An Application of Spatial Econometrics
in Relation to Hedonic House Price Modelling

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This paper applies spatial econometrics in relation to hedonic house price modelling. We present and use some basic acknowledged spatial model alternatives and a battery of relevant tests. Geographically weighted regression, semiparametric analysis, and the mixed spatial Durbin model are also applied. The purpose is to detect missing spatial variables, misspecified functional form and spatial heterogeneity in estimated parameters. Such misspecifications have been shown to give spurious results in relation to some frequently used directional based tests. We achieve significant model improvement, so the paper should be of general interest as an example of practical econometric modelling within the field.

JEL-classification: R21, R31  
Key words: Hedonic house price models, spatial econometrics

1 Introduction

Location characteristics are important determinants of housing prices. Even so spatial modelling is not prevalent in mainstream empirical research on housing markets (Kim et al., 2003). Over the past thirty years, spatial econometrics has advanced from the fringe to a fledgling discipline (Goodchild, 2004). Being a fledgling discipline, and also given the technical and interpretational difficulties of alternative spatial models and test results, the field is not easily available to applied econometricians. The purpose of this paper is to expose researchers in the field to important tools available when testing for and incorporating spatial effects into hedonic house price models. Such an approach is timely, due to the widespread use of hedonic modelling. Building on previous research, we present and use some acknowledged and relative basic spatial econometric model alternatives and estimators. The paper also gives an overview of a battery of tests that may be used in the hedonic modelling process, and the order in which the tests could be conducted. The relevant tests are frequently performed to get consistent estimates on implicit prices and to increase the reliability of ordinary significance hypothesis testing on individual parameters. As we move along, some commonly encountered problems will be discussed. The overall problem that we are dealing with is hence one of diagnostics and significance testing, model specification, and selection. This is an important issue when predicting the partial effects of individual attributes on housing prices, since the choice of spatial model will have an effect on the economic interpretation of the estimated coefficients.
In addition to Dubin (1998), other papers with similar purposes as this are Pace et al. (1998), Dubin et al. (1999), Beron et al. (2004) and Bourassa et al. (in print). A short review of the preceding relevant literature is found in Besner (2002). This paper is, however, more comprehensive, naturally at the expense of the depth of each topic. We achieve significant model improvement. The analysis is hence of general interest as an example of practical spatial econometric modelling in relation to hedonic house price studies. Finally, the paper may help implement the replication methodology developed in Lai et al. (2008) for estimating property values.

In the next section an overview of important ways of modelling spatial effects in econometrics in relation to hedonic house price models will be given. Subsequently, there is a description of the study area, the data and the preliminary base model. This part will not be extensive, since the main focus in the empirical analysis is on the relevant tests and the spatial modelling process itself. On the basis of these tests, we use geographically weighted regression (GWR) and a semiparametric estimator in order to explore the potential for missing spatial variables, spatial trends, or spatial heterogeneity in our initial model.

2 Spatial Effects and Important Spatial Econometric Models

Spatial data is characterised by two key features: Spatial autocorrelation and spatial heterogeneity (Anselin, 1988). Together these features are labelled spatial effects. Can (1990) gives the following explanation of the two characteristics: “Spatial dependence refers to the possible occurrence of interdependence among observations that are viewed in geographic space, and violates the assumption of uncorrelated error terms (...) Spatial heterogeneity (...) refers to the systematic variation in the behaviour of a given process across space, and usually leads to heteroskedastic error terms” (page 256). Although spatial heterogeneity seems to be dominant, it is to be expected that a mixture of these effects will be present in all housing market cross-section data, and will follow from a range of different spatially related phenomena which are not easily discerned. Depending on what is the prevailing problem in the data, one commonly estimates a spatial lag model or a spatial autoregressive error model to account for spatial features. In this section the two main types of spatial econometric models will be presented, in addition to the spatial Durbin model that incorporates the two more commonly used models.
2.1 The Spatial Lag Model

Tobler’s (1979) first law of geography states that “near things” are more related than “distant things”. This empirical regularity is based on the fact that overcoming space is costly, and there is a tendency to economize these costs. “Near and related things” is hence an important starting point in spatial modelling. Based on this, housing prices will be spatially autocorrelated if there is dependency between housing prices. This dependency is related to their relative location to each other, and may diminish as the distance separating the houses increases. The autocorrelation is normally positive and the price obtained on a house will be similar to the prices of neighboring houses. In the housing market literature this pattern of interaction is called the adjacency effect (Can, 1992). There may be various explanations of the adjacency effect. One explanation is that real estate agents, buyers and/or sellers could use similar sales in a neighborhood as a reference for determining a transaction price, for instance due to uncertainties regarding the value of neighborhood characteristics. The price of one house will hence influence the price of other houses located relatively near, and vice versa; see Anselin (2003), Can (1990), or Kim et al. (2003). A related explanation could be spill-over effects. Can and Megbolugbe (1997) uses this interpretation and relate it to “maintenance/repair decisions of neighbors affecting the market value of a given house or the fact that the premium households are willing to pay just for the “snob” value of a particular location” (page 206). Spatial spill-over effects in general are explained theoretically in Brueckner (2003). The core of these effects is that the level of a decision variable one agent chooses (for instance maintenance) will affect the utility of this agent, and the utility of neighboring agents.

A general hedonic price model, incorporating spatial lags is formulated as:

\[ P = \rho WP + X\beta + u \]

In the expression above, \( P \) is an \( n \times 1 \) vector of observations on prices, \( X \) is an \( n \times k \) matrix of observations on independent variables, \( \beta \) is \( k \times 1 \) vector of regression coefficients, \( u \) is an \( n \times 1 \) vector of independent and identically distributed random error terms and \( W \) is the \( n \times n \) exogenous spatial weights matrix that specifies the assumed spatial structure or connections between the observations. Observations that do not interact are represented by zero. Observations assumed to interact are represented by non-zero values. \( WP \) is the spatially
lagged dependent variable included to account for various spatially related dependencies as described above. The parameter $\rho$ is often referred to as the spatial correlation or spatial dependence parameter (Wall, 2004). If $\rho = 0$, (1) is a standard linear regression model. In this way $\rho$ gives the intensity of the dependence between neighboring prices. If the assumed spatial structure is not representative for the existing spatial dependence, the estimated value of $\rho$ will be low or insignificant. Solving (1) for $P$ gives:

\[
P = (I - \rho W)^{-1} X \beta + (I - \rho W)^{-1} u
\]

$I$ is the $n \times n$ identity matrix. In (2) the two inverse expressions are called spatial multipliers (Anselin, 2003). Note that the value of $P$ at one location $i$ depends on the value of $X$ at location $i$ and the value at all other locations in this model.

For $|\rho| < 1$ Kim et al. (2003) show that under a set of specific assumptions, the implicit price of attribute $x_k$ in $X$ will be the matrix $\beta_k[I-\rho W]^{-1}$. $\beta_k$ is the marginal implicit price of $x_k$ in a traditional linear hedonic price function. Thus the marginal effect on house prices consists of both a direct effect due to the change in the amount of the attribute on one house, in addition to induced effects due to marginal changes related to other house prices. These induced effects arise because the price of houses depends on the prices of the neighbors. See also LeSage and Pace (2009) for a thorough discussion of the interpretation of parameter estimates in spatial models.

When spatial autocorrelation in the dependent variable is present but not modelled, ordinary least squares (OLS) is biased and inconsistent (Anselin, 1988). Intuitively, this bias is due to dependence of the error term and all the included variables. In contrast to the time-series case, the inconsistency is due to the multidirectional dependency in the data. This represents an important difference between spatial autocorrelation and autocorrelation related to time series. Since the autoregressive parameter must be estimated simultaneously with other parameters in (2), Maximum Likelihood (ML) is frequently used. This is also the estimator used here.

### 2.2 The Spatial Autoregressive Error Model
If the problem is spatial autocorrelation in residuals, a spatial autoregressive model specification may be used to improve the precision of the estimated parameters. The main idea is to include the a priori most important spatial variables in the model, and leave the more subtle spatial features to the residuals. This is done by specifying a global spatial autoregressive process in the error. A general version of this model can be formulated as (Anselin, 2003):

\[ P = X\beta + \varepsilon \]  
\[ \varepsilon = \lambda W \varepsilon + u \]

\( W \) is the weight matrix, and \( \lambda \) is a spatial autoregressive parameter to be estimated jointly with the regression coefficients. The two vectors of errors are assumed to be uncorrelated and \( u \sim N(0, \sigma^2 I) \). (4) can be solved for \( \varepsilon \), and (3) may be expressed as:

\[ P = X\beta + (I - \lambda W)^{-1} u \]

The implicit price of attribute \( k \) in the linear hedonic model will be equal to \( \beta_k \). Expression (5) shows that the value of the dependent variable for each location is affected by the stochastic errors at all locations through the spatial multiplier \( [I - \lambda W]^{-1} \). The smaller the value of \( |\lambda| \), the smaller the effect of the multiplier. The variance-covariance matrix for the vector \( \varepsilon \) is given by (Anselin, 2003):

\[ \text{Var}[\varepsilon\varepsilon'] = \sigma^2 \left( [I - \lambda W]^{-1} \right) \left( [I - \lambda W]^{-1} \right)' \]

This variance-covariance matrix will usually be heteroskedastic. This problem may be handled by using, for instance, “White-adjusted“ standard errors, which are robust to heteroskedasticity (Florax and De Graaf, 2004).

If the estimated value of \( \beta \) in (5) is strongly affected, compared to an ordinary linear regression model, this may be an indication of the fact that this model may not be the correct one, since spatial autocorrelation in residuals could be a symptom of omitted variables. The
model is therefore not correct in all cases where spatially autocorrelated residuals are detected. If the spatial autoregressive error model is the true model, some minor exogenous and unexplained spatial interaction processes exist. Examples are minor misspecifications, wrong spatial delineations of geographical variables, or omitted less important neighborhood characteristics, the effect of which should diminish with increased distance from a given location. Andrews (2005) gives the spatially correlated errors an economic interpretation by way of common shocks. Examples of shocks in this relation are changes in local infrastructure or renovation of certain areas.

Other spatial processes in the residuals than (4) may be specified. A semiparametric approach could be used as described in Clapp et al. (2002) or the spatial moving average model, which has a local spatial covariance structure (see Anselin, 2003 and Florax and De Graaff, 2004). Instead of specifying a weight matrix, Dubin et al. (1999) estimate the spatial dependence in residuals directly by the interpolation technique kriging. Chica-Olmo (2007) uses kriging and cokriging as an alternative spatial regression model which can be used when observations include both observed and missing values.

2.3 The Spatial Durbin Model

One model that has been developed a long time ago, and has a time-series equivalent, is the spatial Durbin model (7). The model could be developed from the spatial error model (Anselin, 2006) or from a spatial lag model (Bivand, 1984). This is done by additional constraints on the parameters. The spatial Durbin model is specified as:

\[ P = \rho WP + X\beta_0 + \rho WX\beta_1 + \varepsilon \]  

This model includes a spatial lagging of the dependent variable, in addition to a spatial lagging of all the independent variables. The spatial lagging of the dependent variable is included in order to capture effects as described for the spatial lag model. The spatial lagging of the explanatory variables is added so that the characteristics of neighboring houses could have an influence on the price of each house in the sample (Brasington and Hite, 2005). In this way the spatial Durbin model allows both for neighbouring prices to determine the price of a house, in addition to the characteristics of neighbouring houses.
The common factor constraints are formulated as: $\beta_1 = -\rho \beta_0$. Given the constraints, (7) is a variant of the spatial error model. The approach amounts to testing whether the spatial error model is the correct model. If the test cannot reject the null hypothesis, this may be interpreted so that the important spatial variables are included in the model. What is left in the residuals is mainly a nuisance. The spatial Durbin model and the common factor constraint tests are commented upon further when we test for spatial effects.

There exist other modeling strategies than the approach followed in this paper. One possibility is to start with the more comprehensive spatial Durbin model, and the common factor constraints test. Since the spatial Durbin model nests the relevant model variants, the LM tests are less appropriate when following the alternative approach. The strategy followed in this paper where we start with a more parsimonious model is in line with recommendations found in Florax et al. (2003).

2.4 Spatial Heterogeneity

Another assumption when using OLS is the constant variance of residuals. Spatial heterogeneity violates this assumption. This may occur due to structural instability of parameters across space, modelled functional forms that are not spatially representative, or missing variables (Anselin, 1988). One commonly used method to account for broader spatial trends is the spatial expansion method (Casetti, 1972). Local parameter instability could be estimated by GWR or even OLS. These two last-mentioned methods will be used here. GWR can be used when the spatial heterogeneity is assumed to be continuous, and OLS when the instability is assumed to be more discrete.

One example of spatial heterogeneity is house prices falling with increased distance to a central business district (cbd). If the implicit price of distance to the cbd does not vary with the direction from the cbd, the relationship is isotropic, and global parameter(s) may be used in the hedonic model. If the relationship varies with the direction from the cbd, it is anisotropic, giving for instance more or less steep slopes of cbd-gradients, depending on directions from cbd. In this case the hedonic relationship is not constant over space; it is heterogeneous. Other examples are differences in implicit prices in urban and rural areas, inner-city and more suburban areas, or in a main-land versus an island. Can (1992) calls these features neighborhood effects.
One way of looking at spatially varying parameters in relation to hedonic house price models has been through the concept of submarkets. An overview of relevant studies is found in Watkins (2001), Goodman and Thibodeau (2003), and Jones et al. (2004). In this paper it is only the potential for spatial heterogeneity in parameters that is to be considered. For spatial arbitrage to occur, houses in different geographical areas must be considered as appropriate substitutes for each other (Jones, 2002). Search costs and information constraints may impose limits to the degree of spatial substitutability (Jones et al., 2004). These constraints may be incurred by physical barriers between markets. If persistent spatial submarkets exist, there are behavioral structural differences between local markets, which are reflected by the spatial heterogeneity of implicit prices. It is not sufficient to define submarkets as the existence of discrete spatial groupings. According to Straszheim (1974), the structure of demand (due to differences in demography, preferences, or income), supply (due to inelastic supply), or both must be different. If the structure of demand and supply is the same, and no barriers to mobility exist, differences in implicit prices will be arbitrag ed away.

Submarkets may exist in other dimensions than geography. According to Rothenberg et al. (1991), substitutability across all housing characteristics should be considered. The existence of submarkets is hence a broader theme than spatial heterogeneous submarkets. As will be explained below, this analysis focuses on only one housing type. It is therefore argued that it is less likely that the sample consists of any other submarkets, than the potential for spatially related submarkets. One should also be aware of the possibility of finding significant spatial variation in implicit prices that could not be explained by the existence of submarkets. This may only exist as an artefact of the data.

2.5 Preparing the Ground for the Spatial Econometric Analysis

The classical starting point when searching for spatial characteristics of a housing market is to test for spatial effects in the residuals of a given model. Frequently the initial tests convey that some OLS-assumptions are not satisfied. What causes these results is difficult to convey. The test-results may be symptoms of a misspecified model. It may be an open question of how to proceed with the modelling. In a simulation study, McMillen (2003) shows that what appears as spatial autocorrelation in housing prices is caused by a missing explanatory variable such as a subcenter. Consequently, even when spatial autocorrelation in the dependent variable seems to be detected, this demands further investigation. The problem of misspecification is
particularly severe in spatial modelling since theory provides little guidance, and spatial relationships are abundant and often highly nonlinear (McMillen, 2003). In line with this, Fotheringham et al. (2002) state that when looking for spatial variation in estimated parameters, one is almost certain to find it. It is hence necessary to state a priori reasons for the existence of spatial effects in order to find relationships that vary intrinsically across space.

In summary a variety of identification problems related to spatial econometric modelling exist, and spurious interpretations of test results may occur due to various spatially related misspecifications. The identification problem could be particularly important when studying larger regional markets, compared to analyses where a single urban area is studied. In the last mentioned case, the problem of omitted spatially related variables could be smaller, since a single urban area may be more homogenous compared to a regional housing market, which is studied here. For these reasons, prior to final conclusions, hedonic housing price models should be tested and scrutinized from different angles.

3 The Study Area, Base Model and A Priori Expectations

3.1 Description of the Region and the Data.

The study area consists of eight municipalities in the south-western part of Norway (see Exhibit 1). The area is mainly delimited by natural boundaries, and travelling distances are to some degree increased by fjords, islands, and mountains. This is mostly evident for Karmøy, a relatively large island, connected to the main land by one single bridge. The population density is highest in the main city, Haugesund. Travelling distance by car from south to north is approximately 82 minutes, and from west to east approximately 87 minutes. Based on commuting patterns, the internal relationship is far more intense than relationships to regions outside the area. The labor and housing market is therefore relatively self-contained. This is in line with Jones (2002), who argues that housing markets should be defined on criteria linked to spatial labor markets and travel-to-work areas.

(Exhibit 1 about here)

The hedonic models to be estimated are based on 1691 observations of privately-owned single family houses, sold in 1997-2002. The reason for using single family houses is that outside the main city, single family houses are by far the only type of house available. It is also likely that house types such as single-family dwellings, houses in a row, and semi-detached houses
are substitutes. The results could then be fairly representative for the private housing market in the region. Many of the blocks of flats are organized in cooperatives. These apartments could make up a different market segment, for instance due to the sharing of loans, insurance, etc. This way of arguing is in line with, for instance, Adair et al. (1996) who find that submarkets could be defined on the basis of the type of property. Watkins (2001), however, mentions that one overemphasizes the significance of structural attributes if one assumes that people limit their choices to specific property types, irrespective of location. So the results in the research literature on this matter vary.

The data come from different sources. Information on housing sales prices, postal codes, coordinates, site in square metres, and type of building is collected from the national land register in Norway. Information on internal living space, garages, age of building, number of toilets, time of sale, sales price, and number of jobs is collected from Statistics Norway. The number of jobs stems from 2004. The total number of jobs in each municipality is reliable. The more disaggregated information on the number of jobs in each postal code, inside each municipality, is more imprecise. Due to a mismatch between the number of postal codes in the distance matrix and the matrix of information on jobs, some jobs have also been relocated to the closest existing postal code. The travelling time between postal codes has been set by the Norwegian Mapping Authority. See Osland et al. (2007) for a more detailed account of this. The location of every house is identified by a postal code and coordinates. The coordinates are Universal Transverse Mercator projection coordinates in the Euref89/VGS84 system.²

(Exhibit 2 and 3 about here)

3.2 The Base Model
The starting point of the empirical analysis is a parsimonious model based on previous research using data from another Norwegian region (see Osland and Thorsen, 2008). At the outset the following hedonic model is estimated:

² A description of this system can be found at: http://www.fmnh.helsinki.fi/english/botany/afe/map/utm.htm [read: 23.08.2006]
\[
\log P_i = \beta_0 + \beta_1 \log \text{LOTSIZE}_i + \beta_2 (\text{RUR} \cdot \log \text{LOT}_i) + \beta_3 \log \text{AGE}_i + \beta_4 \log \text{sqAGE}_i \\
+ \beta_5 (\text{REBUILD} \cdot \log \text{AGE}_i) + \beta_6 \text{GARAGE}_i + \beta_7 \log \text{LIVAREA}_i + \beta_8 \log \text{TOILETS}_i \\
+ \beta_9 \log \text{MINCBD}_i + \beta_{10} \log \text{ACCESS}_i + \sum_{r=1997}^{2004} \beta_r \text{YEARDUM}_{ir} + \varepsilon_i
\]

Where \(\log(x)\) denotes the natural logarithm of \(x\), \(x > 0\). Overall, the included variables are either related to the house itself (lot size, age, size, number of toilets, and whether it has been rebuilt), its relative location measured by distance to the most central postal delivery zone in Haugesund (cbd), and a potential variable measured by access to workplaces in the area. The definitions of the variables in (8) are given in Appendix 1. \(\text{MINCBD}\) measures driving distance in minutes from the cbd. The interpretation of the implicit price related to this variable is that of urban attraction (Osland and Thorsen, 2008). The variable access to work (\(\text{ACCESS}\)) is a potential variable that is used to account for spatial variation in labor market accessibility and polycentric tendencies in the area. The variable \(\text{RUR}\cdot\text{LOT}\) is included since the implicit price of lots is segmented and varies between rural and non-rural areas. It is assumed that the hedonic function is constant through time, except for parallel shifts due to the time-dummy variables.

### 3.3 A Priori Expectations

Spatial effects are expected to be present in housing price studies due to a range of misspecifications (see Anselin 2003). What misspecifications may occur in the base model (8)? Osland and Thorsen (2006) found that local labor market accessibility may exert a significant effect on house prices in addition to the global accessibility measure in (8). Local labor market subcenters should therefore be searched for. Some segmentation in relation to the island of Karmøy could be found. Spatial barriers could reduce the possibilities of spatial interaction on the island compared to the situation for houses on the mainland. Except for this, the overall assumption is that the base model (8) will perform well in the study-area. This assumption is also based on the fact that the area is different from metropolitan areas that form the basis of many North American studies. In these studies, spatial variations in property values also reflect for instance local tax rates and the quality of local public services. In the study period tax rates are uniformly distributed over the area, crime rate is relatively low, and systematic variation in the quality of primary and middle schools can be ignored. All this forms the basis of using a parsimonious model (8) as a starting point for the spatial econometric analysis.
4 Testing for Spatial Effects

4.1 The Moran’s I Test

When testing for spatial effects, a common first step is the Moran’s I test for residuals:

\[ I = \frac{N}{S} \left[ \frac{\hat{e}'W\hat{e}}{\hat{e}'\hat{e}} \right] \]

In this expression, \( \hat{e} \) denotes a vector of residuals, \( W \) is a exogenous spatial weight matrix, \( N \) is number of observations, and \( S \) is a standardization factor defined as the sum of all elements in the given weight matrix (Anselin, 1988). Three choices have to be made when testing for spatial effects (Bivand, 2006):

- What definition of spatial connectivity should be used in the weight matrix?
- Which weights styles should be used?
- Which methods should one use when estimating the Moran’s I test statistics?

There is no clear cut advice regarding the specification of the neighboring structure and types of weights (Dubin, 1998). Some guidance is, however, found in Griffith (1996). Two kinds of neighboring structures are used here. The first is based on postal delivery zones. All houses inside a postal zone are neighbors. A house is not a neighbor to itself. Postal zones are neighbors if they share a common border, given the existence of a road-transportation network between the zones. On average, each postal zone is linked to three other zones, and the average number of neighbors per house is 212. This neighboring structure is relevant if one does not have more detailed information on the location of houses.

The second type is based on the exact location of each house and uses coordinates. A common way of creating weight matrix in this case is by distance bands. To ensure that all observations have at least one neighbor, a threshold distance is chosen. Since some of the settlements are scattered in the study-area, this definition of a neighbor will include observations within a distance of 14.5 km, which is too large in this case. An alternative is the \( k \)-nearest neighbor. The \( k \)-nearest neighbor is chosen on the basis of metric distances, and distances between neighbors are hence allowed to vary. The weights are asymmetric in the sense that house A may be a neighbour to house B, but the reverse need not be true. For each observation will
have at least one neighbor. This type of weight is explored in Dubin (1998). Note that using only one neighbor could be relevant in this market. Houses are not frequently sold, and there will not be an abundance of houses sold in the immediate neighborhoods of each house in the sample in the study period.

Secondly, a weight style has to be chosen. The effects of weight styles are to give different weights to the specified neighborhood structure (see Tiefelsdorf et al., 1999). The effect of converting the neighborhood matrix to a weights matrix by way of weight styles is to quantify the degree of linkage between zones. An overview of the three different weight styles that could be used is given in Tiefelsdorf et al. (1999). In this case we report the results when using the row-standardized style, since this is the style that is most commonly used in econometrics. Using this style implies that each cell is divided by its row sum, so that more weight is given to observations with relatively fewer neighbors, and vice versa. A more neutral style is given by using a plain binary matrix.

The most common method of estimating the Moran’s I is by the normality approach. An alternative is the saddle point approximation to the underlying distribution (Tiefelsdorf, 2000). The normality approach is used if the underlying variable, the residuals in this case, follow a normal distribution or if number of observations is large, and this is chosen here.

The results from the Moran’s I tests are shown in Exhibit 4. All tests statistics give the same conclusions: the null hypothesis of no spatial effects has to be rejected. These results are qualitatively insensitive to the neighborhood structure, weights styles, and the method used for estimating the Moran’s I. This is in line with Kim et al. (2003).

(Exhibit 4 about here)

4.2 The Lagrange Multiplier Tests

When the Moran’s I test rejects the null hypothesis of no spatial effects, Lagrange Multiplier (LM) tests are frequently used. An overview of these tests and the order in which these tests could be conducted is found in Exhibit 5. As shown in this Exhibit, there are two main variants of these LM-tests. The LM-lag statistic tests the null hypothesis of no spatial autocorrelation in the dependent variable. The LM-error statistic tests the null hypothesis of no significant spatial error autocorrelation. If both hypotheses are rejected, one considers
which value of the test statistics is largest (Florax and de Graaf, 2004). As an alternative or supplement to this, the robust tests (RLM) can be used (see Florax and Nijkamp, 2003). The test procedures in this case are identical to the one described above. The important difference is that the RLM-error test corrects for the presence of local spatial lag dependence. The LM-error test assumes the absence of this kind of autocorrelation. Similarly, the RLM-lag statistic tests the null hypothesis that $\rho$ is zero, correcting for presence of local spatial error dependence. The LM-test is asymptotically distributed as $\chi^2(1)$.

(Exhibit 5 about here)

The results in Exhibit 4 indicate the existence of spatial autocorrelation in the residuals in almost all cases. When $k = 1$, the tests indicate the existence of spatial autocorrelation in the dependent variable. A spatial lag model has hence been estimated using $k = 1$ weights. Spatial autoregressive error models have been estimated for relatively higher numbers of $k$, in addition to weights based on postal codes. When choosing among these spatial autoregressive error models, the Akaike’s information criterion (AIC) can for instance be used. Only the model with the lowest value on the AIC is hence reported.

In both types of spatial models, the spatial lag parameters are significant and positive (see Exhibit 6). The result from the lag model indicates that spatial autocorrelation in house prices exist for houses located very close to each other. It is difficult to know the reason for this dependency, especially since the value of $\rho$ is small. As mentioned earlier, missing variables such as a subcenter could give a positive and significant value of $\rho$.

(Exhibit 6 about here)

The spatial lag and the spatial error models are nested with the ordinary linear regression model. Both spatial models perform better than the linear regression model. The value of the AIC that uses the likelihood ratio may be used as a descriptive measure of the overall model performance when judging non-nested models (Akaike, 1974). The model with the lowest value of the AIC is preferred. In addition to being a measure of fit, the AIC takes model complexity into account. This is useful since parsimonious models often are to be preferred. The AIC is lower in the error model, compared to the lag model.
Another question to consider is how reliable are the tests for spatial effects. The studentized Breusch-Pagan tests reported in Exhibit 6 show that the problem of heteroskedasticity is present. The tests for spatial effects are, however, robust to heteroskedasticity, as long as the heteroskedasticity is spatially uncorrelated (Kelejian and Robinson, 2004). If the heteroskedasticity is related to, for instance, the age variable, and this variable is negatively spatially correlated, one would be less likely to reject the null hypothesis of no spatial effects, when the opposite is true. In a simulation study, Anselin and Rey (1991) show that the LM-error statistic over-rejects the null hypothesis in the presence of heteroskedasticity, whereas the LM-lag statistic is more robust to heteroskedasticity. The results from the LM-lag tests are hence most reliable. The tests mentioned are mainly unaffected by the existence of multicollinearity (Lauridsen and Mur, 2006). The power of the tests is reduced by outliers (Mur and Lauridsen, 2007). This result is particularly relevant in small samples, and the loss of power is most significant in relation to spatial heterogeneity. When accounting for outliers in our data, none of the conclusions are altered. The tests for spatial effects are therefore considered to be reliable in this respect.

4.3 Common Factor Hypothesis Tests
When there is evidence of maintaining the spatial lag model, the spatial Durbin model (7) and the spatial common factor tests may be useful. This approach enables a nesting of the spatial lag and the spatial error model. Given the common factor constraints a spatial autoregressive error model may be formulated so that it contains both the spatially lagged dependent variable and spatially lagged exogenous variables. If the constraints do not hold, there could be an inherent substantial dynamic process in the data. Rejecting the null hypothesis of common factors implies rejecting the spatial error model.

The common factor tests are described in Mur and Angulo (2006): The power of the tests increases substantially with high $R^2$ and with increased sample size. Both $R^2$ and sample size are relatively large in this case. The impact of using a different specification of weights is minor. The results from estimating the spatial Durbin model are found in Exhibit 6, column 5 and 6. Two lagged attribute variables are significant, and it should be noted that lagged lot size gets a relative large negative and significant parameter. Note, however, that in line with the spatial lag model, the individual estimated parameters in the spatial Durbin model do not
equal the marginal effects. This is shown in LeSage and Fischer (2007) and explained more intuitively in Brasington and Hite (2005).

Testing the common factor constraints by a likelihood ratio test using $k=9$, gives a $p$-value of 0.0065. The spatial error model is thus not rejected at the 1% significance level. When using weights so that $k = 1$, the $p$-value is 0.002. The spatial error model is rejected in this case.

### 4.4 Checking the Robustness of Test Results Using a Spatially Denser Sample

So far we have used a regional data set, which implies that the observations are spatially sparse. This is particularly the case for the area outside the municipality of Haugesund. When using denser data, observations that are located closer will be defined as neighbors, and the average variation in for instance house prices or neighborhood characteristics could be smaller. To see whether this has any effect on the test results we use data from the municipality of Haugesund as a robustness check. This implies using 766 observations on 72 km$^2$ so that each observation has 0.1 km$^2$ on average.

Results from some selected LM-tests using this sample are found in Exhibit 4. The results from the previous analysis are confirmed, although the evidence is somewhat stronger in favor of the spatial error model. The lag model is slightly preferred when we use weights so that $k = 1$. The error model is the preferred model when $k$ is higher than 1. Using $k = 8$ gives the lowest AIC for the error model in this case. The common factor constraint test shows that the spatial error model cannot be rejected at the 1% level. This result applies to weights $k = 1$ ($p$-value is 0.02) and when using $k = 8$ ($p$-value is 0.01) in the weights. To be even more certain about which model is correct and to check for misspecifications, there is therefore a need to search for the existence of a subcenter or spatially varying coefficients.

### 5 Further Investigation

#### 5.1 Geographically Weighted Regression (GWR)

Spatial heterogeneity in parameters may be assumed to exist continuously or discontinuously. One common criticism against applying a discontinuous demarcation of the geography is that the study area is sometimes arbitrarily delineated. Various methods have been used to avoid this problem, see for instance Pace and LeSage (2004) and Casetti (1972). In order to avoid this problem we will use GWR to test for spatial parameter heterogeneity. A relevant urban
application of this method is McMillen (1996). GWR is also explained in Fotheringham et al. (2002) by way of hedonic house price modelling, and may be described as a kernel-weighted regression that gives a non-parametric estimate of the regression coefficients:

\[
p_i = \beta_0(u_i, v_i) + \sum_{k=1}^{m} \beta_k(u_i, v_i) x_{ik} + \epsilon_i,
\]

\(\beta_0\) is the intercept, \(u_i\) and \(v_i\) are coordinates, \(x_{ik}\) is the value of the explanatory variable \(k\), and \(\beta_{ik}\) is the coefficient related to variable \(k\), and \(i = 1, \ldots, n\). Using coordinates implies that the model allows for spatially smooth variation in the values of the estimated parameters. The estimator of the parameter vector for regression point \(i\) is:

\[
\beta(u_i, v_i) = (X^T W(u_i, v_i) X)^{-1} X^T W(u_i, v_i) P
\]

The weight function \(W(u_i, v_i)\) is a \(n \times n\) matrix, with off-diagonal elements equal to zero, and diagonal elements given by \(n\) geographical weights, one for each observation. According to Brunsdon et al. (1998) GWR is relatively insensitive to the choice of weight function. The weight function should, however, make the effect of surrounding observations decrease with increased distance from a given observation. Here, the commonly used Gaussian weights will be used:

\[
w_{ij} = \exp\left(-\left(d_{ij} / b\right)^2\right),
\]

d_{ij} denotes Euclidian distance between observation \(i\) and \(j\). A critical element is the choice of bandwidth \(b\) for the given sample, and there is a trade-off between the bias and the variance inherent in the GWR-estimation. Choosing a low bandwidth reduces the bias, but increases the variance since the sample size around each estimated coefficient will be low, which enables the estimator to follow the data more closely. A larger bandwidth gives smoother results. The bandwidth may either be a fixed global bandwidth, or it may be adapted so that it varies depending on how evenly spread the observations are. A smaller bandwidth will hence be used in urban areas and a larger bandwidth in rural areas. The expense is reducing the
degrees of freedom. In both the fixed and adaptive case, bandwidth is chosen so that a cross-validation function is minimized.

In Exhibit 6 the results are shown when using a global bandwidth. The selected global bandwidth of 27 km is large, and is somewhat less than half the distance between the two observations located farthest apart. Consequently, there will be small differences between the estimated global model and the GWR-model. Local variation in coefficients may hence be masked. Since the estimated results are conditional on the choice of bandwidth, the adaptive bandwidth is also used. In this case the weighting function is adapted to the varying density of the spatial distribution of observations. This is done by using a fixed number \( k \) of nearest neighbors around each observation. The number of neighbors is chosen by minimizing a cross-validation function. The effective number of parameters related to the two models is 31 and 73, respectively. See the results in Exhibit 6.

GWR-models may be tested against the linear regression model (see Leung et al., 2000, and Brunsdon et al., 1999). The test found in the last mentioned paper is called BFC. The tests described in Leung et al. (2000) are called F(1) and F(2). An overview of these tests and their hypotheses is found in Exhibit 7. Test procedures are similar to the comparison of two linear regression models. The F-tests could be uncertain in the presence of heteroskedasticity.

(Exhibit 7 about here)

The test F(2) tests the null hypothesis that GWR and OLS describe the data equally well. This hypothesis has to be rejected. The result from F(1) supports the null hypothesis of no significant improvement in fit when using the GWR model compared to the linear regression model. This is also indicated by the fact that the ratio of residual sums of squares of the GWR model and the base model is close to 1. BFC tests the null hypothesis that all the estimated parameters are constant throughout the area. This hypothesis is rejected.

Finally, a null hypothesis of no significant variation in each set of parameters \( \beta_{i} \), \( i = 1, \ldots, n \) related to each explanatory variable against the alternative hypothesis that not all parameters equal is tested (see Leung et al., 2000). The results are shown in Exhibit 6 as (**). When including and excluding different variables in the model, the test shows that depending on the
variables included, most variables vary significantly in one occasion or the other. Changing the order of the variables also alters the estimated results. Therefore the results presented in Exhibit 6 depend upon the ordering of the variables. Brunsdon et al. (1999) discuss meaningful ways of including variables in this respect. Wheeler and Tiefelsdorf (2005) argue that one should practice caution in substantively interpreting spatial variation in GWR-coefficients. They show that even moderate to strong correlation between explanatory variables make the associated local coefficients nearly completely interdependent. Additionally, spatially autocorrelated errors can produce artificially strong dependence between the GWR-coefficients.

Overall the results indicate that GWR does not outperform OLS. All estimated parameters are not, however, constant throughout the area. This implies evidence of spatial heterogeneity. Which of these parameters that vary is not clear.

5.2 Semiparametric Approach

According to McMillen (2003) misspecified functional form is probably the most likely reason for creating spurious spatial autocorrelation. The flexible semiparametric approach may hence be useful as an exploratory tool when studying the existence of non-linearity in the parameters. Hedonic studies using similar approaches are found in Pace (1998), and Bao and Wan (2004). This semiparametric estimation procedure uses a variant of generalized additive models (Hastie and Tibshirani, 1990) and iterative penalized regression smoothing splines. The method is described in Wood (2006). The idea of the method is to estimate the following general model:

\[ P = X\beta + s(z) + \varepsilon \]

\(X\) refers to the matrix of exogenous variables, \(\beta\) is a vector of estimated parameters, \(\varepsilon\) are i.i.d residuals, \(s()\) is the unknown smooth function, and \(z\) is a variable vector, not included in \(X\). \(\beta\) and \(s()\) are estimated by a least square procedure, see Wood (2006) for details. As mentioned above, by including individual variables whose structure is not known a priori, one may study the existence of nonlinearities. The existence of a spatial trend or local variation in house prices could also be studied by including the coordinates in \(s()\). This approach is
chosen here for the purpose of illustration. See also a useful discussion in Beron et al. (2004) who accounts for spatial trends by a parametric approach.

A plot of the partial variation in house prices when moving from south to north is shown in Exhibit 8. The solid line represents the variation around the mean predicted value of the dependent variable. What causes this variation is not identified, and it may be interpreted as a spatially varying intercept. The dashed lines represent approximately 95% confidence levels around the fitted values. The number on the y-axis denotes efficient degrees of freedom of the plotted term, and the equivalent of 11.9 parameters is used in estimating the smooth. A precise definition of efficient degrees of freedom is found in Wood (2006, pages 170-172). If the confidence band includes 0 everywhere this is a sign of the graphed effects not being significant. To some extent this is the case here. The degrees of freedom are also relatively large, so there is a tendency of over fit. The shape of the solid curves in Exhibit 8 is robust to the exclusion of variables from the model. This result is in line with results in Bao and Wan (2004), and represents an advantage of this method compared to OLS, where multicollinearity makes it difficult to identify nonlinearities.

(Exhibit 8 about here)

The analysis shows that there exists some unexplained spatially related variation in the data when moving in the north-south direction, using y coordinates. In Exhibit 8 the island Karmøy is located in the south, with lowest values on the coordinates. There could be a structural break in the data, furthest south, in Karmøy where predicted house prices seems to be higher compared to the situation in the north. This is in line with a priori reasoning. Haugesund is located centrally. The peak in the north represents a local center for municipalities in the most rural area (Ølen). This is also in line with a priori expectations.

6 Improving the Base Model

The results from the semiparametric approach indicate a subcenter in Ølen, which is a center for the most rural municipalities in the area. A dummy variable (SUBCENTERdum) for houses sited in the most centrally located postal codes in Ølen is therefore included. The reason for not using distance from the subcenter is that the postal codes cover quite large areas in the countryside. This makes it difficult to pinpoint how far the subcenter exerts an influence on house prices. Due to a prior reasoning there could be some spatial heterogeneity
on the island of Karmøy. Distance to the mainland is hence multiplied by a dummy variable given the value of 1 when observations are located on this island, else 0. The variable is called $MINCBDisland$. This modelling approach is also in line with the result from the GWR-analysis, although this method did not enable a specific identification of the heterogeneity.

In the final model (Exhibit 6) all the estimated parameters have the expected sign and are significant. The null hypothesis of no spatial effects in the residuals still has to be rejected. The LM-tests show that the problem of spatial effects has been reduced, but none of the previous conclusions has been altered. The corresponding spatial lag and spatial error model for the final model is not reported, since the results are similar to the base model. The values of the spatial parameters for these models are, however, reported in Exhibit 4.

What, then, is our final conclusion? The result of this analysis indicates that mixtures of spatial autocorrelation and spatial heterogeneity are present. There are some autocorrelation in house prices between immediate neighbors. This autocorrelation is not discerned from the tests when including neighbors at a greater distance in the weights. The majority of the LM-tests, however, point to spatial autocorrelation in the residual being the problem. According to LeSage and Pace (2009), divergence between the estimated coefficients in the spatial error model and the OLS-model may arise from misspecifications. In these cases using for instance a spatial lag model may be more appropriate. Our results show that OLS and the spatial error model give similar results on the estimated parameters. A Hausman test could be used to test whether the estimates in these two models are not significantly different (LeSage and Pace, 2009). This test is currently not available in the software used in this paper and is hence not performed.

7 Conclusions

This analysis has shown that the spatial model alternatives have higher explanatory power compared to the initial base model. All the estimated parameters have the expected sign. The parameters and their standard errors are also surprisingly stable, even when estimating the spatial error or the spatial lag model. In this way the results from the ordinary least squares model seem to be very robust. The economic interpretation of the estimated parameters varies depending on which spatial model is the correct one, and the research literature has shown that various misspecifications can give spurious results on the LM-tests. The choice of model is therefore an essential question which should be thoroughly scrutinized.
It is important to experiment with different neighbourhood structures when performing the various LM-tests; since different specifications may give different results. If the specified neighborhood structure is not representative, various spatial features in the data will not be discerned. This problem was not encountered when using the Moran’s I test.

Given the detection of spatial effects in the base model, and given the fact that the spatial lag model could not be completely ruled out as the correct one, GWR and semiparametric analysis, the two traditional spatial econometrics models and the mixed spatial Durbin model have been used in the modelling process. The strength of the common factor hypothesis test is that it nests the spatial lag and the spatial error model. The semiparametric analysis is useful as an exploratory tool, due to its flexibility and robustness to multicollinearity. This simplifies the identification of for instance nonlinearities, submarkets and local subcenters which may be difficult to detect by OLS. GWR seems to be extremely sensitive to multicollinearity. The F-tests however, gave useful information regarding existence of spatial heterogeneity in general, and showed that in this case using GWR did not outperform an ordinary linear regression model. How far one should extend the analysis presented in Exhibit 5 depends on the results from the tests. The initial analysis may show that the spatial autoregressive error model is the correct model. In that case one would stop the analysis at this stage.

When looking for spatial effects, one is almost certain to find some. The difficult part of the analysis is to model the economic important ones. This is achieved in this paper, albeit through a very comprehensive analysis. Among other things the analysis identified a subcenter and a submarket related to the distance to cbd-variable. This was due to a spatial barrier. Estimating the lag model gave a significant but small value of the spatial dependence parameter $\rho$ for one type of neighborhood structure. This indicates that spatial dependence or the adjacency effect to some extent play a role in determining the prices. The dominating feature in the data is, however, spill over effects in the residuals. This may be accounted for by using the spatial error model.

References


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I am grateful to Roger Bivand, Espen Bratberg, Viggo Nordvik, Sylvia Encheva and Jørgen Lauridsen for helpful suggestions during the work on this paper. The anonymous referees and Professor Ko Wang have also provided very useful comments. I am responsible for any remaining errors.
Appendix 1: Definitions of variables in the base model.

\( P \): Dependent variable. Selling price in NOK, deflated by the consumer price index; base year is 1998.

\( LOTSIZE \): Lot size measured in square metres.

\( RUR\cdot LOT \): Lot size multiplied by a dummy variable, which takes the value of 1 if the house is located in a rural area, else 0. A rural area is defined on the basis of population density. All municipalities outside Haugesund are rural areas, except for houses located in the mainland area of Karmøy.

\( AGE \): Age of building, measured in years.

\( sqAGE \): Age of building squared.

\( REBUILD\cdot AGE \): A dummy variable indicating if the building has been rebuilt/renovated or not multiplied by the age-variable.

\( GARAGE \): A dummy variable indicating whether the house has a garage or not.

\( LIVAREA \): Size of house measured in square meters.

\( TOILETS \): Number of toilets in the house.

\( MINCBD \): Distance from the center of the largest city, Haugesund. Distance is measured by minutes between postal codes, travelling by car, accounting for speed limits.

\( ACCESS \): Access to work places is measured by \( S_j = \sum_{k=1}^{\infty} D_k^j \exp(\sigma d_{jk}) \). \( D_k \) represents number of jobs in postal zone \( k \), \( d_{jk} \) represents minutes between zone \( j \) and \( k \), travelling by car. \( \sigma \) and \( \gamma \) are parameters to be estimated by Maximum Likelihood estimation using the base model. The estimated values on the two parameters are -1.172 (4.31) and 0.1107 (1.94), respectively, with robust \( z \)-values in parenthesis. The ACCESS variable is computed based on these imputed values.

\( YEARDUM \): A dummy variable indicating the year the house has been sold (1997-2002). The dummy variable for 1998 is excluded in order to avoid perfect multicollinearity.
Exhibit 1: The division of the region into municipalities and zones.
Exhibit 2: Descriptive statistics. Mean values, standard deviation follow in parentheses.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Whole area</th>
<th>Haugesund</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>1,101,467</td>
<td>1,212,030</td>
</tr>
<tr>
<td></td>
<td>(469,809)</td>
<td>(525,935)</td>
</tr>
<tr>
<td>Price pr square</td>
<td>7,747</td>
<td>8,410</td>
</tr>
<tr>
<td>meter</td>
<td>(2,587)</td>
<td>(2,619)</td>
</tr>
<tr>
<td>Lotsize</td>
<td>813</td>
<td>719</td>
</tr>
<tr>
<td></td>
<td>(504)</td>
<td>(409)</td>
</tr>
<tr>
<td>Size of house</td>
<td>146</td>
<td>147</td>
</tr>
<tr>
<td></td>
<td>(52)</td>
<td>(53)</td>
</tr>
<tr>
<td>Number of toilets</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(0.69)</td>
<td>(0.72)</td>
</tr>
<tr>
<td>Minutes to cbd by</td>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>car</td>
<td>(14)</td>
<td>(2)</td>
</tr>
<tr>
<td>Garage</td>
<td>0.63</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>(0.48)</td>
<td>(0.48)</td>
</tr>
<tr>
<td>Age of house</td>
<td>32</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>(31)</td>
<td>(34)</td>
</tr>
<tr>
<td>n</td>
<td>1,691</td>
<td>766</td>
</tr>
</tbody>
</table>
**Exhibit 3:** Descriptive statistics for the municipalities (Statistics Norway, 2001).

<table>
<thead>
<tr>
<th>Municipality</th>
<th>Area (km²)</th>
<th>Population</th>
<th>Working population</th>
<th>Number of people working relative to number of residents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haugesund</td>
<td>72</td>
<td>30,609</td>
<td>16,166</td>
<td>1.25</td>
</tr>
<tr>
<td>Bokn</td>
<td>48</td>
<td>792</td>
<td>248</td>
<td>0.82</td>
</tr>
<tr>
<td>Karmøy</td>
<td>228</td>
<td>37,145</td>
<td>10,920</td>
<td>0.74</td>
</tr>
<tr>
<td>Tysvær</td>
<td>419</td>
<td>8,909</td>
<td>2,743</td>
<td>0.74</td>
</tr>
<tr>
<td>Sveio</td>
<td>247</td>
<td>4,669</td>
<td>829</td>
<td>0.44</td>
</tr>
<tr>
<td>Vindafjord</td>
<td>444</td>
<td>4,834</td>
<td>1,505</td>
<td>0.76</td>
</tr>
<tr>
<td>Etne</td>
<td>708</td>
<td>3,945</td>
<td>1,057</td>
<td>0.72</td>
</tr>
<tr>
<td>Ølen</td>
<td>185</td>
<td>3,313</td>
<td>1,391</td>
<td>1.06</td>
</tr>
<tr>
<td><strong>In all</strong></td>
<td><strong>2,351</strong></td>
<td><strong>94,216</strong></td>
<td><strong>34,859</strong></td>
<td></td>
</tr>
</tbody>
</table>
**Exhibit 4:** Tests for spatial effects, $k =$ number of neighbours in the weights matrix.

<table>
<thead>
<tr>
<th></th>
<th>Base model:</th>
<th>Final model:</th>
<th>Haugesund model:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = 1$</td>
<td>$k = 5$</td>
<td>$k = 9$</td>
</tr>
<tr>
<td><strong>Postal codes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moran’s I</td>
<td>0.15</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Z</td>
<td>5.03</td>
<td>7.44</td>
<td>9.05</td>
</tr>
<tr>
<td>LMerror</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>LMcov</td>
<td>24.08</td>
<td>50.99</td>
<td>74.30</td>
</tr>
<tr>
<td>RLMerror</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>RLMcov</td>
<td>26.93</td>
<td>30.61</td>
<td>22.45</td>
</tr>
<tr>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td><strong>RLMlag</strong></td>
<td>3.02</td>
<td>22.60</td>
<td>51.90</td>
</tr>
<tr>
<td>(0.08)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td><strong>RLMcov</strong></td>
<td>5.88</td>
<td>2.22</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.14)</td>
<td>(0.83)</td>
<td>(0.00)</td>
</tr>
<tr>
<td><strong>$\rho$</strong></td>
<td>0.071</td>
<td>0.070</td>
<td>0.06</td>
</tr>
<tr>
<td>(5.28)</td>
<td>(5.24)</td>
<td>(3.11)</td>
<td></td>
</tr>
<tr>
<td><strong>$\lambda$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.331)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$n$</strong></td>
<td>1,691</td>
<td>1,691</td>
<td>1,691</td>
</tr>
</tbody>
</table>

$I$ is the estimated Moran’s I statistics, $E(I)$ is the mean and $V(I)$ is the variance, given the null hypothesis of no spatial effects. The null hypothesis is rejected if the estimated $Z$ is larger than 1.64. The LM-tests asymptotically follow a $\chi^2$ distribution, with a critical value of 3.84 at the 5% level and 6.63 at the 1% level. $P$-values follow in parentheses. $\rho$ and $\lambda$ refer to parameters for estimated lag- and error models, respectively. $Z$-values follow in parentheses in these cases.
**Exhibit 5: Overview of basic test procedures**

<table>
<thead>
<tr>
<th>Stage</th>
<th>Test</th>
<th>( H_0 )</th>
<th>( H_1 )</th>
<th>Procedures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Moran’s I test</td>
<td>No spatial effects in the residuals.</td>
<td>Spatial effects of some unspecified kind</td>
<td>If ( H_0 ) is rejected, perform the LM tests.</td>
</tr>
<tr>
<td></td>
<td>LM-error test</td>
<td>No spatial autocorrelation (( \lambda=0 )), given the assumption that (( \rho=0 )).</td>
<td>Spatial autocorrelation (( \lambda \neq 0 )).</td>
<td>If ( H_0 ) is rejected, estimate a spatial error model (3).</td>
</tr>
<tr>
<td></td>
<td>LM-lag test</td>
<td>No spatial autocorrelation (( \rho=0 )), given the assumption that (( \lambda=0 )).</td>
<td>Spatial autocorrelation (( \rho \neq 0 )).</td>
<td>If ( H_0 ) is rejected, estimate a spatial lag model (1).</td>
</tr>
<tr>
<td></td>
<td>Robust LM-error test</td>
<td>No spatial autocorrelation (( \lambda=0 )), correcting for presences of local spatial lag dependence.</td>
<td>Spatial autocorrelation (( \lambda \neq 0 )).</td>
<td>If ( H_0 ) is rejected, estimate a spatial error model (3).</td>
</tr>
<tr>
<td></td>
<td>Robust LM-lag test</td>
<td>No spatial autocorrelation (( \rho=0 )), correcting for the presences of local spatial error dependence.</td>
<td>Spatial autocorrelation (( \rho \neq 0 )).</td>
<td>If ( H_0 ) is rejected, estimate a spatial lag model (1).</td>
</tr>
<tr>
<td>2</td>
<td>Common Factor Hypothesis tests</td>
<td>( \beta_1 = -\rho\beta_0 )</td>
<td>( \beta_1 \neq -\rho\beta_0 )</td>
<td>Estimate a spatial Durbin model (7) and a spatial error model (3). Perform common factor hypothesis tests formulated as a likelihood ratio test. If ( H_0 ) is rejected this is evidence in favour of the unrestricted model. If ( H_0 ) is not rejected this is evidence in favour of the spatial error model (3).</td>
</tr>
</tbody>
</table>

* For all the tests used here one must decide on the definition of spatial connectivity and weights styles. The most common weight style in econometrics is the row-standardized style. \( \rho \) and \( \lambda \) in stages 1-3 refer to the parameters in model (1) and (3) respectively. The LM-tests are asymptotically distributed as \( \chi^2(1) \).
### Exhibit 6: Estimation results for variants of the base model and the final model.

<table>
<thead>
<tr>
<th>Variables</th>
<th>OLS Base model</th>
<th>Lag-model (k = 1)</th>
<th>Error-model (k = 9)</th>
<th>Spatial Durbin explanatory variables not lagged. (z)-values</th>
<th>Spatial Durbin explanatory variables lagged. (k = 9) (z)-values</th>
<th>GWR median values, global bandwidth = 27068 m</th>
<th>GWR median values, (k) about 508</th>
<th>OLS Final model (robust (t)-values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\log{\text{LOTSIZE}})</td>
<td>0.1068</td>
<td>0.0972</td>
<td>0.1178</td>
<td>0.1278</td>
<td>0.1046</td>
<td>0.1100***</td>
<td>0.1156***</td>
<td>0.1151</td>
</tr>
<tr>
<td>(\log{\text{REBUILD}})</td>
<td>–0.0173</td>
<td>–0.0166</td>
<td>–0.0208</td>
<td>–0.0238</td>
<td>–0.0214</td>
<td>–0.0185***</td>
<td>–0.0169***</td>
<td>–0.0185</td>
</tr>
<tr>
<td>(\log{\text{AGE}})</td>
<td>–0.0611</td>
<td>–0.0590</td>
<td>–0.0496</td>
<td>–0.0427</td>
<td>–0.0269</td>
<td>–0.0526***</td>
<td>–0.0540***</td>
<td>–0.0600</td>
</tr>
<tr>
<td>(\log{\text{GARAGE}})</td>
<td>–0.0173</td>
<td>–0.0166</td>
<td>–0.0208</td>
<td>–0.0238</td>
<td>–0.0214</td>
<td>–0.0185***</td>
<td>–0.0169***</td>
<td>–0.0185</td>
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<tr>
<td>(\log{\text{RUR}})</td>
<td>–0.0173</td>
<td>–0.0166</td>
<td>–0.0208</td>
<td>–0.0238</td>
<td>–0.0214</td>
<td>–0.0185***</td>
<td>–0.0169***</td>
<td>–0.0185</td>
</tr>
<tr>
<td>(\log{\text{MINCBDisland}})</td>
<td>0.0209</td>
<td>0.0216</td>
<td>0.0219</td>
<td>0.0212</td>
<td>–0.0165</td>
<td>0.0205***</td>
<td>0.0213***</td>
<td>0.0205</td>
</tr>
<tr>
<td>(\log{\text{AGE}})</td>
<td>0.1026</td>
<td>0.0971</td>
<td>0.1008</td>
<td>0.0991</td>
<td>–0.0389</td>
<td>0.0979***</td>
<td>0.0960***</td>
<td>0.1031</td>
</tr>
<tr>
<td>(\log{\text{LIVARE}})</td>
<td>0.4359</td>
<td>0.4277</td>
<td>0.4092</td>
<td>0.4213</td>
<td>0.1121</td>
<td>0.4555</td>
<td>0.4767</td>
<td>0.4348</td>
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<tr>
<td>(\log{\text{TOILETS}})</td>
<td>0.17065</td>
<td>0.1655</td>
<td>0.1711</td>
<td>0.1652</td>
<td>–0.0869</td>
<td>0.1588</td>
<td>0.1504***</td>
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<td>(\log{\text{MINCB}})</td>
<td>–0.0767</td>
<td>–0.0706</td>
<td>–0.0761</td>
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<td>0.0296</td>
<td>–0.0752</td>
<td>–0.0644***</td>
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<tr>
<td>(\log{\text{ACCESS}})</td>
<td>0.2566</td>
<td>0.2512</td>
<td>0.2745</td>
<td>0.1868</td>
<td>–0.0267</td>
<td>0.2558</td>
<td>0.2426</td>
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<tr>
<td>(\log{\text{YEARUM97}})</td>
<td>–0.1006</td>
<td>–0.0995</td>
<td>–0.1045</td>
<td>–0.1026</td>
<td>0.0898</td>
<td>–0.0975</td>
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<td>(\log{\text{YEARUM99}})</td>
<td>0.1348</td>
<td>0.1381</td>
<td>0.1400</td>
<td>0.1407</td>
<td>–0.0570</td>
<td>0.1389</td>
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<td>0.1362</td>
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<tr>
<td>(\log{\text{YEARUM00}})</td>
<td>0.2309</td>
<td>0.2330</td>
<td>0.2372</td>
<td>0.2392</td>
<td>0.0939</td>
<td>0.2320</td>
<td>0.2369</td>
<td>0.2299</td>
</tr>
<tr>
<td>(\log{\text{YEARUM01}})</td>
<td>0.3139</td>
<td>0.3137</td>
<td>0.3210</td>
<td>0.3195</td>
<td>–0.1491</td>
<td>0.3144</td>
<td>0.3139</td>
<td>0.3110</td>
</tr>
<tr>
<td>(\log{\text{YEARUM02}})</td>
<td>0.4099</td>
<td>0.4121</td>
<td>0.4115</td>
<td>0.4158</td>
<td>–0.077</td>
<td>0.4124</td>
<td>0.4106</td>
<td>0.4042</td>
</tr>
<tr>
<td>(\log{\text{SUBCENTER}})</td>
<td>0.2013</td>
<td>0.2013</td>
<td>0.2013</td>
<td>0.2013</td>
<td>0.2013</td>
<td>0.2013</td>
<td>0.2013</td>
<td>0.2013</td>
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<tr>
<td>(\log{\text{MINCBDisland}})</td>
<td>0.0233</td>
<td>0.0233</td>
<td>0.0233</td>
<td>0.0233</td>
<td>0.0233</td>
<td>0.0233</td>
<td>0.0233</td>
<td>0.0233</td>
</tr>
<tr>
<td>(\rho) or (\lambda)</td>
<td>–0.0707</td>
<td>–0.0707</td>
<td>–0.0707</td>
<td>–0.0707</td>
<td>–0.0707</td>
<td>–0.0707</td>
<td>–0.0707</td>
<td>–0.0707</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
</tr>
<tr>
<td>(R^2)-(adj)</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
<td>0.7052</td>
</tr>
<tr>
<td>(L)</td>
<td>52.04</td>
<td>65.76</td>
<td>80.94</td>
<td>96.91</td>
<td>–127.81</td>
<td>–79.18</td>
<td>–82.00</td>
<td>–87.89</td>
</tr>
<tr>
<td>(AIC)</td>
<td>–70.09</td>
<td>–95.52</td>
<td>–125.86</td>
<td>–127.81</td>
<td>–79.18</td>
<td>–82.00</td>
<td>–87.89</td>
<td></td>
</tr>
<tr>
<td>(RSS)</td>
<td>93.0970</td>
<td>91.5919</td>
<td>88.1979</td>
<td>84.78</td>
<td>84.78</td>
<td>84.78</td>
<td>84.78</td>
<td></td>
</tr>
</tbody>
</table>

*The studentized Breusch-Pagan test in the spatial error model takes the value of \(\lambda\) into account. It is hence the ordinary least square fit that is tested for heteroskedasticity. AIC in the GWR models are so-called AIC-values, which is not directly comparable to the other reported AIC-values. See Fotheringham et al. (2002, page 96). *** significance 0. L denotes log-likelihood function and RSS is residual sum of squares. Total number of observations is 1,691. Columns 5 and 6 show the results of one model, the estimated Spatial Durbin model.*
### Exhibit 7: Overview of F-tests related to the geographically weighted regression analysis.

<table>
<thead>
<tr>
<th>Test</th>
<th>$H_0$</th>
<th>$p$-values using global bandwidth</th>
<th>$p$-values using $k$ about 508</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(1)</td>
<td>No significant improvement in fit when using the GWR estimator compared to the linear regression estimator.</td>
<td>0.439 (1,669;1675)</td>
<td>0.347 (1,648;1,675)</td>
</tr>
<tr>
<td>F(2)</td>
<td>The GWR estimator and the linear regression estimator describe the data equally well.</td>
<td>0.002 (36;1,675)</td>
<td>0.000 (119;1,675)</td>
</tr>
<tr>
<td>BFC-test</td>
<td>All estimated parameters are constant throughout the area.</td>
<td>0.002 (36;1,669)</td>
<td>0.000 (119;1,648)</td>
</tr>
</tbody>
</table>

* F(1) and F(2) are based on Leung et al. (2000). The BFC-test is based on Brundson et al. (1999). For all tests the alternative hypothesis is the rejection of $H_0$. In parentheses we report df1 and df2: (df1;df2)."
Exhibit 8: \( y \) coordinates are included in a smooth function. Movements from left to right represent movements from south to north. The number on the \( y \)-axis is efficient degrees of freedom.
Endnote:

The estimation is performed by using the program R and related packages: spdep (spatial dependence), spgwr (geographical weighted regression) and mgev (multiple generalized cross-validations). The chosen models, tests, and estimators are based on what is currently available via this software. See also Bivand et al. (2008). R is available at: http://www.R-project.org.