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January 2012

Abstract
This paper examines the dynamics of the residential property market in the US between 1960 and 2011. Given the cyclicality and apparent overvaluation of the market over this period, we determine whether deviations of real estate prices from their fundamentals were caused by the existence of two genres of bubbles: intrinsic bubbles and rational speculative bubbles. We find evidence of an intrinsic bubble in the market pre-2000, implying that overreaction to changes in rents contributed to the overvaluation of real estate prices. However, using a regime switching model, we find evidence of periodically collapsing rational bubbles in the post-2000 market.

Keywords: intrinsic bubbles, rational bubbles, rents, US residential real estate market

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1. Introduction

Over the last decade, residential property prices in the United States have attracted much attention from the media. According to the house price index provided by the Federal Housing and Finance Agency, nominal house prices rose by 61% between 1999 and 2009. This sharp rise has attracted the attention of academic researchers, some of whom are interested in establishing whether this increase in property prices is due to the existence of a bubble in the market.

An extended deviation of house prices from their fundamental values could be detrimental to the economy if it leads to a subsequent collapse in the housing market; research shows that homes are the major assets in household portfolios (see Englund et al., 2002). Changes in housing-based wealth are shown to be more important in their effect on the economy than changes in wealth caused by movements in stock prices (Helbling and Terrones, 2003; Rapach and Strauss, 2006). Pavlidis et al. (2009) find that bubbles in the housing market account for a significant share of changes to consumer expenditure. Hence, if house prices are not monitored closely, a crash could have a large adverse impact on the economy. The study of housing bubbles is therefore an important contribution to the literature on future economic development.

Speculative bubbles are certainly not new phenomena, with bubbles identified in commodity and financial markets at least back to the seventeenth century in the Netherlands (Garber, 1990). Researchers have investigated the possible presence of speculative bubbles in numerous asset classes and several new models have been proposed. An important class of such models is based on rational speculative bubbles. These arise when the expectation of the future price of an asset has an abnormally important influence on the asset’s current valuation, which could stimulate demand and thus lead to a deviation of asset prices from their fundamentals. Thus rational bubbles are influenced by extraneous events that are
independent of fundamentals. In the context of a rational bubble, investors are justified in paying ever-higher prices for the asset because they are compensated for the risk that the bubble will collapse by increasing returns. On the other hand, intrinsic bubbles, another genre of speculative bubbles, are driven solely by overreaction to changes in fundamentals (see the following section for more details).

The contributions of this paper to the literature on housing bubbles are two-fold. Firstly, it is the first to simultaneously examine whether intrinsic and rational speculative bubbles exist in the US residential real estate market. The second contribution of this paper is that we test whether or not changes in rents predict future investment returns on residential properties during periods in which intrinsic bubbles are present and during periods of no intrinsic bubbles. This research is the first of its kind as no previous paper has accounted for the presence of bubbles when testing for the predictive power of rents.

2. The Existing Literature

Blanchard and Watson (1982) introduce a model of rational speculative bubbles which can capture the rise and subsequent rapid fall of prices relative to fundamentals, albeit with two key disadvantages. The first is the implicit suggestion that bubbles will grow exponentially and can never be negative, as rational investors would never expect the future price of a stock to fall below zero. The second is that the model assumes two states of nature, with the first state being one in which the bubble survives and the second state representing the bubble collapse. It follows that once the bubble pops, it would never be able to regenerate. Diba and Grossman (1988) further develop this theory and additionally conclude that for a rational bubble to exist, stock prices’ successive differences will be nonstationary.

However, papers by van Norden and Schaller (hereafter vNS, 1993 and 1999) and by Brooks and Katsaris (2005a and 2005b) dispute this approach, arguing that it is theoretically
unjustifiable and leads to empirically implausible results. vNS (1993) propose a model in which bubbles could regenerate after a collapse, as can be seen in observed successions of rallies and crashes in stock markets.

Froot and Obstfeld (1991) introduce a different class of bubbles termed “intrinsic bubbles.” Intrinsic bubbles, unlike rational speculative bubbles, are influenced solely by fundamentals such as dividends (in the case of stock prices), albeit in a nonlinear way. Intrinsic bubbles revert back to their fundamental values periodically. This class of bubbles is also rational and relies on the self-fulfilling expectations of market participants. The difference between intrinsic bubbles and standard rational speculative bubbles is that in the former case, deviations are caused by a nonlinear relationship between fundamentals and prices rather than extraneous factors that would not normally be expected to influence the value of the asset. Froot and Obstfeld show that intrinsic bubbles closely capture the overreaction of stock prices to changes in dividends. Ma and Kanas (2004) provide further evidence supporting the model for intrinsic bubbles by examining the nonlinear cointegration between stock prices and dividends.

There have been a number of studies designed to test for the existence of bubbles in real estate prices, stimulated by the persistent rising trend in house prices which appeared to have been unrelated to economic variables such as construction costs, disposable incomes and unemployment. Björklund and Söderberg (1999) examine the possibility that house prices in Sweden are driven by the presence of speculative bubbles. They observe the dynamics of the Gross Income Multiplier (hereafter GIM), asserting that only the presence of a speculative bubble can cause the GIM to have a different trend from the real estate cycle. Kim and Lee (2000) adopt a cointegration approach to test for the presence of a bubble in the Korean property market. They infer the presence of a bubble in the market on the grounds that, in periods when the bubble is expanding, the variables are not cointegrated and therefore that
there is no observable long-run equilibrium relationship between the variables. Similarly, Mikhed and Zecik (2009) test for bubbles in 23 metropolitan statistical areas (MSAs) in the United States between 1975 and 2006 using unit root tests. In their paper, Mikhed and Zemcik derive the fundamental value of real estate using the present value of future rents. They find evidence of bubbles because fundamental values were stationary while observed prices were non-stationary.

A more econometrically robust test for bubbles is performed by Roche (2001). Using a regime-switching model, he tests for and finds the presence of periodically collapsing rational bubbles in the Dublin residential property market. More recently, Lai and van Order (2010) test for housing bubbles in the U.S. by examining the momentum of house price growth in several MSAs. They find momentum to have increased significantly after 1999, which led to the inception of a bubble.

The only notable test for bubbles in the real estate market that is not based on an econometric approach is conducted by Zhou and Sornette (2006). They apply their test to the housing market of different states in the United States Using an “econophysics” method, their approach examines the speed at which house prices grew. A faster than exponential growth in house prices could be concluded to indicate the existence of a bubble. They find that 22 out of the 50 States in the United States exhibited evidence of bubbles. Another state-level analysis is conducted by Abraham and Hendershott (1996), who observe bubbles in US North East and West coastal cites. Although much of the research is based on the residential market, bubbles are also observed by Hendershott (2000) to be present in the office market in Sydney.

Several other relevant papers have tested for bubbles in the US real estate market, including Case and Shiller (2004), Wheaton and Nechayev (2008), Giliberto (1992), and Goodman and Thibodeau (2008). There is also evidence of speculative bubbles in real estate investment
trusts (REITs) – see, for example, Brooks et al. (2001), Payne and Waters (2005 and 2007), and Jirasakuldech et al. (2006).

All of the above studies in real estate address the existence of rational bubbles. However, existing evidence concerning the presence of intrinsic bubbles is very sparse. Black et al. (2006) and Fraser et al. (2008) represent the only studies that test for the presence of intrinsic bubbles in the real estate market, focusing on the UK and New Zealand property markets. The two papers apply the same methodology in calculating the fundamental values. They measure fundamentals based on the present value of the expected future disposable income of households. By testing for the presence of intrinsic bubbles in the two countries, they investigate whether deviations of house prices from their fundamentals are caused by an overreaction of households to changes in the expected future values of their disposable incomes. Both papers find intrinsic bubbles to be present only in the United Kingdom, whilst deviations from fundamentals in New Zealand appear to have been influenced by price dynamics alone and therefore classified as a rational but not intrinsic bubble.

The bubbles identified by the above literature to be present in the housing market may have contributed to the ambiguous results of an entirely separate strand of real estate research that sought to identify whether price changes could be predicted by rents. Mankiw and Weil (1989) find the relationship between future price changes and the rent/price ratio to be statistically insignificant. Even with the use of an error-correction model, Gallin (2004) shows that the power of the rent/price ratio for predicting future house prices is inconclusive. Hatzvi and Otto (2008) also find no concrete evidence to suggest that an increase in real rents would strongly influence variations in the prices of residential property. Referring to the UK housing market, Baddeley (2005) concludes that modeling the market is more efficient when factors that might destabilize it are accounted for in the analysis. Such factors might include the presence of bubbles, herding, and frenzies. Building on Baddeley’s conclusion, we aim to
establish whether or not there have been intrinsic or rational speculative bubbles in the U.S. housing market and we examine the role of fundamental factors in forecasting future price changes differs between bubble and nonbubble periods. In this paper, our objective is to understand the price dynamics of the US housing market over the last 50 years. In our analysis, we test to see whether house prices differ from their fundamental values, and, if so, whether such deviations are driven by an overreaction of house prices to changes in rents i.e., whether there is an intrinsic bubble or not. Although Black et al. (2006) and Fraser et al. (2008) have, as reported above, tested whether deviations of house prices from their fundamental values were due to an overreaction to changes in fundamentals in the UK and New Zealand, this is the first paper to have addressed the issue in the US market.

3. Data

For this study, we use quarterly data on rents and average purchase/sales prices of houses from 1960 to 2011, provided by Davis et al. (2008) for the Lincoln Institute of Land Policy. To compute the time series of house prices from 1960, Davis et al. employ a two-step procedure. Firstly, they retrieve data from the last five Decennial Housing Census as well as quarterly data from the Freddie Mac Conventional Mortgage House Price Index (CMHPI, a repeat-sales house price index) and the consumer price index (CPI) tenant rent index. They then use interpolation methods to adjust the quarterly changes in the CMHPI and the CPI tenant rent index, so that the series passes through the Decennial Census of Housing benchmark levels. For the post-2000 period, they extrapolate the quarterly house price and rental indices. The discount rate we use in the context of the intrinsic bubble model described below is the federal funds rate which was obtained from the Federal Reserve Economic database. All analysis is conducted using real prices and rents, which are deflated using the CPI, and all series are therefore measured in constant 2011 prices.
There has been a persistent rise in the average house sales price in the United States, as illustrated in Figure 1. The changes in the residential real estate market were mirrored (and influenced) by the history of the Savings and Loan (S&L) industry which experienced huge regulatory changes in the last third of the twentieth century. In the early 1970s the S&L sector was competing for funds, with the banks bidding for deposits at variable interest rates but issuing loans at fixed interest rates. As inflation ballooned, this model began to look fragile. In response to worries about the sustainability of the sector, regulations were progressively loosened – allowing lower capital reserves as well as increased provision for insurance of deposits. The effect of these changes was to fuel the expansionary forces of the S&L sector and thence feed the demand for housing. The number of completions of homes peaked in three phases, in the early and late 1970s and again in the mid-1980s. After a quiet phase in the early 1990s, the housing sector expanded almost continuously until 2004, fueled by low interest rates and easier lending. From the graph, it is clear that there was a faster than average growth in house prices just after 1999, with prices peaking in 2006Q1, when the price of the average house sold in the US was $335,680. This represents a 62% rise in real prices in just 7 years and had been accompanied by an increased proportion of people owning their own homes (69% in 2006 compared with 64% in 1994). The numbers employed in the construction sector reached its highest level in 2006 but ominously the proportion of homes held vacant (either for sale or rent) also had been increasing since the early 1990s and that did not peak until 2008. Over that same period, gross real rents rose by only 4%. Shortly after the peak of 2006, there was a significant decline in house prices, and by 2010, they had fallen 56%, to $215,497, amounting to the largest sustained drop in the housing market over the previous 50 years. This reduction in prices has been attributed to a contraction of credit facilities caused by the well-documented subprime mortgage crisis. This is discussed further below.
The price rent ratio is examined in Figure 2. Before the period of aggressive increases in house prices during 1999–2006, the average price-rent ratio was just 19. However, by the peak of the housing market in 2006, this ratio had risen to 32, implying that house prices had diverged significantly from rents. This divergence may be the result of speculative bubbles. The effective federal funds rate, which represents the cost of borrowing, has continued to fall since the 1980s. Figure 3 shows that, as of 2008, the rate was around 2% compared to 16% in 1981. It has been suggested that these persistent cuts in interest rates contributed to the sharp rise in house prices over this period, although subsequently interest rates remained at extremely low levels even as prices collapsed.

4. Methodology

Examining the price-rent ratio provides information on when the trend in prices departs from the growth trend in rents, which could imply the presence of a bubble (see Mikhed and Zemcik, 2009). The underlying assumption is that the asset price series at some point starts to incorporate a bubble and, at a later point, the bubble bursts. But from casual observation, bubbles are not equally likely to occur at any point in time; rather, they usually occur when the market is in a state predisposed to a bubble. We take the view that bubbles are more likely to occur when the market is more volatile. In order to work within this view, we choose to model the market using a Markov regime switching approach in which the market may be classified into two regimes: low return/high volatility and high return/low volatility. We test for regime switching in the price-rent ratio series. Applying a Markov regime switching methodology, we use the probabilities generated to determine when the price-rent ratio series switches regimes. Using the computed probabilities from the Markov regime switching model, the data are then split into two sub-periods. We separately test for the presence of
intrinsic and periodically partially collapsing rational speculative bubbles in the housing market in both sub-periods.\textsuperscript{ii}

4.1. Locating regimes in the price-rent ratio series

In order to determine the sub-periods, we assume that the price-rent ratio, \(\frac{P}{R}\) can be modeled by the following process:

\[
\left( \frac{P}{R} \right)_t = a + \left( \frac{P}{R} \right)_{t-1} + \zeta_t, \tag{1}
\]

where

\[
\zeta_t \sim i.i.d(0, \sigma^2) \tag{2}
\]

and \(a\) is constant drift term

This means that the price-rent ratio in the current period is equivalent to the previous period’s price-rent ratio plus a constant representing the drift of the process. Re-arranging (1):

\[
\left( \frac{P}{R} \right)_t - \left( \frac{P}{R} \right)_{t-1} = a + \zeta_t, \tag{3}
\]

Applying the Markov regime switching methodology to (3), we obtain (4):

\[
\left( \frac{P}{R} \right)_t - \left( \frac{P}{R} \right)_{t-1} = a_0(1-s_t) + a_1s_t + (\sigma_0(1-s_t) + \sigma_1s_t)\zeta_t, \tag{4}
\]

This is now a Markov switching model,\textsuperscript{iii} with \(s_t\) being a latent state variable that is assumed to follow a first-order Markov chain with constant transition probabilities, which we define as:

\[
\Pr (s_t = 1 | s_{t-1} = 1) = p \tag{5a}
\]
\[
\Pr(s_t = 0 | s_{t-1} = 1) = 1 - p \quad (5b)
\]
\[
\Pr(s_t = 0 | s_{t-1} = 0) = q \quad (5c)
\]
\[
\Pr(s_t = 1 | s_{t-1} = 0) = 1 - q \quad (5d)
\]

This state variable can be either zero or one, representing the two possible regimes of the price-rent ratio series. In this case, the probability of being in one regime is influenced only by the state that prevailed during the previous period.

By implementing this Markov regime switching random walk with a drift model, we could then use the probabilities generated to determine the period in which the price-rent ratio switches to a separate regime with different mean and variance. Hence, we split our data into sub-periods based on these probabilities and apply the intrinsic bubble test to the sub-periods.

**4.2. An intrinsic bubble test**

As described above, Froot and Obstfeld (1991) introduce the concept of intrinsic bubbles which, unlike rational speculative bubbles, are deviations of observed prices from the fundamental price driven by fundamentals in a nonlinear fashion. In contrast with previous studies on intrinsic bubble models that use disposable income as the key fundamental driver of house prices, we use rents as our fundamental measure. The main reason we opt to use rents rather than disposable income is that they directly represent the consumption market for physical space. A shortfall in the supply of space will lead to an increase in rents. But a bubble in the sense of financial market bubbles should not affect the nominal level of rents since the rent is essentially the spot price for immediate space occupation and is not itself an investible asset. From the real estate investor’s viewpoint, rent is analogous in cash flow terms to dividends for equity market investors. Rents provide better fundamental measures
than disposable incomes in countries with easy access to credit facilities, like the United States (historically at least), where the ability of individuals to acquire mortgages did not depend solely on their disposable incomes. In those countries, changes in disposable incomes have not necessarily been associated with fundamental changes in the demand for housing. Furthermore, several studies have shown that rents are fundamental drivers including Campbell et al. (2009), who show that variations in housing valuations are due to growth in rental values, the interest rate, and housing premia changes. There is no mention of disposable incomes.

To carry out the test for intrinsic bubbles in the US housing market, we follow the Froot and Obstfeld approach, but replace dividends with rents. Hence the present value relationship describing house prices is given by:

\[ P_t^{pv} = \sum_{s=t}^{\infty} e^{-ir(s-t+1)} E_t(R_s) \]  

(6)

where \( P_t^{pv} \) is the present value of the house price in period \( t \), \( ir \) is the constant interest rate, \( R_s \) is the gross rents value and \( E \) is the expectation of the market given information at the start of period \( t \). A standard bubble model is given as:

\[ P_t = P_t^{pv} + B_t \]  

(7)

where:

\[ B_t = e^{-ir} E_t(B_{t+1}) \]  

(8)

The actual price of a house is given by \( P_t \) while the bubble term, \( B_t \), is the difference between the actual price and the fundamental value. However, the intrinsic bubble model considers
bubbles that are generated by a nonlinear function of rents. Hence the intrinsic bubble is a function of rents satisfying (8):

\[ B_t(R_i) = cR_i^\lambda \]  

(9)

where \( c \) and \( \lambda \) are constants explained below.\textsuperscript{iv}

The model also assumes that log rents are generated as a martingale. Hence, the process for log rents, \( r_t \), must follow a random walk with a drift \( \mu \):

\[ r_{t+1} = \mu + r_t + \varepsilon_{t+1} \]  

(10)

where:

\[ \varepsilon_{t+1} \sim N(0,\sigma^2) \]

From (6), the house price’s present value ought to be proportional to rents if the rent during period \( t \) is known:

\[ P_t^{pv} = \kappa R_t \]  

(11)

where:

\[ \kappa = (e^{\mu} - e^{\mu+\sigma^2/2}) \]  

(12)

The sum in (6) is assumed to converge, thereby implying that \( \delta r \) must be greater than \( \mu+\sigma^2/2 \). Also, this condition is required to ensure that we do not yield negative present values. Therefore, if a bubble is present, the observed house price, \( P_t \), can be thought of as the sum of \( P_t^{pv} \) and \( B_t(R_i) \):

\[ P(R_t) = P_t^{pv} + B_t(R_t) = \kappa R_t + cR_t^\lambda \]  

(13)
where $c$ is an arbitrary constant and $\lambda$ is the positive root of (14), which is obtained from the inequality condition $i_r > \mu + \sigma^2/2$:

\[
\frac{\sigma^2}{2} \lambda^2 + \mu \lambda - i_r = 0
\]  (14)

Based on (14), the inequality $i_r > \sigma^2/2 \lambda^2 + \mu \lambda$ is used to indicate that $\lambda$ must be greater than one at all times, and it is this explosive nonlinearity that allows house prices to overreact to changes in rents. Like Froot and Obstfeld, we then divide (13) by $R_t$ due to the presence of collinearity, using:

\[
\frac{P}{R_t} = \kappa + cR_t^{\lambda-1} + \zeta_t
\]  (15)

to test for the presence of intrinsic bubbles in the housing market. The null hypothesis in this case is that $c = 0$, implying that there is no intrinsic bubble present.

4.3. A rational speculative bubble test

To determine whether real estate prices between 1960 and 2011 were instead driven by extraneous events that are independent of fundamentals, we employ the vNS (1993 and 1999) model. Note that this approach to testing for rational speculative bubbles was also used by Roche (2001) to test for bubbles in the Irish housing market. This model tests for the presence of periodically collapsing rational speculative bubbles in an asset. In order to fully understand the vNS specification, it is important to briefly discuss the Blanchard and Watson (1982) model. According to their model, prices are decomposed into two parts: the fundamental value and the bubble component, as seen in (7). In this model, the bubble component grows at an exponential rate and cannot be negative in value, implying that observed prices must always equal or exceed the fundamental value. The model also assumes
that two bubble states exist, the first being one where the bubble survives \((S)\) and the other being a state where the bubble collapses \((C)\), implying the bubble process to be:

\[
E_t(B_{t+1} \mid S) = \frac{(1 + ir)}{q} B_t \quad \text{with probability } q
\]

\[
E_t(B_{t+1} \mid C) = 0 \quad \text{with probability } 1 - q
\] (16)

where the probabilities of the bubble surviving and collapsing are \(q\) and \(1 - q\), respectively.

The process in (16) implies that the bubble component in period \(t+1\) is expected to grow at a faster rate than the real rate of return if the bubble does not collapse, thus compensating the investor for risk-taking.\textsuperscript{vii} However, in the event of the bubble collapsing, its value immediately falls to zero and the observed price would equal the fundamental value. This model also concludes that the bubble cannot regenerate once it collapses. Therefore, these assumptions are not realistic as there have been significant episodes of bubble crashes and regenerations.

The vNS approach is an extension of the Blanchard and Watson model, eliminating the assumption of a bubble being unable to regenerate following a collapse. vNS also allow the probability of a bubble collapsing to depend on its size relative to the asset price \(b_t = B_t / P_t\), i.e. an increase in the bubble relative to the asset price increases the probability of a collapse. Lastly, they allow for the gradual collapse of speculative bubbles, unlike Blanchard and Watson’s model, which assumes the value of a bubble to equal zero once it has collapsed. Therefore, the vNS stochastic bubble process is given as:

\[
E_t(B_{t+1} \mid S) = \left(\frac{1 + ir}{q(b_t)} - 1 - \frac{q(b_t)}{q(b_t)}\right) B_t - \frac{u(b_t)}{q(b_t)} P_t \quad \text{with probability } q(b_t)
\]

\[
E_t(B_{t+1} \mid C) = u(b_t) P_t \quad \text{with probability } 1 - q(b_t)
\] (17)
where \( u(b_t) \) is a continuous and everywhere differentiable function:

\[
0 \leq \frac{\partial u(b_t)}{\partial b_t} \leq 1
\]  

(18)

Here, the expected size of the bubble in the collapsing regime does not immediately reduce to zero. Instead, it decreases gradually. Note that the probabilities \( q \) are now represented as \( q(b_t) \), implying that it is dependent on the size of the bubble relative to prices. To calculate the fundamental value of housing, we use the average price-rent ratio multiplied by the rent during the selected time period:

\[
P_t^f = \frac{\bar{P}}{R} R_t
\]

(19)

Therefore, the nonfundamental or rational speculative bubble component of the stock is given by simply subtracting the actual price of the stock from its fundamental component – i.e. the proportion of observed price that is not explained by rents, as shown in (20):

\[
B_t = P_t - \frac{\bar{P}}{R} R_t
\]

(20)

Under this bubble specification model, vNS note that returns to a “bubbly” asset ought to be state dependent such that periodically collapsing bubbles induce regime switches in asset returns:

\[
W_{t+1}^c = \beta_{c,0} + \beta_{c,1} b_t + u_{c,t+1}
\]

\[
W_{t+1}^r = \beta_{c,0} + \beta_{c,1} b_t + u_{c,t+1}
\]

\[
q(b_t) = \Phi(\beta_{q,0} + \beta_{q,1} |b_t|)
\]

(21)

where \( W \) represents the return on investing in real estate, the unexpected returns in the collapsing and surviving regimes are represented by \( u_{c,t+1} \) and \( u_{s,t+1} \) respectively, and they
have constant variance and zero mean. Φ represents the standard normal cumulative distribution function and so Φ(β_{q,0}) is the average probability of being in the surviving regime. On the other hand, β_{q,1} measures how the probability of being in the surviving regime changes with respect to the absolute bubble size as a proportion of the actual price.

To estimate the parameters in (21), we maximize the following log-likelihood function (l):

$$
\ell(W_{t+1} | \zeta) = \sum_{t=1}^{T} \ln \left[ q(b_t) \frac{\phi\left( \frac{W_{t+1} - \beta_{s,0} - \beta_{s,1}b_t}{\sigma_s} \right)}{\sigma_s} + (1 - q(b_t)) \frac{\phi\left( \frac{W_{t+1} - \beta_{c,0} - \beta_{c,1}b_t}{\sigma_c} \right)}{\sigma_c} \right]
$$

(22)

Here, the set of parameters to be estimated in (22) is represented by ζ, and these comprise β_{s,0}, β_{s,1}, β_{c,0}, β_{c,1}, β_{q,0}, β_{q,1}, σ_s and σ_c. Note that the notation φ represents the standard normal probability density function, and σ_s and σ_c are the disturbances’ standard deviations in the surviving and collapsing regimes, respectively. Given these estimates, it is possible to make inferences on whether or not bubbles exist.

For the rational speculative bubble model to provide a good fit to the data in the real estate market, four conditions must be met:

(i) β_{s,0} ≠ β_{c,0}: The mean returns in the surviving and collapsing regimes must differ
(ii) β_{c,1} < 0: The bubble should yield a negative return in the collapsing regime
(iii) β_{s,1} > β_{c,1}: The bubble must yield a higher return in the surviving regime than in the collapsing regime
(iv) β_{q,1} < 0: The larger the size of the bubble relative to the price, the greater the probability of it collapsing.

As a robustness check, the vNS bubble model is tested against three other stylized alternatives of returns, namely the volatility regime, mixture-normal and fads models,
proposed by Schwert (1989), Akgiray and Booth (1988), and Cutler et al. (1991), respectively. In the volatility regime model, regimes in returns are caused by changes in volatility but the bubble component has no effect on the returns themselves. Like the volatility regime model, in the mixture-normal model, the bubble component does not influence returns but returns are constant over the two regimes. As for the fads model, returns may be predictable by the bubble component but they do not change across regimes.

5. Results

5.1. Regime switching structural break test

To check for regime switches in the price-rent ratio series, we first implement the Markov regime switching methodology discussed in the previous section, estimating the parameters of equation (4) by maximum likelihood, and these are presented in Table 1. The results show that there are two distinct states: State 0 represents a regime with relatively low volatility ($\sigma_0^2$) and a positive mean ($a_0$). State 1, on the other hand, represents a regime which has a negative mean ($a_1$) and higher volatility ($\sigma_1^2$) in the price-rent ratio series. All the variables except for $a_1$ are statistically significant at the 1% level and $a_1$ is almost significant at the 10% level. Based on the transition probability matrix calculated from the Markov regime switching model, we compute the smoothed states probabilities, which locate the probability of being in the high volatility state in the price-rent ratio series, and these are presented in Figure 4.

From the Markov regime switching model, the probabilities generated indicate that price-rent ratio is likely to have changed regime around 2000Q1 because the probability of being in State 1 starts to increase from 2000Q1 onwards. Based on these findings, we split our data into two sub-periods. Sub-period ‘A’ represents the period 1960–1999, while sub-period ‘B’ (2000–
2011) represents the period corresponding to an increased probability of the price-rent ratio coming from the high volatility regime.

Evidence of a very large departure of house prices from rents is also given in Figure 5 which, unlike Figure 1, shows house prices and rents in real terms and is rescaled so that both series take the value 100 at the start of the sample period in 1960. It shows that house prices and rents grew at almost the same rate over the years until the late 1990s, and that thereafter house prices began to grow at a faster rate. The increase in the growth rate of house prices may have led them to deviate from their fundamentals. Since the growth rate of rents was relatively stable, this rise in prices may not have been caused by an overreaction to changes in rents; rather, it could have been due to other exogenous factors. This unusually fast growth in house prices lasted until 2006, and then prices fell 56% by 2010. Therefore, we test for the presence of intrinsic bubbles in the two sub-periods. We also test for the presence of rational speculative bubbles between in the sub-periods, as these two genres of bubbles are mutually exclusive.

5.2. The intrinsic bubble test

In this sub-section, we aim to find out whether or not intrinsic bubbles exist in the two sub-periods. Following the steps in (6) to (15), we compute the parameters, all of which are estimated using ordinary least squares (OLS, on 16) except for $\lambda$, which is the positive root of the quadratic equation given in (15). Table 2 represents information on the estimated parameters in sub-period ‘A’, whilst Table 3 represents results from the intrinsic bubble test in sub-period ‘B’. It shows that between 1960 and 1999, $\mu$, which represents the rents’ trend growth rate, is calculated to be approximately 0.31% per quarter, with a standard deviation of 0.53%, indicating little fluctuation in rents. With these parameters estimated, we compute the value of $\lambda$, the quadratic solution of (15), which we find to be 3.1878. $^x$
Using OLS to carry out the hypothesis test in (16), we find $\kappa = 16.4687$ and $c = 0.0262$, with corresponding $t$-statistics of $68.5055$ and $11.4031$, respectively. Therefore, there is strong evidence to reject the null hypothesis of the absence of intrinsic bubbles in the US housing market over the first sub-period. The positive and significant value of $\kappa$ signifies that, as the cost of renting increases, house prices increase at a faster rate than rental growth.

As explained earlier, the presence of an intrinsic bubble in the housing market implies that house prices diverge from their fundamentals due to an overreaction to changes in the values of rents, and this intrinsic bubble could have been sustained by a widely held belief by investors/homebuyers that house prices would continue to rise based on the ever-increasing cost of rents. Also, the availability of credit facilities to borrowers with poor credit-ratings (subprime borrowers) may have led many renters to buy houses rather than to continue renting because, as rents continued to rise, more individuals were encouraged to apply for mortgages.

Table 3 presents the result of the intrinsic bubble test for the second sub-period, 2000–2011. The trend growth in rents, $\mu$, is 0.0014 (0.14% per quarter), which is 0.17% less than in the first sub-period. The standard deviation obtained from a random walk with drift model of log-rents is 0.67%. Using these estimates, we calculate $\lambda$ to be $3.3520$.

Table 3 shows that the coefficients $\kappa$ and $c$ are both statistically insignificant, which therefore implies that there is considerable evidence to suggest the absence of intrinsic bubbles in the housing market between 2000 and 2011. This means that price rises during the more recent period cannot be attributed to an overreaction to changes in the cost of renting, but may have been influenced by changes to other exogenous factors such as easily accessible credit and the increased popularity of mortgage-backed securities products.
5.3. Rational speculative bubbles

As discussed above, it is possible that in periods with no evidence of intrinsic bubbles, rational speculative bubbles, on the other hand, may have influenced the dynamics of the residential property market, implying that deviation of real estate prices from their fundamental values could have been caused by other exogenous factors independent of rents. Using the formulae given in (19) and (20), we construct the fundamental values and the rational speculative bubble component and compare it with observed real estate prices over time.

From Figure 6, it is evident that there have been prolonged periods of undervaluation and overvaluation in the real estate market between 1960 and 1990. Between the early 1970s and 1980s, high inflation rates and relatively slow house price growth led to the cyclical trend in real estate prices. From 1990 to 1999, prices gradually rose and the market was slightly overvalued by an average of 7% relative to fundamentals. However, from 2000 onwards, the momentum of house price growth and the size of the rational speculative bubble component relative to prices increased significantly. This was due to the aggressive cuts in interest rates following the collapse of the “dot-com” bubble in 2000. According to Bankrate.com, the national average interest rate on 30-year fixed home mortgages fell by over 300 basis points between 2000 and 2005. In those five years, the Mortgage Bankers Association noted that the number of mortgage applications almost doubled, increasing the demand for homes. Similar to previous bubble episodes, individuals began to “flip” properties, as they believed that prices would continue to rise.\textsuperscript{xii} The number of mortgages given to borrowers who were unable to qualify for conventional mortgages (nonprime or subprime borrowers) increased, and so at its peak in 2005 the real estate market was overvalued by 38%.

However, from 2005, the Federal Reserve System began increasing interest rates to reduce the inflationary pressure on the economy. The volume of residential real estate transactions
fell as mortgage rates rose, thus slowing down the growth in prices. By 2009, foreclosures as a percentage of the total number of loans had more than trebled. Since then, the market has been undervalued.

To determine whether the deviation of prices from fundamental values was due to the presence of rational periodically collapsing speculative bubbles in the real estate market in either of the two sub-periods, we apply the vNS regime switching bubble model presented above. Table 4 provides results from the vNS bubble model. The parameter estimates for the period 1960–1999 met only the first of the four rational speculative conditions listed in above (see the second panel of Table 4), and thus there is strong evidence to conclude that rational speculative bubbles did not influence the market during this period. The period 2000–2011, on the other hand, had clear signs of the existence of rational speculative bubbles as three of the four restrictions are met and the fourth is almost met at the 10% significance level.

Focusing on the sub-period 2000–2011, average quarterly growth rate of real estate prices in the surviving regime ($\beta_{s,0}$) is 1.90% compared to negative 2.64% (i.e., 1–0.9736) in the collapsing regime. There is statistical evidence to show that mean returns differ significantly across the two regimes. Also, the bubble in the surviving regime yields greater returns than in the collapsing regime, as can be seen by the signs of the coefficients of $\beta_{s,1}$ and $\beta_{c,1}$. Lastly, the significant negative estimate of $\beta_{q,1}$ implies there is evidence that an increase in the size of the rational speculative bubble component relative to price increases the probability of a collapse occurring. It is also important to note that the volatility of returns in the market is greater in the collapsing regime ($\sigma_c = 0.0254$). Furthermore, rejections of the other stylized alternatives provide stronger evidence to suggest that a rational speculative bubble influenced the real estate market between 2000 and 2011 and that this model fits the data better than these simpler approaches.
Therefore, there is significant information to conclude that intrinsic bubbles existed in the real estate market between 1960 and 1999. Post-2000, however, only rational speculative bubbles influenced the market. This period was characterized by increasing government policy to encourage loans to low-income borrowers by setting higher targets for Fannie Mae to facilitate such loans. Interest rates fell and by 2003 Fannie Mae and Freddie Mac were deeply involved in the financing of subprime mortgages. It is interesting to note that there were arguably only two significant bubble bursts – in 1968 and in 2006. Both of these had roughly the same sort of anatomy – that is, a gradual run up in prices followed by a faster decline in prices before they stabilized some three years or so later.

In order to support the findings above concerning whether a rational speculative bubble existed in the residential property market post-1999, a separate test is conducted. Based on the positive feedback theory, a rational speculative bubble arises when previous increases in house prices alone influence present investments in the residential property market i.e., when rational speculative bubbles exist in the market, house price changes are not moved by key macroeconomic factors but by past price increases only. Considering the two sub-periods, we run OLS regressions on each separately where the dependent variable is the current increase in price ($\Delta p_t$). The explanatory variables are past price changes ($\Delta p_{t-1}$), changes in interest rates ($\Delta ir_t$) and in the unemployment rate ($\Delta un_t$). These explanatory variables are chosen because they are important in determining the demand for houses. The interest rate level represents the cost of borrowing, which should have a negative effect on house prices – see Jud and Winkler (2002). The unemployment rate should also have a negative impact on house prices, as it high unemployment levels would suppress housing demand. A rational speculative bubble would be implied if none of the fundamental variables were significant in the regression and only the changes in past prices explained current price changes.
Table 5 shows the results of the test for the presence of rational speculative bubbles in the housing market. Pre-2000, changes in interest rates and changes in past prices significantly influence changes in current prices. The parameter attached to changes in the unemployment rate, which had a negative relationship with house price changes as expected, is statistically insignificant. However, in the second regime (between 2000 and 2011), the only variable that is statistically significant at even the 10% level is the past price increase. Neither interest rates nor changes in the level of unemployment had an impact on house price changes, implying that house prices were only moved by changes to prices in the previous period and they no longer even have the correct signs. A Wald test is implemented to check for the joint significance of interest rates and unemployment in both sub-periods. We find a very low probability value in sub-period A, indicating that unemployment levels and interest rates jointly influence changes in prices; however, in sub-period B, we find no evidence of joint significance. Hence, there is evidence that a rational speculative bubble existed in the housing market during that period, as only appreciations in lagged prices significantly explain price growth. House prices, based on the average price-rent ratio methodology for computing the fundamental value of houses, were overvalued by as much as 46% in 2006. The subsequent collapse of the rational speculative bubble saw house prices fall by 37% between 2006 and 2009.

6. Do Rents Predict Changes in Real Estate Prices?

Following the tests aimed at distinguishing between the types of bubble, we proceed to investigate the predictive nature of returns in the housing market. As outlined above, our aim is to establish whether or not changes in rents predict price growth in times where intrinsic or rational bubbles exist. To perform this analysis, we first test for unit roots in the variables using the augmented Dickey–Fuller (hereafter, ADF) test. If the variables have unit roots, then it could be the case that they are cointegrated. Cointegration implies that a linear
combination of the two series would be integrated of order zero, or I(0), if the two variables are non-stationary. If both variables are non-stationary but not cointegrated, we would implement a vector autoregressive (hereafter, VAR) model on the first differences. Otherwise we would use the vector error correction model (hereafter, VECM). This is consistent with Granger’s (1986) conclusion that when variables are cointegrated, a VAR model in differences would be mis-specified whereas the VECM has the ability to capture the deviations from the long-term equilibrium. Then a Granger causality test is employed to determine whether or not changes in rents predict the returns on investing in the US housing market.

In order to test whether rents can predict returns in the housing market, the data are again split into the same two sub-periods as above: ‘A’ representing the period 1960–1999 and ‘B’ representing 2000–2011. Sub-period ‘A’ corresponds to the time of an intrinsic bubble while ‘B’ is for the period where rational periodically partially collapsing speculative bubbles existed in the residential property market. This step will allow us to shed additional light on whether the traditional relationship between house prices and rents broke down post-2000 when the rational bubble was in existence.

Firstly, we employ the ADF test on the log changes of both rents and real estate prices in the two sub-periods. The lag length for the ADF unit root test is automatically selected using the Akaike Information Criterion (AIC). The test results are reported in Table 6. From the table, it is clear that all the variables analysed are non-stationary in the log-levels but stationary in the first differences of the log-levels, as the null hypothesis of unit root is not rejected in Panel A but rejected in Panel B. Therefore, we proceed in Table 7 to test for cointegration between the price and rent series in their log-levels forms using the Johansen approach.
There is significant evidence to suggest that real estate prices and rents are found to be cointegrated in sub-period A as the null hypotheses are rejected, indicating that there is at most one co-integrating equation. The Trace test on sub-period B, on the other hand, indicates no cointegration at the 5% level. Hence, we use a VECM model to analyse the first sub-period and a VAR for sub-period B.

Using the coefficients obtained from the VAR and VECM models estimated, we aim to determine whether log changes in rents can predict real estate price growth by applying a Granger causality test. Note that the optimal lag length for the two models is determined by the AIC. We find that the optimum lag lengths for the VAR models in sub-periods A and B are four and one, respectively. Table 8 presents the results from the Granger causality tests on the two sub-periods.

It is very clear that changes in rents Granger-cause changes in real estate prices only during the period where there was an intrinsic bubble in the housing market. However, post-1999, there appears to be no causal relationship between changes in rents and the returns to investing in the residential housing market. This shows that changes in rents can predict future real estate price changes only when there is an intrinsic bubble; when only rational speculative bubbles exist, changes in rents do not predict returns. These findings confirm that house prices are more sensitive to changes in rents during periods when an intrinsic bubble is present.

To further investigate the relationship between house prices and rents, we examine the impulse response function of real estate prices to a positive unit shock in rents in the two sub-periods for twenty quarters ahead. Figures 7(a) and 7(b) focus on the degree of sensitivity of real estate prices to changes in rents in the two sub-periods. The graphs provide further evidence supporting the intrinsic bubble model for the earlier sub-period. In the period 1960–
1999, unexpected increases in rental cost cause prices to fall for the first seven quarters. From the eighth quarter onwards, the effect of rent innovation on real estate prices becomes positive. The effect eventually fizzles out around the twentieth quarter. For the period 2000–2011, the effect of unexpected changes in rental cost is far smaller than for the intrinsic bubble period, as prices do not react to any economically meaningful extent to changes in rents. It also takes a much shorter period (around nine quarters) for the effect of innovations to rental cost on real estate prices to die down to zero.

7. Summary
In this paper, we set out to establish whether the residential real estate market in the United States has been influenced by either intrinsic bubbles or rational speculative bubbles, or both, over the last fifty-one years. By applying a Markov switching model, we identify that there was a structural break in the price-rent ratio series, as the housing market appeared to have switched from a regime of low price-rent ratios to that of relatively high ratios, at or around 2000. Given these findings, we split our data set into two separate sub-periods, pre-determined by the regime switch, and we test for the presence of intrinsic bubbles. From our analyses, we conclude that in the first period (1960–1999), the market behaved in a manner consistent with an intrinsic bubble, with buyers overreacting to changes in the cost of renting. The second period, however, displayed no signs of intrinsic bubbles. Using a separate test for rational speculative bubbles, we find significant evidence to suggest that this genre of bubble existed in the market after 2000 only, with 2005Q4 being the time when housing was most overvalued. Therefore, we conclude that, between 1960 and 1999, the overvaluation of real estate prices was driven by an overreaction to changes in rental costs (and possibly by exogenous factors as well). However, post-2000, the systematic deviation of real estate prices from their fundamental values was caused solely by factors exogenous to rents. In other
words, buyers ceased to be influenced by underlying fundamental factors and instead became fixated on nonlinearly extrapolating the historical growth in house prices.

Previous studies aimed at determining the predictive power of rents on house prices have found differing results, and that may have been a result of differences in the time-periods studied combined with a change in the drivers of price changes. While some papers studied the impact of rents and other factors on house prices during a bullish housing market (e.g. pre-2005 in the United States), others investigated periods of downturns. In this paper we have been able to shed light on why previous research has found conflicting or insignificant results. Our tests have shown that the type of bubble is important in determining whether rents influence returns in the housing market. Using a Granger causality test on estimates from VAR models, we examined how important rent changes were in determining changes in house prices. Our results showed that changes in rents Granger-cause, and could therefore predict, future returns in the housing market only when there is an intrinsic bubble.

Even though real estate bubbles have been extensively scrutinized over the years, there is further research to be conducted on establishing the factors that led to the inception and subsequent collapse of rational speculative bubbles. There is widespread speculation regarding the true causes of the house price rises that occurred between the late 1990s and 2006. Some attribute this to the consistent decline in interest rates whilst others conclude that it was caused by aggressive subprime lending, especially in states such as California and Florida. To date, there is no cohesive conclusion on this matter. Hence, further research might reveal the relative importance of the possible causes of the rational speculative bubble and the extent to which bubbles arose in certain specific regions or were endemic to the entire national market.
Finally, there are policy implications arising from this research. Some states in the US have adopted tax policies on home purchase (e.g., Texas) that discourage speculative activities. Although some regions are more prone to speculation-induced bubbles than others, consideration could be given to rolling out such policies nationally. Also, stricter policies on home refinancing could be adopted in order to curb the re-emergence of real estate bubbles in the future.
References


Kim, K.-H. and H.S. Lee, Real estate price bubble and price forecasts in Korea, Department of Economics, Sogang University, 2000.


**Acknowledgments**

The authors would like to thank four anonymous reviewers for their helpful suggestions. We would also like to thank Apostolos Katsaris for providing the Matlab code to estimate the van Norden and Schaller model.
Table 1: Maximum likelihood estimates from the Markov regime switching model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Standard Errors</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0^2$</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_1^2$</td>
<td>0.0011</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.0027</td>
<td>0.0005</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-0.0081</td>
<td>0.0049</td>
<td>0.1026</td>
</tr>
</tbody>
</table>

Log Likelihood: 660.5768

Notes: This table presents the parameter estimates, their standard errors and the associated $p$-values from the Markov switching model:

\[
\left(\frac{P}{R}\right)_t - \left(\frac{P}{R}\right)_{t-1} = a_0(1 - s_t) + \alpha_1 s_t + (\sigma_0(1 - s_t) + \sigma_1 s_t) \zeta_t,
\]

where $P/R$ is the price to rent ratio, $s_t$ is the state at time $t$, and $\zeta_t$ is an error term.
Table 2: Results from the test for intrinsic bubbles in the US housing market between 1960 and 1999

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Coefficients</th>
<th>T-stats</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.0031</td>
<td>7.1963</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>3.1878</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>16.4687</td>
<td>68.5055</td>
<td>0.0000</td>
</tr>
<tr>
<td>$c$</td>
<td>0.0262</td>
<td>11.4031</td>
<td>0.0000</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.4611</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table presents estimates of the parameters and their associated t-ratios and p-values, with the model being estimated using OLS (on equation 16) except $\lambda$, which is the positive root of the quadratic equation given in equation (15).
Table 3: Results from the test for intrinsic bubbles in the US housing market between 2000 and 2011

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Coefficients</th>
<th>T-stats</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.0014</td>
<td>1.4917</td>
<td>0.1422</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>3.3520</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>15.8507</td>
<td>1.7071</td>
<td>0.0941</td>
</tr>
<tr>
<td>$c$</td>
<td>0.0002</td>
<td>0.9370</td>
<td>0.3534</td>
</tr>
</tbody>
</table>

$R^2$ \[0.0176\]

Notes: This table presents estimates of the parameters and their associated t-ratios and p-values, with the model being estimated using OLS (on equation 16) except $\lambda$, which is the positive root of the quadratic equation given in equation (15).
Table 4: Results from the vNS rational speculative bubble model

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>REAL ESTATE MARKET</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{s,0}$</td>
<td>1.0070 (0.0000)</td>
</tr>
<tr>
<td>$\beta_{s,1}$</td>
<td>0.0243 (0.7246)</td>
</tr>
<tr>
<td>$\beta_{c,0}$</td>
<td>0.9966 (0.0000)</td>
</tr>
<tr>
<td>$\beta_{c,1}$</td>
<td>-0.0466 (0.0023)</td>
</tr>
<tr>
<td>$\beta_{q,0}$</td>
<td>1.1400 (0.0274)</td>
</tr>
<tr>
<td>$\beta_{q,1}$</td>
<td>-4.4549 (0.5603)</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0.0064 (0.0000)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.0157 (0.0000)</td>
</tr>
</tbody>
</table>

**LIKELIHOOD RATIO TESTS FOR RATIONAL SPECULATIVE BUBBLE CONDITIONS**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{s,0} \neq \beta_{c,0}$</td>
<td>5.1610 (0.0231)</td>
<td>4.9445 (0.0262)</td>
</tr>
<tr>
<td>$\beta_{c,1} &lt; 0$</td>
<td>0.1392 (0.7091)</td>
<td>3.6021 (0.0577)</td>
</tr>
<tr>
<td>$\beta_{s,1} &gt; \beta_{c,1}$</td>
<td>1.1972 (0.2739)</td>
<td>6.0794 (0.0137)</td>
</tr>
<tr>
<td>$\beta_{q,1} &lt; 0$</td>
<td>0.3455 (0.5567)</td>
<td>2.4641 (0.1165)</td>
</tr>
</tbody>
</table>

**ROBUSTNESS CHECK: TEST AGAINST OTHER STYLIZED ALTERNATIVES**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility Regime</td>
<td>26.8489 (0.0000)</td>
<td>18.7179 (0.0000)</td>
</tr>
<tr>
<td>Mixture Normal</td>
<td>34.2569 (0.0000)</td>
<td>23.8440 (0.0000)</td>
</tr>
<tr>
<td>Fads</td>
<td>29.8665 (0.0000)</td>
<td>19.8340 (0.0000)</td>
</tr>
</tbody>
</table>

Notes: This model presents the parameter estimates with p-values in parentheses for the van Norden and Schaller model given by equation (21) estimated using equation (22). The p-values are calculated using the inverse of the Hessian matrix. As discussed, the likelihood ratio test is used to test the four conditions that must be met in order to conclude the existence of rational speculative bubbles. The robustness check is to ensure that returns are not better modelled using other, simpler alternative approaches.
Table 5: Test for the presence of rational speculative bubbles in the sub-periods.

<table>
<thead>
<tr>
<th>Sub-periods</th>
<th>Variable</th>
<th>Coefficients</th>
<th>t-stats</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1960 - 1999</strong></td>
<td>Constant</td>
<td>0.001</td>
<td>1.165</td>
<td>0.246</td>
</tr>
<tr>
<td></td>
<td>$\Delta ir_t$</td>
<td>-0.002</td>
<td>-4.254</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>$\Delta un_t$</td>
<td>-0.001</td>
<td>-0.788</td>
<td>0.432</td>
</tr>
<tr>
<td></td>
<td>$\Delta p_{t-1}$</td>
<td>0.899</td>
<td>20.892</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>2000 - 2011</strong></td>
<td>Constant</td>
<td>-0.001</td>
<td>-0.205</td>
<td>0.839</td>
</tr>
<tr>
<td></td>
<td>$\Delta ir_t$</td>
<td>0.010</td>
<td>0.162</td>
<td>0.252</td>
</tr>
<tr>
<td></td>
<td>$\Delta un_t$</td>
<td>0.007</td>
<td>0.523</td>
<td>0.604</td>
</tr>
<tr>
<td></td>
<td>$\Delta p_{t-1}$</td>
<td>0.624</td>
<td>4.417</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: This table presents the parameter estimates and their associated $t$-ratios and p-values for estimation of the model: $\Delta p_t = Constant + \Delta ir_t + \Delta un_t + \Delta p_{t-1} + u_t$ using OLS, where the variables $\Delta p_t$, $\Delta ir_t$, and $\Delta un_t$ represent changes in prices, interest rates and unemployment respectively, and $u_t$ is an error term. 

Table 6: Augmented Dickey-Fuller unit root test for the sub-periods

Panel A: Tests on the log levels

<table>
<thead>
<tr>
<th>Sub-periods</th>
<th>Variables</th>
<th>t-Statistics</th>
<th>5% Critical Value</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: 1960 - 1999</td>
<td>Price</td>
<td>-0.7488</td>
<td>-2.8801</td>
<td>0.8300</td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>-1.2968</td>
<td>-2.8797</td>
<td>0.6305</td>
</tr>
<tr>
<td>B: 2000 - 2011</td>
<td>Price</td>
<td>-2.4848</td>
<td>2.9458</td>
<td>0.1247</td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>-2.0500</td>
<td>2.9297</td>
<td>0.2653</td>
</tr>
</tbody>
</table>

Panel B: Tests on the Changes in the log-levels

<table>
<thead>
<tr>
<th>Sub-periods</th>
<th>Variables</th>
<th>t-Statistics</th>
<th>5% Critical Value</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: 1960 - 1999</td>
<td>Price</td>
<td>-4.7631</td>
<td>-3.4391</td>
<td>0.0008</td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>-6.8679</td>
<td>-3.4385</td>
<td>0.0000</td>
</tr>
<tr>
<td>B: 2000 - 2011</td>
<td>Price</td>
<td>-1.8952</td>
<td>-1.9496</td>
<td>0.0562</td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>-4.8031</td>
<td>-1.9487</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Notes: This table presents the unit root test statistics along with the appropriate critical values at the 5% significance level and the associated p-values. The null hypothesis that the variable has a unit root; and Mackinnon et al. (1999) p-values are employed.
Table 7: Johansen’s unrestricted cointegration rank (Trace) test for the sub-periods

<table>
<thead>
<tr>
<th>Sub-periods</th>
<th>Null</th>
<th>Trace Statistics</th>
<th>5% Critical Value</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: 1960 - 1999</td>
<td>(d = 0)</td>
<td>20.8478</td>
<td>15.4947</td>
<td>0.0071</td>
</tr>
<tr>
<td></td>
<td>(d \leq 1)</td>
<td>2.2033</td>
<td>3.8415</td>
<td>0.1377</td>
</tr>
<tr>
<td>B: 2000 - 2011</td>
<td>(d = 0)</td>
<td>10.2812</td>
<td>15.4947</td>
<td>0.2597</td>
</tr>
<tr>
<td></td>
<td>(d \leq 1)</td>
<td>0.6487</td>
<td>3.8415</td>
<td>0.4206</td>
</tr>
</tbody>
</table>

**Notes:** The table presents the Johansen cointegration trace test statistics, the appropriate 5% critical values and the associated p-values; \(d\) is the number of cointegrating vectors under the null hypothesis; Mackinnon-Haug-Michelis (1999) \(p\)-values are employed.
Table 8: Granger causality test for the sub-periods with changes in house prices being the dependent variable

<table>
<thead>
<tr>
<th>Sub-periods</th>
<th>Chi-Sq</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: 1960 - 1999</td>
<td>13.4649</td>
<td>0.0092</td>
</tr>
<tr>
<td>B: 2000 - 2011</td>
<td>0.0018</td>
<td>0.9660</td>
</tr>
</tbody>
</table>

Notes: This table presents the chi-squared critical values and associated p-values for Granger causality tests constructed using a VAR model. The optimal lag lengths for the two models are determined by the AIC, and are found to be four and one in sub-periods A and B respectively.
Figure 1: Average price and annual rent (US$)
Figure 2: Price/rent ratio
Figure 3: Federal funds rate
Figure 4: Probabilities of being in State 1
Figure 5: House price vs rent (1960 = 100)
Figure 6: Actual price, fundamental value, and the relative bubble size
Figure 7: Response of real estate price changes to rent changes

(a) Intrinsic bubble period: 1960–1999

(b) No intrinsic bubble period: 2000–2011
More information on the construction of the data can be found at Land and Property Values in the US, Lincoln Institute of Land Policy. [http://www.lincolninst.edu/resources/](http://www.lincolninst.edu/resources/). While there approach to constructing the data is very sensible, it is possible that the interpolation and extrapolation methodology could cause some biases.

It is an interesting question as to whether intrinsic and periodically, partially collapsing rational bubbles can co-exist at the same time. Given that intrinsic bubbles are based on a nonlinear function of fundamental prices, and yet periodic, partially collapsing rational bubbles are based on the difference between actual prices and fundamentals, it seems as if they should be mutually exclusive, which is what we find empirically in the paper.

Although Markov switching regression models date back to Goldfeld and Quandt (1973), we use the Hamilton (1994) approach, whereby the transition from one state to another is modeled using a Markov chain. See Hamilton (1989 and 1994) for more details on how the probabilities are generated.

Of course, other nonlinear functional forms could be used here, and it is possible that a different functional form could lead to different results. However, we believe that the exponential form that is universally used in the current intrinsic bubble literature is not only the most intuitive, it is also fairly flexible and covers a wide range of possible shapes when both $c$ and $\lambda$ are allowed to vary.

This implies that if $\lambda$ satisfies equation (14), then (8) clearly satisfies the definition of the bubble in (7). Therefore, this can be defined as an intrinsic bubble. Froot and Obstfeld prove this mathematically in equation (13) on p. 1192 of their paper.

There would be collinearity amongst the explanatory variables if we decided to test for the presence of an intrinsic bubble using the formula $P(R_t) = \kappa R_t + c R_t^\lambda$.

The derivation of equation (16) is fairly straightforward. Blanchard and Watson (1982) show that the expected size of the bubble in the next period is given as:

$$E_r(B_{t+1}) = E_r(B_{t+1} | S) q + E_r(B_{t+1} | C) (1 - q)$$

This is then re-arranged to give the stochastic process in (16).

Note that in this study, the estimation of v NS’s bubble model is conducted using Matlab 7.9 and we use the Broyden et al. (BFGS) method for solving optimisation problems that are nonlinear.

This is in line with Lai and van Order’s (2010) finding that there was a regime shift in house prices which saw a rise in momentum from 1999 onwards. In addition, it appears as if there could be another regime switch around 1970 but unfortunately, this is too close to the start of the overall sample period and so splitting the data here would leave an insufficient number of observations before that date to estimate the models.

We must find $\lambda$ to be greater than one to show that a nonlinear explosive relationship exists between the rents and bubbles. This would mean that house prices may overreact to changes in rents.

Prior to performing the intrinsic bubble test, we rescale the price and rent data, dividing each data point by 1,000 so that the parameter estimates will be easier to interpret. This will, of course, not change any of the significance levels or conclusions from the analysis.

The term “flip” is defined as purchasing and selling an asset within a short period of time. An online article provided proof of this phenomenon occurring in the US housing market: [http://online.barrons.com/article/SB111905372884363176.html](http://online.barrons.com/article/SB111905372884363176.html).

For sub-period A, the value of computed $F$-statistic is 3.371 with a corresponding probability value of 0.047. Hence there is evidence to reject the null hypothesis of the joint insignificance of interest rates and unemployment in predicting changes in house prices. In sub-period B, however, the $F$-statistic is 1.973 with a probability value of 0.139 and so we cannot reject the null hypothesis.

Note that the scale is approximately ten times smaller for Figure 7(b) than Figure 7(a).

Miller et al. (1988) examine a very intense, localised speculative episode that occurred in Hawaii during the late 1980’s.