utility function is no longer piecewise: the agent interprets any possible offer as a gain (if $r \leq x_{\text{min}}$), or she interprets any possible offer as a loss (if $r \geq x_{\text{max}}$). When $r \in \text{Int}(I)$, there exists at least one possible offer strictly lower than $r$ (this offer would be perceived as a loss) and at least one possible offer strictly higher than $r$ (this offer would be perceived as a gain), so the utility function follows the piecewise formula above. We examine both cases in turn.
2.1 Exterior reference point

When the reference point is outside the range of possible offers, it implies that the utility function is no longer piecewise. If the reference point is lower than all possible offers, the agent is never in her loss domain, and \( u(x, r) = x - r \). The corresponding value function is

\[
V(x, r) = \max \left\{ u(x, r), \beta \int_I u(y, r) f(y) dy \right\}
\]

\[
= \max \left\{ (x - r), \beta \int_I (y - r) f(y) dy \right\}
\]

Let us define \( \bar{x} \) as the reservation value in the standard case and \( x^*(r) \) as the reservation value associated with the reference point \( r \) when the agent manifests loss aversion. \( x^*(r) \) is defined by

\[
u(x^*(r), r) = \beta \int_I u(y, r) f(y) dy
\]

which we can rewrite as

\[
x^*(r) - r = \beta \int_I (y - r) f(y) dy
\]

And

\[
x^*(r) = (1 - \beta)r + \beta E(y) = (1 - \beta)r + \bar{x}
\]

Conversely, if the reference point is higher than all possible offers, the agent is always in her loss domain and \( u(x, r) = \lambda(x - r) \), so we have

\[
u(x^*(r), r) = \beta \int_I u(y, r) f(y) dy
\]

\[
\Rightarrow \lambda(x^*(r) - r) = \beta \int_I \lambda(y - r) f(y) dy
\]

Simplifying yields

\[
x^*(r) = (1 - \beta)r + \beta E(y) = (1 - \beta)r + \bar{x}
\]

Both cases result in the same formula. We can see that the loss aversion coefficient \( \lambda \) is absent in this formula: the reference point has an impact
on the reservation value, but this effect is independent from the asymmetry between gains and losses.

Moreover, with realistic values for $\beta$ (e.g. $\beta = 0.99$ for moderately high frequency offers\(^2\)), this effect is quantitatively extremely small and cannot match the observed relationship between previous buying prices and new asking prices.

This result is in line with Wakker (2010), who finds that “loss aversion only concerns mixed prospects. Loss aversion does not affect preferences between pure gain prospects nor preferences between pure loss prospects” (p259).

2.2 Interior reference point

We have seen that when the reference point is lower than any possible offer, we have, by construction,

$$u(x^+(r), r) = x^+(r) - r$$  \hspace{1cm} (10)

and when the reference point is higher than any possible offer, we have

$$u(x^-(r), r) = \lambda(x^+(r) - r)$$  \hspace{1cm} (11)

When the reference point is an interior point of I, some possible offers are framed as losses and other are framed as gains. When the reference point $r$ is sufficiently low, the reservation value $x^+(r)$ is higher than $r$; conversely, when $r$ is sufficiently high, $x^-(r)$ is lower than $r$ by construction: we have $x^+(x_{\min}) \geq x_{\min}$ and $x^-(x_{\max}) \leq x_{\max}$ by property of the reservation value, and the reservation value is continuous with respect to the reference point. We will show below that whenever $x^+(r) \geq r$, $\frac{dx^+}{dr} < 0$ and whenever $x^-(r) \leq r$, $\frac{dx^-}{dr} > 0$, which means that $x^*$ is a decreasing function of $r$ until the unique point $\hat{r}$ such that $x^*(\hat{r}) = \hat{r}$ and then it is increasing (but remaining below the diagonal $x^*(r) = r$).

\(^2\)As a comparison, Carrillo (2012) uses values of 0.9998, 0.994 and 0.93 respectively for the daily, monthly and annual rates.
In the first case (low reference point), we have

\[ x^*(r) - r = \beta \int u(y, r) f(y) dy \]

\[ = \beta \left[ \int_{x_{\text{min}}}^{r} \lambda(y-r) f(y) dy + \int_{r}^{x_{\text{max}}} (y-r) f(y) dy \right] \]  \hspace{1cm} (12)

we can isolate \( x^*(r) \):

\[ x^*(r) = r + \beta \left[ \int_{x_{\text{min}}}^{r} \lambda(y-r) f(y) dy + \int_{r}^{x_{\text{max}}} (y-r) f(y) dy \right] \]  \hspace{1cm} (13)

We want to know if introducing loss aversion (i.e. taking \( \lambda > 1 \)) will increase or decrease the reservation value, so we simply take the derivative of the reservation value with respect to \( \lambda \), at the baseline value \( \lambda = 1 \):

\[ \frac{dx^*(r)}{d\lambda} |_{\lambda=1} = \beta \int_{x_{\text{min}}}^{r} (y-r) f(y) dy < 0 \]  \hspace{1cm} (14)

Therefore introducing loss aversion decreases the reservation value.

In the second case (high reference point), we have

\[ \lambda(x^*(r) - r) = \beta \left[ \int_{x_{\text{min}}}^{r} \lambda(y-r) f(y) dy + \int_{r}^{x_{\text{max}}} (y-r) f(y) dy \right] \]  \hspace{1cm} (15)

We simplify by \( \lambda \) and isolate \( x^*(r) \):

\[ x^*(r) = r + \beta \left[ \int_{x_{\text{min}}}^{r} (y-r) f(y) dy + \int_{r}^{x_{\text{max}}} \frac{1}{\lambda}(y-r) f(y) dy \right] \]  \hspace{1cm} (16)

And therefore the derivative of the reservation value with respect to \( \lambda \) is

\[ \frac{dx^*(r)}{d\lambda} |_{\lambda=1} = \left[ \beta \left( \frac{1}{\lambda^2} \right) \int_{r}^{x_{\text{max}}} (y-r) f(y) dy \right] |_{\lambda=1} \]

\[ = -\beta \int_{r}^{x_{\text{max}}} (y-r) f(y) dy < 0 \]  \hspace{1cm} (17)

We see that from the baseline situation when \( \lambda = 1 \) (there is no asymmetry between gains and losses), introducing asymmetry by increasing \( \lambda \) actually decreases the reservation value. To get a complete picture of the effect of reference-dependent preferences on the agent’s behavior, we now need to consider the effect of the variation of the reference point on the reservation
value. Indeed, many articles in the literature, including Genesove and Mayer (2001), assume that the larger the distance between the reference point and the considered consumption, the higher the effect of loss aversion.

In order to study the effect of the variation of the reference point on the reservation value, we first need to define the value $\hat{r}$ such that $x^*(\hat{r}) = \hat{r}$; that is

$$u(x^*(\hat{r}), \hat{r}) = 0 = \int_{x_{min}}^{\hat{r}} \lambda(y - \hat{r}) f(y)dy + \int_{\hat{r}}^{x_{max}} (y - \hat{r}) f(y)dy$$

$$= \lambda \int_{x_{min}}^{\hat{r}} y f(y)dy + \int_{\hat{r}}^{x_{max}} y f(y)dy - F(\hat{r}) \lambda \hat{r} - (1 - F(\hat{r})) \hat{r}$$

The existence of $\hat{r}$ can be inferred by continuity of the reservation value function: first, $x^*(x_{min}) \in [x_{min}, x_{max}]$ by property of the reservation value, and hence $x^*(x_{min}) \geq x_{min}$; we can show similarly that $x^*(x_{max}) \leq x_{max}$. By continuity of the reservation value with regard to the reference point, there exists a point such that $x^*(r) = r$, and as we will show below, this point is unique.

When the reference point is lower than $\hat{r}$, we have

$$\frac{dx^*(r)}{dr} = 1 - \lambda \beta \int_{x_{min}}^{r} f(y)dy - \beta \int_{r}^{x_{max}} f(y)dy$$

$$= 1 - \beta - (\lambda - 1) \beta F(r)$$

For $\lambda = 1$, this derivative is positive but almost negligible. As soon as we introduce loss aversion ($\lambda > 1$), this derivative becomes negative: the reservation value is a decreasing function of the reference point. Moreover, the cross-derivative with regard to $r$ and $\lambda$ is

$$\frac{d^2 x^*(r)}{d\lambda dr}|_{\lambda=1} = -\beta < 0$$

This means that as the reference point increases, the negative effect of loss aversion on the reservation value gets stronger.

Conversely, when the reference point is higher than $\hat{r}$, we have

$$\frac{dx^*(r)}{dr} = 1 - \beta F(r) - \beta/\lambda(1 - F(r)) = 1 - \beta/\lambda - \beta F(r) \left( \frac{\lambda - 1}{\lambda} \right)$$

$$= 1 - \beta/\lambda - \beta F(r) \left( \frac{\lambda - 1}{\lambda} \right)$$

$$= 1 - \beta/\lambda - \beta F(r) \left( \frac{\lambda - 1}{\lambda} \right)$$

This completes the proof that the reservation value decreases as the reference point increases.
For $\lambda = 1$, this derivative is again positive but negligible. The cross-derivative with regard to $r$ and $\lambda$ is

$$\frac{d^2 x^*}{drd\lambda}|_{\lambda=1} = \frac{\beta}{\lambda^2} - \beta F(r) \left( \frac{1}{\lambda^2} \right) = \beta(1 - F(r)) > 0$$

(22)

This means that with loss aversion,

$$\frac{dx^*}{dr}(r) > 0$$

(23)

We can note that $x^*(r)$ is non-derivable at $\hat{r}$: it is decreasing on the left of this point and increasing on the right.

These results mean that the reservation value as a function of the reference point reaches its minimum at $\hat{r}$. In other words, the effect of loss aversion on the reservation value is maximal for $\hat{r}$. This further undermines the notion that loss aversion has a stronger effect for an “extreme” (very high or very low) reference point.

To conclude this demonstration, now that we have calculated the derivatives, we can prove that $\hat{r}$ is unique. First, the right derivative of $x^*$ is less than one at $r = \hat{r}$:

$$\frac{dx^*}{dr}|_{r+} = 1 - \beta/\lambda - \beta F(\hat{r}) \left( \frac{\lambda - 1}{\lambda} \right) < 1$$

(24)

Moreover, the second derivative of $x^*$ with respect to $r$ is negative on the right of $\hat{r}$:

$$\frac{d^2 x^*}{dr^2}(r) = -\beta \left( \frac{\lambda - 1}{\lambda} \right) f(r) < 0$$

(25)

This means that after crossing the 45-degree line at $\hat{r}$, $x^*$ will never increase fast enough to cross it again, so there is no other point such that $x^*(r) = r$. This proves the uniqueness of $\hat{r}$.

As a guide for intuition, we computed the reservation value of an agent drawing offers from the uniform distribution on $[0; 100]$ with $\beta = 0.9$ for various values of $\lambda$ (from the standard case $\lambda = 1$ to $\lambda = 4$). Figure 1 plots the reservation value (ordinate) against the reference point (abscissa). The diagonal line is the identity application ($x^* = r$). As we can see, the higher the $\lambda$, the lower the reservation value.
Figure 1: Reservation value as a function of reference point for several values of the loss aversion coefficient $\lambda$
3 Finite time model

In this section, we show that the results we have seen in the previous section extend to any model in finite time, using backward induction. Let us define for the penultimate period

\[ V_1(x, r) = \max \left\{ u(x, r), \beta \int_I u(y, r) f(y) dy \right\} \] (26)

In this case, the formula is similar to the one in the two-period case. And for any other period

\[ V_t(x, r) = \max \left\{ u(x, r), \beta \int_I V_{t-1}(y, r) f(y) dy \right\} \] (27)

with \( t \) being the number of periods remaining before the search ends. For any given reference point \( r \), the reservation value when there are \( t \) periods remaining, \( x_t^*(r) \), is defined by

\[ u(x_t^*(r), r) = \beta \int_I V_{t-1}(y, r) f(y) dy \] (28)

Using the same line of reasoning as in the previous section, we consider successively the situation of a “low” reference point and of a high reference point.

If the reference point is low enough, we have \( x_t^*(r) \geq r \) and \( u(x_t^*(r), r) = x_t^*(r) - r \). We can isolate \( x_t^*(r) \):

\[ x_t^*(r) = r + \beta \int_I V_{t-1}(y, r) f(y) dy \] (29)

We take the derivative with regard to \( \lambda \), the coefficient of loss aversion:

\[ \frac{dx_t^*}{d\lambda}(r) = \beta \int_I \frac{d}{d\lambda} V_{t-1}(y, r) f(y) dy \] (30)

For any couple \((x, r)\) and any \( t \), either

\[ \frac{d}{d\lambda} V_t(x, r) = \frac{d}{d\lambda} u(x, r) \leq 0 \quad \text{or} \quad \frac{d}{d\lambda} V_t(x, r) = \beta \int_I \frac{d}{d\lambda} V_{t-1}(y, r) f(y) dy \] (31)

So by induction, we always have

\[ \frac{d}{d\lambda} V_t(x, r) \leq 0 \] (32)
and therefore
\[ \frac{dx^*}{d\lambda}(r) \leq 0 \] (33)
because we have shown it to be true for \( t = 1 \) in the two-period case, and if it is true for \( t - 1 \) it is true for \( t \).

If the reference point is high enough, we have \( x^*_t(r) \leq r \) and \( u(x^*_t(r), r) = \lambda(x^*_t(r) - r) \). We can isolate \( x^*_t(r) \):
\[ x^*_t(r) = r + \frac{1}{\lambda} \beta \int_1^V V_{t-1}(y, r)f(y)dy \] (34)
We take the derivative with regard to \( \lambda \), the coefficient of loss aversion:
\[ \frac{dx^*_t}{d\lambda}(r) = -\frac{1}{\lambda^2} \beta \int_1^V V_{t-1}(y, r)f(y)dy + \frac{1}{\lambda} \beta \int_1^V \frac{d}{d\lambda} V_{t-1}(y, r)f(y)dy \] (35)
The same reasoning applies as in the case of a low reference point, hence
\[ \frac{1}{\lambda} \beta \int_1^V \frac{d}{d\lambda} V_{t-1}(y, r)f(y)dy \]
is always negative, and therefore
\[ \frac{dx^*_t}{d\lambda}(r) \leq 0 \] (36)
because we have shown it to be true for \( t = 1 \) in the two-period case, and if it is true for \( t - 1 \) it is true for \( t \).

This concludes the proof. Anticipating a bit on the next section, we can see that the longer the time horizon, the weaker the effect, because \( \frac{dx^*_t}{d\lambda}(r) \) is multiplied by \( \beta < 1 \) at each period.

4 Infinite time model
We now consider the extension of the previous model to infinite time with endogenous entry. The agent may decide to enter the market, or to stay in her house and receive a utility we will normalize to zero. If she enters the market, at each period she draws an offer, which she has to accept or reject; if she rejects it, she waits for another period. The assumption of endogenous entry is subject to discussion, as homeowners may be forced to sell their
house by exogenous factors (e.g. the need to relocate to another city). But in infinite time, an agent forced to enter the market can virtually withdraw from the market by refusing any possible offer forever, which yields a utility of zero; the assumption of endogenous entry has the same consequences, but makes clearer the choice faced by the agent between really searching for an offer and withdrawing (effectively or virtually) from the market.

We will consider first the case with standard utility, then the case with loss aversion.

4.1 Standard Utility

With standard (linear) utility, once the agent is in the market, her value function for having the offer \( x \) at hand is

\[
V(x) = \max \left\{ u(x); \beta \int_I V(y)f(y)dy \right\} = \max \left\{ x; \beta \int_I V(y)f(y)dy \right\}
\]

The reservation value is characterized by

\[
V(\bar{x}) = \bar{x} = \beta \int_I V(y)f(y)dy
\]

Or equivalently

\[
\bar{x} = \frac{\beta}{1 - \beta F(x)} \int_{\bar{x}}^{x_{max}} yf(y)dy
\]

Let

\[
\Gamma(x) \equiv x - \frac{\beta}{1 - \beta F(x)} \int_{x}^{x_{max}} yf(y)dy
\]

The reservation value of the agent cancels out \( \Gamma \), by construction. We have

\[
\Gamma(x_{min}) = x_{min} - \beta E(y) \text{ and } \Gamma(x_{max}) = x_{max}
\]

If \( \Gamma(x_{min}) \geq 0 \), then \( x_{min} \geq \beta E(y) \) and the agent accepts the first offer she draws, however low it might be. If \( \Gamma(x_{min}) < 0 \), as we have \( \Gamma(x_{max}) > 0 \), there exists \( \bar{x} \) such that \( \Gamma(\bar{x}) = 0 \), so the reservation value exists.

In the standard case, the expected utility of entering the market is positive: the worst possible offer is \( x_{min} \), which yields utility \( u(x_{min}) = x_{min} \geq 0 \),
so the agent always enters the market. The problem of the agent is therefore well-defined: she always enters the market, and either accepts the first offer she draws, or draws new offers until she reaches an acceptable one.

### 4.2 Loss Aversion

We now consider an agent subject to loss aversion. We have

\[
V(x, r) = \max \left\{ u(x, r); \beta \int_I V(y, r) f(y)dy \right\}
\]

(42)

Let us consider the different possible situations regarding the reference point:

1. a reference point lower than all possible offers,
2. a reference point higher than all possible offers,
3. a reference point inside the interval of possible offers.

**Lower reference point** If \( r < x_{\min} \), we have \( u(x, r) = x - r \) for all \( x \).

Then the reservation value \( x^*(r) \) is characterized by

\[
x^*(r) - r = \beta \int_{x_{\min}}^{x^*(r)} [x^*(r) - r] f(y)dy + \beta \int_{x^*(r)}^{x_{\max}} [y - r] f(y)dy
\]

(43)

Which leads to

\[
x^*(r) - r = \frac{\beta}{1 - \beta F(x^*(r))} \int_{x^*(r)}^{x_{\max}} (y - r) f(y)dy
\]

(44)

We can make two remarks: first, for \( r = 0 \) we find the standard result which was to be expected; second, \( \lambda \) does not appear in the expression, so loss aversion does not affect the reservation value of the agent.

**Higher reference point** If \( r > x_{\max} \), any offer would yield a negative utility, so the agent does not enter the market.
Interior reference point  If \( r \in [x_{\min}, x_{\max}] \), the agent may or may not enter the market. We define \( r_{\text{max}} \) as the maximum threshold for the reference point regarding entry: if \( r \geq r_{\text{max}} \), the agent will not enter the market. The expected value of entering the market with a reference point \( r \) is

\[
EV(r) = \int_{I} V(y, r) f(y) dy \tag{45}
\]

Hence

\[
EV(r_{\text{max}}) = 0 \tag{46}
\]

And on the other hand we have \( EV(x_{\min}) > 0 \) and \( EV(x_{\max}) < 0 \) In the first case, all offers are framed as gains, and in the second case, all offers are framed as losses. So by continuity and monotonicity of the value function with respect to its second argument, we can confirm that indeed \( r_{\text{max}} \in ]x_{\min}, x_{\max}[ \).

Let us calculate the reservation value associated with \( r_{\text{max}} \):

\[
V(x^*(r_{\text{max}}), r_{\text{max}}) = u(x^*(r_{\text{max}}), r_{\text{max}}) = \beta \int_{I} V(y, r_{\text{max}}) f(y) dy = 0 \tag{47}
\]

by definition of \( r_{\text{max}} \), which implies that

\[
x^*(r_{\text{max}}) = r_{\text{max}} \tag{48}
\]

That is, \( r_{\text{max}} \) is the unique fixed point of the function \( x^* \).

We have \( x^*(x_{\min}) > x_{\min} \) and \( x^*(r_{\text{max}}) = r_{\text{max}} \), which means that by monotonicity of the value function,

\[
\forall r \leq r_{\text{max}}, \ x^*(r) \geq r \tag{49}
\]

therefore

\[
V(x^*(r), r) = x^*(r) - r = \beta \int_{I} V(y, r) f(y) dy
\]

\[
= \beta \int_{x_{\min}}^{x^*(r)} V(x^*(r), r) f(y) dy + \beta \int_{x^*(r)}^{r_{\text{max}}} (y - r) f(y) dy \tag{50}
\]
And finally we get the same expression as before:

\[ x^*(r) - r = \frac{\beta}{1 - \beta F(x^*(r))} \int_{x^*(r)}^{x_{max}} (y - r) f(y) dy \]  \hspace{1cm} (51)

The expression \( x^*(r) \) is not affected by \( \lambda \). The interpretation is that with endogenous entry, an agent can secure at least a zero utility by staying put instead of entering the market; hence, she has no reason to accept an offer lower than her reference point, and loss aversion never arises.

One might object that this result depends on the assumption of endogenous entry, whereas in reality people may be forced by external circumstances to sell their house. However, like we said before, if we force an agent to enter the market with an infinite time horizon, she can still virtually withdraw from the market by rejecting any offer; this strategy offers an expected utility of zero. And if we force an agent to enter the market with a finite time horizon, we get back to the results of the previous sections.

To conclude this section, we have found that in infinite time, introducing a reference point influences the decision to enter the market or not, but loss aversion per se has no effect whatsoever on the reservation value of the agent.

5 Discussion

To summarize the results, in finite time, loss aversion decreases the reservation value of the seller; in infinite time, loss aversion does not intervene, but the presence of a reference point is enough to prevent the agent from ever selling at a loss, given that she can always secure a zero utility by withdrawing from the market.

A possible behavioral interpretation of this phenomenon is that when the agent knows she will have to sell her house soon, and she may be forced to incur a loss, caution dominates, which drives the reservation value down. Conversely, when the agent may withdraw from the market or keep her house on the market without selling for long enough, she indefinitely postpones incurring a loss.
The first part of this explanation is consistent with the classical interpretation of prospect theory: loss aversion makes the agent more reluctant to take a bet (here, rejecting an offer and waiting for another one) and more inclined to take a certain value (here the offer at hand). The second part of this explanation is consistent with the theory of intertemporal substitution: the agent prefers to postpone any loss.

These results are broadly consistent with Sun (2008), who also analyzes from a theoretical standpoint the results from Genesove and Mayer (2001). He finds that in infinite time, the existence of a reference point increases the reservation value, without the need for loss aversion. However, he uses a different modeling strategy, which makes direct comparison of our results difficult. In particular, he sets his framework in continuous time and assumes that the cost of waiting is a constant not proportional to the value of the house, whereas we use a framework set in discrete time with a cost of waiting which is proportional to the value of the house (through the discount rate). Which framework is more realistic is debatable, but the discrete-time framework allows us to draw more conclusions in the general case, whereas in continuous time, Sun (2008) finds that “although the solving process is straightforward, equation 2.5 to 2.6 are too general to give us any clear implications on the relationship between a seller’s asking price and the reference value” (p18) and he resorts to numerical simulations.

The results we found contradict the conclusions of Genesove and Mayer (2001). We offer several possible interpretations why this is the case, from the simplest to the most convoluted. First it is empirically difficult to disentangle the effect of loss aversion from the effect of liquidity constraints, and it is possible that in spite of their controls Genesove and Mayer (2001) actually observed the effect of liquidity constraints. Second, instead of being caused by loss aversion, the reluctance to sell at a loss may simply reveal the imperfect adjustment of the seller’s assessment of the market conditions.
This interpretation would be consistent with the idea of anchoring (Furnham and Boo (2011)). Third, in our model the seller is subject to loss aversion but is otherwise perfectly rational; it is possible that when deciding between accepting the offer at hand and waiting for another possible offer, the agent fails to anticipate the possibility of a worse offer and the pain loss aversion would then inflict upon her. This interpretation would be consistent with the idea of myopia.

Unfortunately, we do not know of formal models of anchoring or myopia that could be directly compared to the model of loss aversion in the Prospect Theory. Therefore, we cannot distinguish between those competing explanations from a theoretical standpoint. However, it should be possible to distinguish between loss aversion and anchoring empirically. A possible way to do that would be to test in the laboratory whether the initial buying price influences the selling price when the subjects do not have an emotional link to the initial buying decision; for example, if the subjects are told that they act as agents for a principal, or that they inherited a house that they need to sell, anchoring could be a factor but loss aversion should not.

Another possible explanation for the phenomenon observed by Genesove and Mayer (2001) is the one offered by Sun and Seiler (2013), namely hyperbolic discounting. In their words, “we show that asymmetry in the value function is not necessary to achieve the same results as Genesove and Mayer (2001). As such, loss aversion is only potentially responsible for explaining seller behavior in their model. To demonstrate our supposition, we show that the ’loss aversion’ behavior observed in Genesove and Mayer (2001) can be driven by an inter-temporal choice problem.” Their article complements ours: it shows that loss aversion is not necessary to create the observed behavior, whereas ours shows that loss aversion is not sufficient for it. Thus, the hypothesized link between loss aversion and the anomalous behavior observed in the housing market is completely severed.
6 Conclusion

In this article, we discussed the impact of loss aversion on the sellers’ behavior in the housing market. The main, and rather unexpected, result is that the impact depends on the time horizon: for a very short time horizon, loss aversion tends to make the agent risk-averse and more willing to accept a lower offer (lest she ends up with a worse one the next time); for a long or infinite time horizon, the agent is loath to accept any offer framed as a loss and she would rather defer the choice or exit the market.

These results suggest that the reluctance to sell at a loss cannot be ascribed to loss aversion per se: this phenomenon could be caused by loss aversion in conjunction with myopia, or simply be an example of anchoring, not to mention the obvious effect of liquidity constraints. Therefore, more empirical research is necessary to disentangle these different competing explanations.

On a broader level, we modeled the housing market as a search problem from the point of view of a representative seller. Therefore, the conclusions we have drawn also apply more generally to any model of search. For example, in a model of consumer search for the lowest price, this suggest that loss aversion lead consumers to accept higher prices than they would without loss aversion.

References


