The Speculative Value of Farm Real Estate*

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Abstract

Farm real estate in the Midwest has increased in value at rates not seen since the 1970s. The combination of low interest rates, an increase in the domestic demand for corn for ethanol production, and higher world demand for grain in conjunction with lower stocks have all pushed grain prices higher for commodities grown on farmland. A question that naturally arises is whether or not there is speculation in farm real estate and if so, to what extent speculation has contributed to the increase in farmland values? In this paper, a simple dynamic and stochastic farmland valuation model is developed. Calibrating the model to current economic conditions for corn grown on high quality Iowa farmland suggests that there is a modest speculative component in current farmland value, but higher corn prices would be necessary for speculation to be a significant component of total value.

Keywords: dynamic, entropy, farmland, speculation, valuation

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* Any errors or omissions are the sole responsibility of the authors.
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Introduction

Farm real estate in the Midwest has increased in value at rates not seen since the 1970s. For example, in Iowa, after bottoming out in 1986 the average nominal value of high quality farmland has increased 22 out of the last 25 years (see Figure 1). Since 2000, the annual rate of growth in the value of high quality Iowa farmland has averaged 12.5%. This, in conjunction with an average annual yield of 4.4% from cash rent over the same period, and it is clear that farm real estate has been an attractive investment.

The combination of low interest rates, an increase in the domestic demand for corn for ethanol production, and higher world demand for grain in conjunction with lower stocks have all pushed grain prices higher for commodities grown on farmland. Higher grain prices tend to lead to more profitable agricultural operations giving farmers the economic signal that expanding their operations would likely be profitable. As farmers can then bid larger amounts to acquire control of the land resource, farm real estate prices increase. Consistent with these ideas, Henderson and Gloy (2009) find that recent higher crop prices have quickly translated into higher farmland values and that farmland values are highest in locations approximate to ethanol plants.

One question that naturally arises in this setting is whether or not farm real estate prices have increased more than fundamentals would suggest. If they have, it can be argued there may be a speculative component to farm real estate values that could signal the formation of a bubble in Midwestern farmland markets.\(^1\) The popular press has begun asking these questions (e.g. see Zumbrun (2011), Mohindru (2011), Blas (2012), Gustin (2012), Love (2012), Hoak (2012), or Denning (2012)) but has provided little in the way of substantive answers.

The academic literature on farm real estate values offers a mixed bag of results. For example Burt (1986) develops a detailed econometric model of the capitalization formula that appears to describe Illinois farmland prices well. Engsted (1998) uses vector auto-regression to model the present value model of farm real estate prices. Clark \textit{et al.} (1993) reject the notion that farmland rents and prices have the same time series representation while Tegene and Kuchler (1993) reject the notion of speculative bubbles in their study of farmland prices in the North Central region of the U.S. during the 1970s. This is in contrast to research Plaxico (1979) and Featherstone and Baker (1987) who concluded that the

\(^1\) As Nneji \textit{et al.} (2012) state, speculative bubbles are certainly not a new phenomenon and have been investigated in numerous asset classes.
The 1970s were characterized by speculative mania rather than market fundamentals. Falk (1991) also found that farm real estate prices tend to rise (fall) faster than rents when both are rising (falling). Lavin and Zorn (2001) find no evidence of rational expectations bubbles in Iowa and Nebraska farm real estate but do find evidence that farm real estate price changes are asymmetric.

The research cited above is almost singularly concerned with farm real estate markets leading up to and including the 1970s. Only Lavin and Zorn (2001) investigate farm real estate markets more recently (up through 1995). This suggests that a more up to date analysis might help shed light on whether there is speculation in current farm real estate markets. To that end, the purpose of this research is to determine whether there is evidence that current farmland prices contain a speculative component and if so, determine its magnitude. In the next section, we specify an analytic farmland valuation model. It is then shown that the magnitude of one parameter of the model in particular can help determine the existence of capital gain speculation. In a subsequent section of the paper, the results of empirically estimating the model parameters using Iowa farmland data are presented. A discussion of the results and conclusions are then presented.

**Farmland Valuation**

As noted in Capozza et al. (2004) and Wheaton and Nechayev (2008), there is consensus in the literature on housing prices that as employment and population grow, rents and prices increase. For farm real estate markets, it is more likely that the prices and yields of commodities produced on the farmland are the key determinants of rents and values. In what follows, we assume the price of a commodity grown on farmland is the principal source of uncertainty affecting farm real estate values in the mind of potential buyers.

Consistent with this idea, we model the commodity price as a diffusion of the form

\[ \frac{dp_t}{p_t} = \mu dt + \sigma dz_t, \]

where \( \mu \) and \( \sigma \) are (constant) drift and diffusion terms for the price. In (1), \( dz_t \) is the increment of a \( \mathbb{P} \)-Brownian motion (i.e. under the natural measure \( \mathbb{P} \)) with the properties that \( \mathbb{E}^\mathbb{P}(dz_t) = 0 \) and \( \text{var}(dz_t) = dt \). It follows that the risk neutral analog to equation (1) is

\[ \frac{dp_t}{p_t} = \delta dt + \sigma dw_t, \]

where \( \delta \) is the risk premium.
where $\delta = \mu - \lambda \sigma$, $\mathbb{E}^Q(dw_t) = \lambda dt$ and $\text{var}(dw_t) = dt$. In this setting the interpretation of $\lambda$ is that it is the market price of risk for the commodity in question. It is important to point out at this point, that speculation in commodity markets, while not of any particular interest given our focus on speculation in farm real estate markets, is not precluded at this stage. However, arbitrage in the commodity markets is precluded with our specification and, as shown below, this has implications for farm real estate markets.

The valuation model is specified next. Let $p_L > 0$ be a constant lower bound on the price of the commodity. In this context, $p_L$ may be an exogenous commodity support price or simply a long run average price associated with the production of the commodity on farmland. Further, let $V_t = V_t(p_L)$ be the value of farmland and let the equilibrium return on the farmland be equal to the expected capital gain and cash flow accruing to the owner of the farmland. Therefore, we have $\mathbb{E}^Q(dV_t) + \pi(p_t) dt = rV_t dt$, where $r$ is the risk free rate, and $\pi(p_t) = \max(p_t, p_L) y - c$ for constant yield $y$, and constant variable cost of production $c$.

It follows that the solution to the second order ordinary differential equations (ODEs) given by

\[
\left(\frac{\sigma^2}{2}\right)p_t^2 V_{pp} + \delta p_t V_p - rV + p_t y - c = 0 \quad p_t > p_L
\]

\[
\left(\frac{\sigma^2}{2}\right)p_t^2 V_{pp} + \delta p_t V_p - rV + p_L y - c = 0 \quad p_t \leq p_L
\]

characterize the value of the farmland on either side of the price boundary given by $p_t = p_L$. We make the further assumption that $p_L$ is no less than the average variable cost of production, $c/y$. This assumption ensures that farmland will always have a nonnegative value since when $p_t \leq p_L$ we have $p_t y \geq c$, and non-negative cash flows accrue to the owner of the farmland.

The general solution to the ODEs in (3) when $p_t > p_L$ is

\[
V_t(p_t) = A_1 p_t^{\beta_1} + A_2 p_t^{\beta_2} + \frac{p_t y}{r - \delta} - \frac{c}{r},
\]

while the solution when $p_t \leq p_L$ is

\[
V_t(p_t) = B_1 p_t^{\beta_1} + B_2 p_t^{\beta_2} + \frac{p_L y - c}{r}.
\]

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2 While yields and costs are stochastic in the real world, they would generally be assumed to be fixed under best production practices for the purposes of valuing farmland. Maintaining this assumption also aids in keeping the model relatively simple for the analytics and empirical analysis that follows.
In either case, \( \beta_{1,2} = \frac{-\left( \delta - \frac{\sigma^2}{2} \right) \pm \sqrt{\left( \delta - \frac{\sigma^2}{2} \right)^2 + 2\sigma^2 r}}{\sigma^2} \), with \( \beta_1 > 1 \) and \( \beta_2 < 0 \). Notice that as indicated above, the no arbitrage condition in the commodity market carries over to the farm real estate market in that \( \delta \) is a function of \( \lambda \), the market price of risk. In (4) and (5), \( A_1, A_2, B_1, \) and \( B_2 \) are constants whose values need to be determined. The value matching and smooth pasting conditions applicable at the price boundary given by \( p_t = p_L \) yield two equations for helping to determine the magnitude of these constants. However, with a total of four unknowns and only two equations, the problem is ill posed having an infinite number of solutions.\(^3\)

There are instances when assumptions can be made to reduce the number of unknown parameters. For example, as \( p_t \to 0 \), it is reasonable to expect that \( V_t(p_t) \to 0 \) since zero is an absorbing barrier for the commodity price. In the current model, however, \( V_t(p_t) \to \frac{p_L y - c}{r} \) as \( p_t \to 0 \) since the owners of farmland would tend to value the farmland with \( p_L \) in mind, that is, as if \( p_t < p_L \) was a short term (and deterministic) phenomenon. This would imply that \( B_2 = 0 \) so that speculation in farmland values for excessively low commodity prices is precluded.\(^4\)

Speculation attributable to high commodity prices are also typically precluded by assumption by specifying that \( A_1 = 0 \) so that as \( p_t \to \infty \), \( V_t(p_t) \to \frac{p_L y}{r-\delta} - \frac{c}{r} \). In this case, as commodity prices increase the value of the farmland increases but approaches the farmland’s intrinsic or fundamental value given by the capitalization formula. It is this particular assumption that is relaxed in the present context. That is, as supply and demand conditions conspire to increase the equilibrium price of a commodity grown on farmland, \( A_1 \neq 0 \) allows for an equilibrium solution that permits capital gain speculation in farm real estate. Since speculation attributable to low commodity prices is not consistent with current commodity prices, nor the focus of this paper, we retain the assumption that \( B_2 = 0 \).

To see more specifically how capital gain speculation can arise, let \( p_t \gg p_L \) so that \( A_2 p_t^{\beta_2} \approx 0 \) since \( \beta_2 < 0 \). Further, let \( V_t(p_t) = S_t(p_t) + F_t(p_t) \) where \( S_t(p_t) = A_1 p_t^{\beta_1} \) is the speculative component of farmland value and \( F_t(p_t) = \frac{p_L y}{r-\delta} - \frac{c}{r} \) is the intrinsic or

\(^3\) Technically, there are five unknowns if we count the market price of risk parameter given by \( \lambda \).

\(^4\) Notice, however, that if \( p_L = \frac{c}{y}, V_t(p_t) \to 0 \) as \( p_t \to 0 \) when \( B_2 = 0 \) and the farmland has no value as the commodity price approaches zero.
fundamental value of the farmland. In this context, $S_t(p_t)$ represents the component of total farm real estate value, $V_t(p_t)$, that is in excess of its fundamental value, $F_t(p_t)$.

Since $\beta_1 > 1$, it follows that $V_t(p_t) \to \infty$ as $p_t \to \infty$ and an investor anticipating high enough commodity prices might value farmland above its fundamental value if there is the expectation of a large enough capital gain from acquiring the farmland and selling it at a later date. This type of speculation is consistent with, for example, the type of rational speculative bubble discussed in Nneji et al. (2012) and can happen with $A_1 \neq 0$ since an asset valued at $S_t(p_t) = A_1 p_t^{\beta_1}$ yields its risk adjusted return from its capital gain alone (Dixit and Pindyck).

To see this, apply Ito’s lemma to $S_t(p_t)$ which yields

$$
(6) \quad \frac{dS_t}{S_t} = \beta_1 \frac{dp_t}{p_t} + \frac{1}{2} \beta_1 (\beta_1 - 1) \left( \frac{dp_t}{p_t} \right)^2.
$$

Substituting using equation (1) above and simplifying gives a first order stochastic differential equation describing the evolution of the speculative component of farmland value

$$
(7) \quad \frac{dS_t}{S_t} = \left[ \left( \frac{\sigma^2}{2} \right) \beta_1^2 + \left( \mu - \frac{\sigma^2}{2} \right) \right] dt + \beta_1 \sigma dz_t.
$$

In equilibrium however, the quadratic equation giving rise to the solution $\beta_1$ (given above) holds so that (7) may be rewritten more simply as

$$
(8) \quad \frac{dS_t}{S_t} = (r + \theta) dt + \beta_1 \sigma dz_t.
$$

From equation (8), the expected capital gain equals a risk adjusted return that is proportional to the speculative value of farmland and consists of the risk free rate, $r$, plus a risk premium, $\theta$, which incidentally equals $\beta_1 \lambda \sigma > 0$. This risk premium is positively related to $\beta_1$, the market price of risk, and commodity price volatility. In the next section of the paper, we seek to confirm whether there is empirical evidence that $A_1 \neq 0$ and if so, what value of $\theta = \beta_1 \lambda \sigma$ is supported by actual farm real estate price data.

**Empirical Application**

In this section an empirical application of the model from the previous section is presented assuming corn grown on high quality Iowa farmland. In addition to the three unknowns $A_1$, $A_2$, and $B_1$ in (4) and (5) above, we also simultaneously estimate the market price of risk, $\lambda$, while calibrating the model parameters to current Iowa farmland market conditions. As
there are more unknowns than structural equations, we make use of a maximum entropy econometric formulation based on the work of Golan et al. (1996) and designed specifically for situations where the economic problem is underdetermined and ill posed. In what follows, we discuss the data, the econometric model, and the results of the estimation.

Data

Yield per acre, \( y \), is assumed to be 200 bushels with a variable cost of production, \( c \), equal to $395/acre. These values are consistent with yields and actual variable production costs for high quality farmland in Iowa (Duffy). The exogenous lower bound on the price of corn, \( p_L \), is assumed to be $2.52/bushel so that \( p_L \geq \frac{c}{y} = $1.975/bushel\). The parameters of the price diffusion equation (1) were estimated to be \( \mu = 2.79\% \) and \( \sigma = 16.36\% \). The risk free rate of interest is assumed to be \( r = 3\% \) throughout.

To estimate the remaining parameters of the model, the parameters estimated above are used with observed Iowa corn prices and farmland values in a maximum entropy econometric specification. Observed farmland value estimates are reported by the Realtors Land Institute who conducts a bi-annual Iowa Land Trends and Values Survey. In the survey, participants are asked to estimate average values for bare, unimproved farmland where the sale price is reported on a cash basis. Participants in the survey are realtors who deal almost exclusively in farmland. Estimates for high, medium, and low quality cropland are reported as well as pasture and timberland. The quality of the cropland (i.e. high, medium, or low) is determined by its Corn Suitability Rating, a land classification system that ranks the quality of cropland by its potential corn yield.

Monthly average cash corn prices per bushel received by Iowa farmers were used to compute a six-month average corn price. The six-month average price is assumed to be the average price farmland investors use when deciding how much to bid for farmland and coincides with the bi-annual nature of the farmland value estimates. A total of 13 six-month average corn prices and farmland value estimates are available for calibrating the model ranging from September 2005 to September 2011. These data are presented in Figure 2 and show the general increase in low, medium, and high quality farmland value as well as the extent to which these increases are related to monthly and six-month average corn prices.

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5 According to Iowa State University Extension, the average annual calendar year cash corn price in Iowa over the last 25 years is $2.4672/bu while the average annual marketing year cash corn price in Iowa over the same period is $2.5696/bu. The midpoint of these two values is $2.5184/bu so \( p_L = $2.52/bu \) is assumed.
A maximum entropy econometric specification is proposed due to the fact that the technique is particularly adept at robust estimation of parameters from ill posed or underdetermined economic problems (Golan, et al.). The current model is underdetermined because the value matching and smooth pasting conditions associated with equations (4) and (5) result in only two equations from which the four unknowns $A_1, A_2, B_1$, and $\lambda$ must be estimated. In addition, calibrating the model to market data requires at least the estimation of a correlation coefficient thereby adding one additional parameter to the estimation. As such, there are an infinite number of solutions to the problem and the maximum entropy principle offers a nonlinear inversion procedure for selecting the most probable solution to the problem that is consistent with the information (i.e. data) presented.

**Econometric Model**

In what follows, $\bm{\psi}$ represents a matrix of probabilities chosen to maximize the total entropy given by

$$
\max H(\bm{\psi}, A_2, B_1) = - \sum_{t=1}^{T} \sum_{i=1}^{N_v} \psi_{ti}^{\alpha} \ln(\psi_{ti}^{\alpha}) - \sum_{i=1}^{N_A_1} \psi_i^{A_1} \ln(\psi_i^{A_1}) - \sum_{i=1}^{N_\lambda} \psi_i^{\lambda} \ln(\psi_i^{\lambda}).
$$

subject to the model consistency equations

\begin{align}
(10.1) \quad A_1 & = \sum_{i=1}^{N_A_1} \psi_{ti}^{A_1} \eta_i, \\
(10.2) \quad \lambda & = \sum_{i=1}^{N_\lambda} \psi_{ti}^{\lambda} \zeta_i, \\
(10.3) \quad \rho_{\psi V} & = \sum_{i=1}^{N_\psi} \psi_{ti}^{\psi} \kappa_i, \\
(10.4) \quad \nu_t & = \rho_{\psi V} V_t(A_1, A_2, B_1, \lambda) + \sum_{i=1}^{N_V} \psi_{ti}^{V} \epsilon_{ti}, \quad \forall \ t \\
(10.5) \quad \sum_{t=1}^{T} \sum_{i=1}^{N_v} \psi_{ti}^{V} \epsilon_{ti} & = 0,
\end{align}

value matching equation

\begin{align}
(11) \quad A_1 p_1^{\beta_1} + A_2 p_1^{\beta_2} + \frac{p_1 Y}{r-\delta} & = B_1 p_1^{\beta_1} + \frac{p_1 Y}{r}.
\end{align}

smooth pasting equation
(12) \( \beta_1 A_1 p_L^{\beta_1-1} + \beta_2 A_2 p_L^{\beta_2-1} + \frac{y}{r - \delta} = \beta_1 B_1 p_L^{\beta_1-1} \),

and normalization equations

(13.1) \( \sum_{i=1}^{N} \psi_{vi}^t = 1, \quad \forall t \)

(13.2) \( \sum_{i=1}^{N_{A_1}} \psi_{A_1}^t = 1, \)

(13.3) \( \sum_{i=1}^{N_A} \psi_{\lambda}^t = 1, \)

(13.4) \( \sum_{i=1}^{N_p} \psi_{\rho}^t = 1. \)

In (9), the entropy is taken over the sum of all the estimated probabilities associated with the unknown parameters and errors in the model consistency equations. The model consistency equations presented in (10.1) through (10.3) show the probability estimates are multiplied by their respective parameter supports and summed giving the expected value of \( A_1, \lambda, \) and \( \rho_{ev} \), the correlation coefficient between observed \((v_t)\) and model \((V_t)\) estimated farmland values. In equation (10.4) the estimated correlation coefficient is used to reconcile the two farmland values with the possibility of error where the error term is the sum-product of the error support values, \( \epsilon_{ti} \), and their estimated probabilities. In equation (10.5), the sum of the estimated errors across time must be zero so that the empirical solution fits the data as well as possible.

The extent to which \( A_1 \) is nonzero is an indication of the strength of the speculative component of farmland value. This is because, as noted above, as corn prices rise, \( A_1 = 0 \) would imply that the value of farmland approaches its fundamental value. Similarly, the magnitude of \( \rho_{ev} \) gives some indication of how well the analytic model presented as the solutions (4) and (5) fit observed farmland value data. An estimated correlation coefficient that is very near one with \( A_1 \neq 0 \) offers joint support for the analytic model and the existence of a speculative component of farmland value. Lastly, the estimate of the market price of risk will help give some indication of the speculative premium associated with farmland as noted above in equation (8).
The remaining equations in the econometric specification are also model consistency equations associated with the value matching (11) and smooth pasting (12) conditions\(^6\) as well as normalization constraints, (13.1) to (13.4), that ensure that the estimated probabilities sum up to one. Not shown are the constraints ensuring that all the probabilities are proper probabilities (i.e. non-negative) or the constraints that bound the correlation coefficient to between -1 and +1. In theory, prior distributions for the probabilities can be specified resulting in a cross entropy econometric model (see for examples Golan, et al., or Lee and Judge). However, if no such information is available, uniformly distributed prior distributions are implicit in the specification above with the data suggesting whether a departure from a uniform distribution assumption is warranted. It should also be noted that while it is the estimated probabilities that give rise to estimates of the parameters \(A_1, \lambda,\) and \(\rho_{vV},\) point estimates of the parameters \(A_2\) and \(B_1\) are also a part of the solution.

Using vector notation, the parameter supports that are used in the empirical analysis are: \(\eta = \xi = [0.00 \ 0.05 \ 0.10 \ 0.15 \ 0.20]\) and \(k = [0.0 \ 0.25 \ 0.5 \ 0.75 \ 1.0]\) while the error support is assumed constant across \(t\) and is given by: \(\epsilon = [-2.500 \ -1.250 \ 0 \ 1.250 \ 2.500]\). Therefore, if the distribution of probabilities is uniform, \(A_1 = \lambda = 0.1\) and \(\rho_{vV} = 0.5\) are used as starting values for the nonlinear inversion with the data determining the extent to which the probabilities should be adjusted to maximize the entropy. Notice also that with a starting value of \(A_1 = 0.1,\) the data will determine whether adjustments to the probabilities \(\psi^{A_1}_i\) are warranted with \(\psi^{A_1}_1 = 1\) and \(\psi^{A_1}_i = 0\) for all \(i \neq 1\) a possibility. In this case, \(A_1 = 0\) would result and there would be no data supported speculative component in the observed value of farmland.

**Empirical Results**

The results of the estimation are presented in Tables 1 and 2 and suggest that \(\rho_{vV}\) equals 0.977 and \(A_1\) equals 0.037. Taken together, these results offer strong support for the analytic model and reasonably strong support for a non-zero speculative component. Further, the solution yields estimates of: \(A_2 = 16,231, B_1 = 1.136, \lambda = 3.48\%, \beta_1 = 5.75,\) and \(\beta_2 = -0.39.\) Using these values, the speculative premium for farmland (see equation (8)

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\(^6\) These equations are a direct result of matching equations (4) and (5) and their first derivatives evaluated at \(p_t = p_L.\)
above) is \( \theta = 3.28\% \) which implies that in equilibrium, the expected rate of growth in the speculative component of farmland value is \( r + \theta = 6.28\% \).

Shown in Tables 1 and 2 are the estimated probabilities along with their associated entropy, normalized entropy, and the percentage reduction in information uncertainty associated with each piece of information. Since maximum entropy is not a parametric estimation, conventional tests of significance are not applicable. However, as shown by Courchane et al. (2000), normalized entropy is a measure of the importance of each piece of information in reducing overall information uncertainty. Maximum entropy uncertainty results when the consistency constraints are not used and the associated distribution of probabilities is uniform. This results in an entropy estimate equal to \( \ln(5) = 1.609 \) since there are five probabilities to be estimated (for each \( r \) as well as for the three parameters). Normalized entropy is calculated by expressing the estimated entropy relative to this uniform entropy and shows the relative value of introducing each of the consistency constraints. The percentage reduction in uncertainty is equal to one minus the normalized entropy and is merely another way of expressing the usefulness of the consistency constraints by showing how much they reduce information uncertainty.

The percentage reduction in information uncertainty associated with the speculation parameter \( A_1 \) is 27.3\% and offers reasonably compelling evidence that there is a speculative component inherent in Iowa farmland values. In addition, the percentage reduction in information uncertainty associated with the correlation coefficient is 80.5\% which offers very strong evidence that the analytic model fits the data well (i.e. the model estimated and actual farmland values are highly positively correlated). Similarly, the market price of risk parameter has a reduction in information uncertainty of 29.5\% and again, offers reasonably strong evidence that the distribution of probabilities is not uniform.

As shown, very modest reductions in information uncertainty are noted for many of the consistency constraints associated with the errors. These values are especially small for the more distant farmland value data, the exception being the 30.9\% reduction from September 2008. In the six month period preceding September 2008, the price of corn in Iowa increased about 12\% over the previous six month average (see Figure 2) and the entropy model seizes on the importance of this information. After that time, larger reductions are noted culminating in a 33.7\% reduction for the most recent datum in September 2011. As shown in Figure 1, the average Iowa corn price and high quality farmland estimate were both at their highest point over the sample period.
Depicted in Figure 3 is the model solution in graphical format. As shown, the fundamental farmland value (i.e. \( A_1 = A_2 = B_1 = B_2 = 0 \)) is constant until a corn price of \$7.50/bu. is obtained at which point farmland value increases linearly with the price of corn. Given the variable cost and yield assumed, corn prices less than \$7.50/bu. result in cash flow that when capitalized at the risk adjusted rate is less than the capitalized value of the cash flow based on the lower bound price, \( p_L \). More clearly, for \( p_t \leq \$7.50, \frac{p_L Y - c}{r} > \frac{p_L Y - c}{r - \delta} - \frac{c}{r} \) so that a lower bound on farmland values is \$3,634/acre. It is important to point out that these results are conditional on the specific \( p_L \) used in the model. Recall, \( p_L \) less than or equal to \$1.98/bu. implies a lower bound on the value per acre equal to \$0. In addition, with the parameter restriction noted, this solution offers no component of value attributable to any other terms in equations (4) and (5).

When \( A_1 = B_2 = 0 \), but with \( A_2 \neq 0 \) and \( B_1 \neq 0 \), the resulting curve in Figure 3 shows the contribution to value (over strict fundamental valuation) from a dynamic and stochastic specification. However, given these parameter restrictions no additional value attributable to speculation related to excessively low or excessively high corn prices is possible. The curve lies entirely above the fundamental value curve and asymptotically approaches the fundamental value curve as the price of corn increases. Therefore, the interpretation of the distance between the two curves is the contribution to farmland values or premium that arises from explicitly considering the stochastic nature of corn prices.

The final curve shown in Figure 3 is the solution when \( B_2 = 0 \), but with \( A_1 \neq 0, A_2 \neq 0 \), and \( B_1 \neq 0 \). As shown, the value of farmland increases at an increasing rate. For high corn prices, the resulting farmland value curve lies above the other two curves and diverges as the price of corn increases. Given the behavior of the other two curves in Figure 2 for high corn prices, this result suggests that the magnitude of the speculative component increases as the price of corn increases.

This feature of the model is presented graphically in Figure 4 which shows the proportion of total farmland value, \( V_t(p_t) \), attributable to \( S_t(p_t) \) and \( F_t(p_t) \). For example, at a corn price per bushel \$3, about 86% of the value of farmland is attributable to capitalized cash flow alone with about 13% attributable to the dynamic and stochastic specification of corn prices.

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7 In Figure 3, the curve labeled “Fundamental” is the proportion of total farmland value that is attributable to only capitalized cash flow while the curve labeled “Speculative” is the proportion of total farmland value attributable to the speculative component. The remaining curve labeled “Dynamic” is the remaining proportion of value that is attributable to a dynamic model specification without speculation.
the valuation model and, therefore, a negligible amount attributable to speculation. At a corn price equal to $4/bu, only 2% of the value of farmland is speculative with the remaining value split between fundamental (about two-thirds) and dynamic (about one-third) values.

At a corn price of about $6/bu, the speculative component is still only about 12% of total farmland value. Therefore, while the model fits the data exceptionally well and there is empirical support for a speculative component in farmland values, recent market conditions suggest that the average value of Iowa farmland is not overwhelmingly speculative by any means. Even so, it is important to point out that as shown in Figure 4, the speculative component increases rapidly as corn prices per bushel increase. For comparison purposes, at a price of about $7/bu, the model predicts a value for farmland that is about 20% speculative while a price of $8/bu suggests that farmland values is about 33% speculative. Anticipation of higher prices, therefore, not only increases the fundamental value of farmland, but increases the proportion of that value that is attributable to speculation.

To determine the extent to which these results hold for Iowa farmland that is not high quality, the parameters were re-estimated using medium and low quality farmland values also provided by the Realtors Land Institute Survey. Yields are necessarily lower for lower quality farmland as are some of the variable costs of production. On lower quality farmland, lower seeding rates can be expected as well as less applied phosphate and potash. Due to lower yields, hauling, drying, and grain handling expenses are lower as well. For low (medium) quality farmland, a yield of 156 bu/acre (178 bu/acre) is assumed with variable costs of production equal to $340/acre ($368/acre).

Shown in Figure 5 is the speculative proportion of total farmland value for low, medium, and high quality farmland in Iowa related to the price of corn per bushel. As shown, high quality and medium quality farmland have generally the same speculative proportion of total farmland value while low quality farmland has a smaller speculative component. This result suggests that in Iowa, there is less speculation in low quality farmland than in medium or high quality farmland. These results are also reflected in the parameter estimates for the speculative risk premium, $\theta$. For low quality land $\theta = 1.53\%$ while for medium (high) quality land $\theta = 2.80\%$ ($\theta = 3.28\%$).

**Summary and Conclusions**
In this paper, an analytic model of farmland prices is developed and solved numerically using data for farmland in Iowa. The solution demonstrates that one parameter of the model determines the extent of capital gain speculation. As a result, an empirical estimation of the model parameters can be used to gauge whether the data support the idea that current farmland values have a speculative component. Our results suggest that while there does appear to be some capital gain speculation in current farmland prices in Iowa for high and medium quality farmland, the amount is not overwhelming. There appears to be little if any empirical support for speculation in low grade Iowa farmland. In general, corn prices in excess of $7 to $8 per bushel may be required before the percentage of observed farmland value that is speculative in nature exceeds 20%.
Figure 1. Average nominal value of high, medium, and low grade farmland in Iowa (1950-2011) Source: Iowa State University Extension.

Figure 2. Monthly corn price ($/bu), six-month average corn price ($/bu), and low, medium, and high farmland values ($1,000s/acre) in Iowa, March 2005 to September 2011.
Figure 3. Farmland value per acre as a function of corn price per bushel.

Figure 4. Proportionate farmland value per acre as a function of corn price per bushel.
Figure 5. Proportion of farmland value that is speculative for low, medium, and high quality farmland.
Table 1. Estimated error support probabilities, entropy, normalized entropy, and percentage reduction in information uncertainty.

<table>
<thead>
<tr>
<th>Error Support Values ((\epsilon_{it}))</th>
<th>Probabilities</th>
<th>Entropy</th>
<th>Normalized Entropy</th>
<th>Percentage Reduction</th>
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Table 2. Estimated parameter support probabilities, entropy, normalized entropy, and percentage reduction in information uncertainty.

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References


