Presales, Leverage Decisions and Risk Shifting

Su Han Chan  
Professor  
Carey Business School  
Johns Hopkins University, USA  
schan@jhu.edu

Fang Fang  
Associate Professor  
School of Public Economics and Administration  
Shanghai University of Finance and Economics  
Email: ffang@mail.shufe.edu.cn

Jing Yang*  
Professor  
Department of Finance  
Mihaylo College of Business and Economics  
California State University at Fullerton, USA  
jyang@fullerton.edu

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* Contact Author
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Abstract

We set up a game-theoretic model for a market where the presale method (to sell a property before its completion) can be used together with construction loans and mortgages as a developer’s financing tools for project developments. This model captures the interactions and dynamics among four players (developer, buyer, mortgage lender and construction loan lender) involved in the presale market. We find that the use of the presale method together with an increase in the leverage (by using construction loan or mortgage) have risk shifting effects to other players in the market. In our model setup, the developer is the one who benefits if the mortgage lender and construction loan lender do not adjust their interest rates based on the developer’s presale strategy.

Keywords: Property presale · Real option · Risk shifting · Real estate financing
1 Introduction

The development of a real estate project is a complicated process, but has received little attention from researchers. In the U.S. and some other countries with mature financing systems, a developer normally takes out a construction loan (which sometimes includes a land loan) to finance the construction of a development project. After the construction is completed, the developer will pay back the construction loan either by borrowing a permanent loan (mortgage) or by using the sale proceeds of the project. The construction loan interest rate is normally adjusted by the perceived risk of the development and the credit history of the borrower. Under this setup, since the developer and construction loan lender only need to deal with each other, the system seems to work well.

However, recently the presale method (where a property is sold to buyers before its completion or even before the construction starts) has gradually gained momentum in some cities in the U.S. and Canada, with a concentration on high-rise condominium markets. Because mortgage lenders are reluctant to lend on a condominium unit if other units in the same condominium are vacant, some lenders might require developers to presell some of the units as part of the lending criteria. (For example, as of November 2010, the presale requirement for new condominium projects is 51%-70% by Fannie Mae, 70% by Freddie Mac and 30% by FHA.1) With the presale in place, at a given time there could be four players (developer, buyer, construction loan lender and mortgage lender) with different claims on a development project. In a normal market with a positive property appreciation rate, this presale mechanism can still work smoothly since buyers and developers will not default on the payment nor abandon the project. However, in a depressed market where it is in the best interest of the developer to abandon the project, it might be challenging to work out a solution among these four players.

1See an article titled “GSE and FHA Condo Rules” in January 17, 2013 Government Affairs Update.
The presale method was initially used by developers in Asia (for example, Taiwan and Hong Kong) several decades ago as a way to finance a development project. Since developers were not able to get a construction loan from a lender at that time, the developer presold the projects after the construction of a project started (or sometimes even before the start of construction) and buyers began to pay installments on the presale price to the developer after the contract was signed. In many cases, a significant portion of the presale price was paid to the developer before the project was completed. This method seemed to work, although it is frequently observed that a developer abandoned a presale project or buyers decided not to complete the presale contract.

However, the presale system has become much more complicated in Asian countries (for example, China) in recent decades. In China, mortgages can be obtained before a development project is completed. It is not uncommon to see that the buyer of a presale contract paid in the full presale price, which includes both the buyer’s equity and the mortgage, to the developer before the project is near completion. This type of payment schedules, which seems to offer little protection to the mortgage lender and the buyer, is a quite normal practice in many cities in China.\(^2\)

It should also be noted that, in a typical presale contract, a buyer can default on the presale payments when the future spot price is sufficiently lower than the remaining equity payment of the presale contract. (This is the buyer’s default option.) The developer can abandon the project when the remaining construction cost is unexpectedly high and/or the future spot price for the completed property is unexpectedly low. (This is the developer’s abandonment option.) In the presale market of China, the use of a construction loan and mortgage during the construction stage makes it more difficult to estimate the values of the default and abandonment options. For example, holding

a presale contract constant, the use of a construction loan will increase the probability for the developer to exercise the abandonment option because he now has less capital in the development project. The developer’s abandonment decision, in turn, will affect the payoffs of the buyer and the mortgage lender.

Similarly, holding the terms of the construction loan constant, an increase in the buyer’s payment or mortgage payment to the developer in the early stage of the development process will increase the probability that the developer defaults on the construction loan and, therefore, affect the construction loan lender’s profitability. Furthermore, the use of a mortgage can also increase the incentive for the buyer to default, as the buyer can now use the mortgage payment as part of his initial downpayment. A reduction in a buyer’s downpayment in a presale contract means there is less to lose when she defaults on the presale contract (and, therefore, it is more likely for her to default). When a buyer defaults, it might reduce the developer’s incentive to continue the project. This will, in turn, affect the profitability of the construction loan lender. Clearly, in a presale market with multiple players, the action of one player can shift the risk to other players and change the potential payoffs among players. Who will benefit from these complicated interrelationships and how can we make the market work more efficiently? These are the research questions that beg for answers.\(^3\)

Given the size of the real estate development market, it is surprising that research on the presale method and construction loans is still quite limited. The handful studies on construction loans concentrate their efforts on the risk factors and the impact of credit availability on the construction industry. Lusht and Leidenberge (1979) find that the residential construction lending risk is driven by the unavailability of materials, inflationary cost, borrower’s development experience, and lender’s

\(^3\)We thank Professor William Hardin for raising this research topic at the 2008 AsRES Annual Meeting in Shanghai.

In this paper we explore the impact of a decision from one player on the payoffs of all the four players (developer, buyer, mortgage lender, and construction loan lender) in a presale market. Specifically, we want to see how the decision of one player can create a risk shifting effect and change the payoffs to the other players. In our model, we show that a developer will set the presale price at a level to ensure that the buyer is indifferent from either buying the property at the presale market or purchasing it at the future spot market. Within this framework, we are able to show the risk shifting effects when a presale contract changes the downpayment ratios of the buyer, mortgage lender or construction loan lender. We demonstrate that developers or buyers can benefit from an increase in leverage; however, mortgage and construction loan lenders are the ones who suffer. Our next section introduces the model framework. The third section sets up the model framework while section four derives the equilibrium solutions for the four players in the presale market. The fifth section discusses the model results and the last section contains our conclusions.
2 Model Framework

To facilitate comparisons, our model framework and notations follow closely the presale pricing model developed by Chan, Wang and Yang (2012). Different from their study that only describes a game between a presale buyer and a developer, we model the situation in which a developer will build a single-unit property with his own equity, presale proceeds from the buyer, construction loan and mortgage loan. Specifically, we add a construction loan lender and a mortgage lender into the model framework. Our model assumes that the project is a positive NPV project and the developer has already decided to use the presale method as the selling tool. The two decisions the developer needs to make are a presale price \( p_p \) today and an abandonment decision in the future.

In our model, the developer will set the presale price at the level that the buyer will be indifferent between either purchasing the property today at the presale price \( p_p \) or purchasing an identical unit at the future spot price \( p \).

We use three dates \((t = 0, t = 1\) and \(t = 2\)) to model the developer’s decisions. Since it is a positive NPV project, the developer will launch the first-phase construction at \( t = 0 \). At this time, the developer will decide on the presale price \( p_p \). We assume that, at \( t = 0 \), both the developer and the buyer do not know exactly what the realized construction cost and the future spot price will be. However, they both know the probability distribution of the construction cost \( \ddot{c} \sim \Phi [\bar{c} - \theta, \bar{c} + \theta] \) and the probability distribution of the future spot price \( \ddot{p} \sim \Omega [\bar{p} - \delta, \bar{p} + \delta] \). Here, \( \bar{c} \) is the expected construction cost and \( \bar{p} \) is the expected future spot price. The other two parameters, \( \theta \) and \( \delta \), indicate the magnitudes of the variations from the expectations. For simplicity and without loss of generality, we assume both \( \ddot{c} \) and \( \ddot{p} \) follow uniform distributions.

For simplicity, we assume that the buyer’s payment schedule is exogenously determined. The
developer will presell the property to the buyer based on the payment schedule. We assume that, at the time of presale, the developer has already reached a construction loan agreement with a construction loan lender on the loan interest rate $i$, loan to expected construction cost ratio $\beta$, and the first-phase construction loan release ratio $\gamma$ (which is based on the expected construction loan amount $\beta \overline{c}$). It should be noted that this assumption implies that all the parameters ($i$, $\beta$ and $\gamma$) related to the construction loan are exogenously determined. We also assume that, at the presale stage, the buyer is able to obtain a mortgage with interest rate $r$, loan to expected spot price ratio $l$, and initial payment schedule factor $m$ (which is the percentage of the total mortgage loan amount $l \overline{p}$). Since the loan to value ratio is typically determined by the appraisal value, this assumption reflects the real world practice. We assume that there is no distinction on the terms of the mortgage ($r$, $l$ and $m$) if the mortgage is obtained at the presale market or at the spot market. In other words, these parameters ($r$, $l$ and $m$) are also exogenously determined.

If the buyer will not default on the presale contract, the buyer will make a total of three payments to the developer. The first two payments are proportional to the presale price $p_p$. It should be noted that, since the mortgage payments are based on the expected future price $l \overline{p}$ (while the equity payments are based on the presale contract price $p_p$), the buyer will also need to pay a third payment $l(p_p - \overline{p})$ to make sure that the total payment from the buyer and the mortgage lender equals the presale price. Given this, the buyer’s initial equity downpayment (at $t = 0$) is a portion $d$ of the sum of the first two payments $(1 - l)p_p$.

At $t = 0$, with the two probability distributions ($c$ and $\overline{p}$), the buyer’s equity downpayment ratio $d$, the terms for mortgage and construction loan ($i$, $\beta$, $\gamma$, $r$, $l$ and $m$),\footnote{To simplify the presentation, we assume that the developer is risk neutral and the real interest rate is zero. In this regard, the interest rates of the construction loan and the mortgage can be considered as the rates above the real interest rate. Our model results are the same without this assumption, but the presentation will be much more complicated.} and the two embedded options
(the developer can abandon the project and the buyer can default on the remaining payment) in mind, the developer sets the presale price $p_p$ at the level that the buyer is willing to buy the property at the presale stage. Exhibit 1 shows the decision flows of this model.

[Insert Exhibit 1 here]

After the presale price $p_p$ is set and accepted by the buyer, the developer receives a presale down-payment $d(1 - l)p_p$ from the buyer. The mortgage lender pays the developer an initial mortgage downpayment $ml\overline{p}$. The developer also receives a first-phase construction loan $\gamma \beta \bar{c}$ from the construction loan lender. With these three payments, the developer will complete the first-phase construction of the project at $t = 1$. At $t = 1$, the information on the actual construction cost $c$ and spot price $p$ will be realized and revealed to all players. At this stage, the buyer will decide if she wants to continue with the presale contract. She will continue with the presale contract if she knows that the developer will continue the project and it is beneficial for her to continue too. If this happens, the mortgage lender will pay the remaining mortgage amount $(1 - m)l\overline{p}$, and the buyer will pay the second equity payment $(1 - d)(1 - l)p_p$, followed by the third payment $l(p_p - \overline{p})$ to conclude the contract. At $t = 2$, the buyer will pay off the mortgage and its interest, $l\overline{p}(1 + r)$, and obtain the property. However, if the buyer decides not to continue with the presale contract, she will lose the initial downpayment of the presale contract and will not obtain the property at $t = 2$ (even if the developer eventually finishes the project). In our model, we assume that the buyer will not pay back the mortgage lender the mortgage downpayment that totals $ml\overline{p}(1 + r)$ if she defaults on the presale contract or if the developer abandons the project. Clearly, the buyer can exercise a default option at $t = 1$ once she knows the actual construction cost $c$ and spot price $p$. 

7
At $t = 1$, the developer can decide whether to continue with the construction based on the realized construction cost $c$ and the realized spot price $p$. If the developer chooses to continue with the construction, he will borrow an additional amount of construction loan $(1 - \gamma)\beta\bar{c}$ from the construction loan lender to finance the rest of the construction and the project will be completed at $t = 2$. Under this circumstance, the developer will pay off the construction loan $\beta\bar{c}(1 + i)$ at $t = 2$. If at $t = 1$, the developer decides to abandon the project, the buyer will lose the presale downpayment, the construction lender will lose the initial-phase construction loan, the mortgage lender will lose the mortgage downpayment, and the developer will no longer have a claim on the property anymore.

Both the mortgage lender and construction loan lender are passive players in the model. The mortgage lender loses the initial mortgage downpayment $ml\bar{p}$ when the buyer defaults on the presale contract or the developer abandons the project, but will receive $lp(1 + r)$ at $t = 2$ if both the developer and the buyer continue with the presale contract. If the developer continues the project, the construction loan lender will receive $\beta\bar{c}(1 + i)$ at $t = 2$ when the project is completed. If the developer exercises his abandonment option, the construction loan lender will lose the first-phase construction loan $\gamma\beta\bar{c}$. Exhibit 2 reports the payoffs of the four players (developer, buyer, mortgage lender and construction loan lender) under the four possible courses of actions (both developer and buyer do not exercise their options, only buyer exercises the default option, only developer exercises the abandonment option, and both exercise their options).

[Insert Exhibit 2 here]
2.1 Decision rules for the options

We need to take two steps to derive our model results. First, at \( t = 0 \), the developer decides on the presale price, \( p_p \), which must be acceptable to the buyer. Second, at \( t = 1 \), the developer decides on whether to abandon the project at \( t = 1 \) and the buyer decides on whether to commit on the remaining presale payments. This means that, before we can derive the developer’s optimal presale price at \( t = 0 \), we need to use a backward induction procedure to solve for the buyer’s default decision rule and developer’s abandonment decision rule. These decision rules will allow us to calculate the payoff of each player. Lemma 1 reports the results.

**Lemma 1**

(1) The buyer will not exercise the default option if

\[ p \geq \hat{p} = [1 - d(1 - l)] p_p + r \bar{p}. \]  

(2) The developer will not exercise the abandonment option if the buyer will not default and

\[ c \leq \hat{c} = \frac{1}{(1 - h)} \{[1 - d(1 - l)] p_p - lm \bar{p} - \beta \bar{e}(\gamma + i)\}. \]  

(3) The developer will not exercise the abandonment option if the buyer will default and

\[ c \leq \hat{c}' = \frac{1}{(1 - h)} [p - \beta \bar{e}(\gamma + i)]. \]

**Proof.** See Appendix A.  ■

The decision rules derived in Lemma 1 are very intuitive. A buyer should default if the realized spot price \( p \) is lower than her remaining presale payment plus mortgage payment, which is \([1 - d(1 - l)] p_p + r \bar{p}\). Similarly, a developer should abandon the project if the realized remaining construction cost \((1 - h) c\) exceeds the net benefits from the completion of the project, which is \([1 - d(1 - l)] p_p - lm \bar{p} - \beta \bar{e}(\gamma + i)\) if the buyer commits on the presale payment and \( p - \beta \bar{e}(\gamma + i) \) if
the buyer defaults. Note that $-\beta \bar{c}(\gamma + i) = \beta \bar{c}(1 - \gamma) - \beta \bar{c}(1 + i)$, where $\beta \bar{c}(1 - \gamma)$ is the second-phase construction loan payment, and $\beta \bar{c}(1 + i)$ is the total payoff for the construction loan.

### 2.2 Pay-off functions of the four players

With the decision rules specified in Equations (1), (2) and (3), we can now derive the expected payoffs of all the four players in our model. We start with the buyer’s expected cost for the presale contract. It should be noted that the buyer faces two choices: to accept the presale contract at $t = 0$ or to buy the property at a future spot price at $t = 2$. Equation (4) calculates the expected presale purchase cost of a buyer $E(C(p_p))$ for accepting a presale price $p_p$ offered by the developer. The first term in Equation (4) is the buyer’s expected purchase cost if both the buyer and the developer do not exercise their default and abandonment options. The second term describes the buyer’s purchase cost if the developer abandons the project. The last term calculates the buyer’s purchase cost if the buyer defaults on the remaining payment (regardless of if the developer abandons the project).

$$E(C(p_p)) = \int_{\bar{p}}^{\bar{p} + \delta} \int_{\bar{c}}^{\bar{c} + \theta} (p_p + rl\bar{p}) d\Phi(\bar{c})d\Omega(\bar{p})$$

$$+ \int_{\bar{p}}^{\bar{p} + \delta} \int_{\bar{c}}^{\bar{c} + \theta} [d(1 - l)p_p + \bar{p}] d\Phi(\bar{c})d\Omega(\bar{p})$$

$$+ \int_{\bar{p} - \delta}^{\bar{p}} \int_{\bar{c} + \theta}^{\bar{c}} [d(1 - l)p_p + \bar{p}] d\Phi(\bar{c})d\Omega(\bar{p}).$$

Alternatively, the buyer can choose to reject the presale offer and wait till $t = 2$ to buy an identical property in the future spot market. Her expected purchase cost under this scenario is

$$E(C)|_{\text{reject}} = \int_{\bar{p} - \delta}^{\bar{p} + \delta} \int_{\bar{c} - \theta}^{\bar{c} + \theta} \bar{p}d\Phi(\bar{c})d\Omega(\bar{p}) = \bar{p},$$

where $\bar{p}$ is the expected future spot price. Since we know that the developer has to offer a presale price $p_p$ that ensures $E(C(p_p))$ is no more than $E(C)|_{\text{reject}}$ (so that the buyer will accept the presale
contract), the necessary and sufficient condition for a buyer to accept a presale price \( p_p \) is

\[
g(p_p) = E(C(p_p)) - E(C)_{\text{reject}} \leq 0. \tag{6}
\]

With the buyer’s decision rule specified in Equation (6),

\[5\] for a given presale price \( p_p \), the developer’s expected profit from the presale contract is

\[
E(\pi) = \int_{\hat{p}}^{p_p+\delta} \int_{\tau-\theta}^{\tilde{c}} (p_p - \tilde{c} - i\beta\tau) d\Phi(\tilde{c})d\Omega(\tilde{p})
\]

\[\quad + \int_{\hat{p}}^{p_p+\delta} \int_{\tilde{c}}^{\tau+\theta} (d(1-l)p_p + ml\tilde{p} + \gamma\beta\tau - h\tilde{c})d\Phi(\tilde{c})d\Omega(\tilde{p})
\]

\[\quad + \int_{\hat{p}}^{\hat{p}} \int_{\tau-\theta}^{\tilde{c}} (d(1-l)p_p + ml\tilde{p} + \tilde{p} - \tilde{c} - i\beta\tau)d\Phi(\tilde{c})d\Omega(\tilde{p})
\]

\[\quad + \int_{\hat{p}}^{\hat{p}} \int_{\tau-\theta}^{\tilde{c}} (d(1-l)p_p + ml\tilde{p} + \gamma\beta\tau - h\tilde{c})d\Phi(\tilde{c})d\Omega(\tilde{p}). \tag{7}
\]

The first, second, third and fourth terms in Equation (7) describe the expected payoff of the developer under four different situations: if none of the two players exercise their options, if only the developer exercises the abandonment option, if only the buyer exercises the default option and if both the developer and the buyer exercise their options. Note that when the developer abandons the project, we assume that he will not pay back the downpayments from the buyer, the mortgage lender and the construction loan lender.

With the mortgage interest rate \( r \) and the loan to price ratio \( l \) exogenously determined, for a given presale price \( p_p \) selected by the developer, the expected return of the mortgage lender is

\[
E(Rp) = \int_{\hat{p}}^{p_p+\delta} \int_{\tau-\theta}^{\tilde{c}} rml\tilde{p}d\Phi(\tilde{c})d\Omega(\tilde{p}) + \int_{\hat{p}}^{p_p+\delta} \int_{\tilde{c}}^{\tau+\theta} (-ml\tilde{p})d\Phi(\tilde{c})d\Omega(\tilde{p}) + \int_{\hat{p}}^{\hat{p}} \int_{\tau-\theta}^{\tilde{c}} (-ml\tilde{p})d\Phi(\tilde{c})d\Omega(\tilde{p}). \tag{8}
\]

The first, second and third terms in Equation (8) describe the expected mortgage return \( E(Rp) \) under three possible situations: if no party defaults, if only the developer abandons the project

\[5\] It should be noted that since the project is a positive NPV project, when the presale price is set to be at least the same as the expected future spot price, the presale contract should also be a positive NPV project.
and if the buyer defaults on the remaining payment (regardless of whether or not the developer abandons the project). Finally, since the construction loan interest rate $i$ and the loan to price ratio $\beta$ are also exogenously determined, for a given presale price $p_p$ selected by the developer, the expected return of the construction loan is

$$E(Bp) = \int_{\hat{p}}^{\hat{p}+\delta} \int_{\tau-\theta}^{\hat{c}} i\beta\tau d\Phi(\hat{c})d\Omega(\hat{p}) + \int_{\hat{p}}^{\hat{p}+\theta} \int_{\hat{c}}^{\hat{c}+\theta} (-\gamma\beta\tau)d\Phi(\hat{c})d\Omega(\hat{p})$$

(9)

The four terms in Equation (9) describe the expected construction loan return $E(Bp)$ with the same four possible situations used in Equation (7) for calculating the developer’s expected profit $E(\pi)$.

### 3 The Equilibrium Contract

The developer will select a presale price that maximizes his profit as specified by Equation (7), subject to the constraint that the presale price $p_p$ must be accepted by the buyer (or when the buyer’s expected purchase cost $E(C(p_p))$ based on the presale price $p_p$ is no more than the expected future spot price $\bar{p}$). Given this, the optimal presale price $p_p^*$ satisfies

$$p_p^* \in \{p_p|g(p_p) = 0\} \text{ and } E(C(p_p^*)) = \bar{p},$$

(10)

where $g(p_p)$ is defined in Equation (6). This means that, in equilibrium, the developer will offer a presale price that makes the buyer just indifferent between accepting the presale price today and going for a spot sale in the future. Since the developer’s expected presale profit $E(\pi)$ is increasing in the presale price $p_p$, the optimal presale price $p_p^*$ selected by the developer must be the highest price that can be accepted by the buyer. Since the buyer’s expected purchase cost $E(C(p_p))$ is also increasing in $p_p$, the highest boundary of the expected purchase cost that can be accepted by
the buyer must be at $E(C(p_p)) = \bar{p}$. Given this, the developer’s optimal price $p_p^*$ must satisfy the condition $E(C(p_p^*)) = \bar{p}$.\footnote{Equations (29), (30), (40), (44) in appendix B will formally prove that both the buyer’s expected presale cost $E(C)$ and the developer’s expected presale profit $E(\pi)$ are increasing in the presale price $p_p$.}

While there is a closed-form solution of the equilibrium price $p_p^*$ for Equation (10), the result of this constrained optimization problem is too lengthy and complicated to draw any inference. Given this, we adopt the approach used by Chan, Wang and Yang (2012) and start our analyses with two extreme cases: the case when only the buyer’s default option is valuable and the case when only the developer’s abandonment option is valuable. If the signs of these two extreme cases are consistent, it is very likely that the conclusions can be applied to the general case.

### 3.1 Conditions for the two extreme cases

We first analyze the case when the value of the developer’s abandonment option is zero while the value of the buyer’s default option is still positive. This case, Case 1, requires three conditions:

$$\bar{c} + \theta \leq \bar{c} = \frac{1}{(1-h)} \{ [1 - d(1-l)] p_p^* - lm\bar{p} - \beta\bar{c}(\gamma + i) \} , \quad (11)$$

$$\bar{p} - \delta < \hat{p} = [1 - d(1-l)] p_p^* + rl\bar{p}, \text{ and} \quad (12)$$

$$\bar{c} + \theta \leq \bar{c}'|_{p=p-\delta} = \frac{1}{(1-h)} [\bar{p} - \delta - \beta\bar{c}(\gamma + i)] . \quad (13)$$

Specifically, Equation (11) is the condition to ensure that the abandonment option value under a presale is zero. The developer will never abandon a project if the maximum expected remaining construction cost $(1-h)(\bar{c} + \theta)$ is lower than the net presale payoff from continuing the project,

$$[1 - d(1-l)] p_p^* - lm\bar{p} - \beta\bar{c}(\gamma + i) = (1-d)(1-l)p_p^* + l(p_p^* - \bar{p}) + (1-m)l\bar{p} + \beta\bar{c}(1-\gamma) - \beta\bar{c}(1+i), \quad (14)$$

where $(1-d)(1-l)p_p^* + l(p_p^* - \bar{p})$ is the total remaining payment from the buyer, $(1-m)l\bar{p}$ is the remaining payment from the mortgage lender, $\beta\bar{c}(1-\gamma)$ is the second-phase construction loan
injection, and $\beta \bar{c}(1 + i)$ is the construction loan payback at the end.

Equation (12) is the condition to ensure that the buyer still has the option to default on the presale contract. From Equation (1), we know that a buyer’s default option has value when the lower bound of the spot price distribution ($\min\{\bar{p}\} = \bar{p} - \delta$) is lower than the buyer’s remaining payment

$$\hat{p} = [1 - d(1 - l)] p^*_p + rl\bar{p} = (1 - d)(1 - l)p^*_p + l(p^*_p - \bar{p}) + l(1 + r)\bar{p},$$

which includes her total remaining equity payment $(1 - d)(1 - l)p^*_p + l(p^*_p - \bar{p})$ and the amount of the mortgage payback $l(1 + r)\bar{p}$. Finally, Equation (13) is the condition to ensure that the developer’s abandonment option value under a spot sale is also zero. The developer will never abandon a project if the maximum expected remaining construction cost $(1 - h)(\bar{c} + \theta)$ is lower than the minimum net spot sale payoff from continuing the project $\bar{p} - \delta - \beta \bar{c}(\gamma + i)$, where $\bar{p} - \delta$ is the minimum spot sale price, and $\beta \bar{c}(\gamma + i)$ is the developer’s net payment to the construction loan lender (which is the same as that under a presale).

The second case, Case 2, is when the value of the developer’s abandonment option is positive while the buyer’s default option has no value. Intuitively, the required conditions are exactly opposite to Equations (11) and (12), or

$$\bar{c} > \hat{c} = \frac{1}{(1 - h)} \left\{ [1 - d(1 - l)] p^*_p - lm\bar{p} - \beta \bar{c}(\gamma + i) \right\} \quad \text{and}$$

$$\bar{p} - \delta \quad \geq \quad \hat{p} = [1 - d(1 - l)] p^*_p + rl\bar{p}.$$  

3.2 Model Results

To derive equilibrium results, we first prove that the developer’s optimal presale price must satisfy Equation (10). Once this condition is satisfied, we can derive the closed-form solution of the optimal presale price of a presale contract. After that, we impose the conditions displayed in Equations
(11), (12) and (13) to derive the equilibrium results for Case 1 (when only the buyer’s default option has value) and impose the conditions derived in Equations (16) and (17) to derive the equilibrium results for Case 2 (when only the developer’s abandonment option has value). Proposition 1 reports the equilibrium results of the two cases.

**Proposition 1** This proposition reports the equilibrium price of a presale contract $p_p^*$, the buyer’s probability to default voluntarily $P_{def}$, the buyer’s total probability to default (including voluntary default and default due to developer’s abandonment decision) $TP_{def}$, the developer’s probability to abandon the construction $P_{ab}$, the buyer’s expected cost of the presale contract $E(C)$, the developer’s expected presale profit $E(\pi)$, the mortgage loan expected return $E(Rp)$ and the construction loan expected return $E(Bp)$:

<table>
<thead>
<tr>
<th>Case 1: only positive value for default option</th>
<th>Case 2: only positive value for abandonment option</th>
</tr>
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<tbody>
<tr>
<td>$p_p^*$</td>
<td>$\frac{(1 - rl)Y + \delta - 2A}{T}$</td>
</tr>
<tr>
<td>$P_{def}$</td>
<td>$\frac{TP_p^* + \delta - (1 - rl)p}{2\delta}$</td>
</tr>
<tr>
<td>$P_{ab}$</td>
<td>0</td>
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<tr>
<td>$TP_{def}$</td>
<td>$\frac{TP_p^* + \delta - (1 - rl)p}{2\delta}$</td>
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<tr>
<td>$E(\pi)$</td>
<td>$\frac{1}{T} { [2p_p^* - (1 + i\beta)\bar{c}] [(1 - rl)p + \delta - p_p^*T] + [p_p^*T - (1 - rl)p\bar{c} + \delta] [p_p^*Y + (\bar{p} - \delta) + l(2m + r)p - 2\gamma(1 + i\beta)] }$</td>
</tr>
<tr>
<td>$E(Rp)$</td>
<td>$\frac{1}{2} { {Y + [TP_p^* - \bar{c}F - lm\bar{p}]^2 + 2(1 - h)\theta[Y_p^* - \bar{c}G + lm\bar{p}]} }$</td>
</tr>
<tr>
<td>$E(Bp)$</td>
<td>$\frac{1}{2} { [TP_p^* - \bar{c}F - lm\bar{p}] + \theta(1 - h)(r - m) }$</td>
</tr>
</tbody>
</table>

Here $A = \sqrt{d(1-l)\delta[T(1-lr) + \delta]}$, $T = 1 - d(1-l)$, $Y = 1 + d(1-l)$, $Z = 1 + l(m-r)$, $F = 1 - h + \beta(i+\gamma)$, $G = 1 + h + \beta(i-\gamma)$, and $W = \sqrt{[(1 - h)\theta Y - T(\bar{c}F + \bar{p}Z)]^2 - 4T^2\bar{p}(1 - lr) [lm\bar{p} + \bar{c}F - (1 - h)\theta]}$.

**Proof.** See Appendix B. ■

It should be noted that the equilibrium price $p_p^*$ reported in Case 1 and Case 2 of Proposition 1 can be reduced to the $p_p^*$ derived by Chan, Wang and Yang (2012) if we take the construction loan...
and mortgage loan out of the game (or by setting $l = 0$ and $\beta = 0$). Similar to Chan, Wang and Yang (2012), we did not report the equilibrium price $p_p^*$ for the general case as the result is lengthy and does not carry any intuition.

With the results reported in Proposition 1, we are now in a position to analyze how these results can be affected by changes in decision parameters. Specifically, we study how the changes in the downpayment ratios ($d$ for the buyer’s equity, $m$ for the mortgage, and $\gamma$ for the construction loan) affect the equilibrium results of $p_p^*$, $P_{def}$, $P_{ab}$, $T_{P_{def}}$, $E(\pi)$, $E(C)$, $E(Rp)$ and $E(Bp)$.

### 4 Model Implications

Compared to Chan, Wang and Yang (2012), our model is unique in that, in addition to the developer and the buyer, we also include mortgage and construction loan lenders. Correspondingly, we will focus our analyses on how the change in the downpayment ratios (from presale contract $d$, mortgage $m$ and construction loan $\gamma$) affect the equilibrium presale price $p_p^*$, the three players’ expected profits ($E(\pi)$ for developer, $E(Rp)$ for mortgage lender, and $E(Bp)$ for construction loan lender) and the exercise decisions of the players (characterized by the buyer’s probability to default voluntarily $P_{def}$, the buyer’s total probability to default including voluntary default and default due to developer’s abandonment decision, $T_{P_{def}}$, and the developer’s probability to abandon the construction $P_{ab}$). It should be noted that in our model $E(C)$ equals to a constant $\overline{p}$ (the expected future price). Consequently, $E(C)$ will not be affected by any parameter other than $\overline{p}$. Given this, we will not report the comparative results of the buyer’s expected cost $E(C)$.

**Proposition 2** This proposition reports the effects of the downpayment ratios (from buyer, $d$, mortgage lender, $m$, and construction loan lender, $\gamma$) on the equilibrium outcomes of the presale price $p_p^*$, the buyer’s probability to default voluntarily $P_{def}$, the buyer’s total probability to default...
(including default voluntarily and default due to developer’s abandonment decision) $TP_{\text{def}}$, the developer’s probability to abandon the construction $P_{\text{ab}}$, the developer’s expected presale profit $E(\pi)$, the mortgage lender’s expected return $E(R_p)$ and the construction loan lender’s expected return $E(B_p)$:

### Exhibit 3 Effects of Downpayment Ratios ($m, \gamma$ and $d$) on Equilibrium Outcomes

<table>
<thead>
<tr>
<th>Ratios:</th>
<th>$p^*_b$</th>
<th>$P_{\text{def}}$</th>
<th>$P_{\text{ab}}$</th>
<th>$TP_{\text{def}}$</th>
<th>$E(R_p)$</th>
<th>$E(B_p)$</th>
<th>$E(\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$: Mortgage Downpayment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>−</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Case 2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>−</td>
<td>+ if $\Delta &gt; 0$</td>
</tr>
<tr>
<td>$\gamma$: Initial-Phase Construction Loan</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Case 2</td>
<td>−</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>−</td>
<td>+ if $\Delta &gt; 0$</td>
</tr>
<tr>
<td>$d$: Buyer’s Downpayment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>−</td>
<td>−</td>
<td>0</td>
<td>−</td>
<td>+</td>
<td>0</td>
<td>NA</td>
</tr>
<tr>
<td>Case 2</td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>NA</td>
</tr>
</tbody>
</table>

Here, Case 1 occurs when Equations (11), (12) and (13) hold (when only the default option has a positive value); Case 2 occurs when Equations (16) and (17) hold (when only the abandonment option has a positive value); $\Delta = (\bar{c} + \theta - \hat{c}) - \frac{(\hat{p} - \overline{p})[\hat{c} - (\bar{c} - \theta)T + 2d\theta(1 - l)]}{WT}$, with $\hat{c}$ and $\hat{p}$ defined in Equations (16) and (17).

**Proof.** See Appendix C. □

### 4.1 The effects of the mortgage downpayment ratio $m$

We will start our analysis with the effects the mortgage downpayment ratio $m$ have on the equilibrium outcomes of the four players. We analyze Case 1 (only the default option has a positive value) first. From Equation (1) we know that the mortgage downpayment $m$ does not affect the buyer’s default decision, as the buyer’s default decision is based on her remaining presale payment, total mortgage payment (a change in the mortgage downpayment $m$ does not affect the total mortgage
payment at the end), and the realized spot price of the property. Given this, if the developer does not abandon the project, the mortgage initial payment $m$ does not affect the buyer’s expected property purchase cost under a presale contract. When this cost is unaffected, the equilibrium presale price $p_p^*$ must be unaffected as well (see Equation (10)). The mortgage downpayment ratio $m$ will not affect the construction loan lender’s expected return either when the developer does not abandon the project. Therefore, as shown in Case 1 of Proposition 2, a change in the mortgage downpayment $m$ has no effect on the buyer’s probability of voluntary default $P_{def}$, the buyer’s total probability to default $TP_{def}$ (which includes voluntary default and default due to developer’s abandonment decision), and the developer’s probability to abandon the project $P_{ab}$. Additionally, because it does not affect the developer’s abandonment probability, it does not affect the construction loan lender’s return $E(Bp)$. A large mortgage initial downpayment ratio $m$ will decrease the mortgage lender’s profit $E(Rp)$ and in the meantime increase the developer’s presale profit $E(\pi)$. This is true because when the buyer defaults, a large mortgage downpayment means that the mortgage lender has more to lose while the developer has more to gain.

Case 2 of Proposition 2 reports the results when the buyer will not default and the value of the developer’s abandonment option is positive. Developer will abandon a project if the benefits he will receive (which include the remaining mortgage payment) upon completion of the project is less than the additional construction cost he needs to put in. Given this, while an increase in the mortgage downpayment ratio $m$ has no effect on the buyer’s probability of voluntary default $P_{def}$, it will increase the likelihood that the developer will abandon the project $P_{ab}$. An increase in the likelihood for the developer to exercise the abandonment option will decrease the equilibrium presale price $p_p^*$ because the buyer will price this additional risk. Furthermore, a higher abandonment risk will increase the buyer’s total probability to default $TP_{def}$ (because the buyer has to default when
the developer abandons the project). Given this increased total profitability for the buyer to default $TP_{def}$, the mortgage loan expected return $E(R_p)$ drops. By the same token, a higher abandonment risk will bring down the construction loan expected return $E(B_p)$. If this is a zero sum game, when both the construction loan lender and the mortgage lender suffer and the position of the buyer stays the same, the developer’s expected presale profit $E(\pi)$ should increase. Mathematically, this happens when $\Delta > 0$.\(^7\)

Our analyses indicate that when a fixed rate mortgage is used, there is a strong incentive for the developer to get as high a mortgage downpayment as possible. With an increase in the mortgage downpayment, both the mortgage lender and construction loan lender will suffer because of the increase in the probabilities of default and abandonment. The developer will most likely be the only player who benefits from this leverage increase, as the buyer will be indifferent about the arrangement (through the adjustment of the presale price $p_p^*$). It should be noted that, at least in our model, the mortgage lender and the construction loan lender are not compensated for the risk shifted to them from the use of more mortgage. This demonstrates that a mortgage lender should adopt a risk-adjusted mortgage pricing system when a presale contract is used.

While this problem (the use of mortgage at the presale stage) is less relevant to the presale market in the U.S. (where the buyer will not obtain a mortgage until the project is complete, or the mortgage downpayment $m = 0$), the use of mortgage is very common in other presale markets of the world. For example, in China, it is not uncommon that the developer requires the buyer to pay the full presale price (using equity and mortgage) before the project is far from completion. Clearly, mortgage lenders should understand the risk they are taking in this type of presale market.

\(^7\)We run sensitivity analyses with reasonable parameters. The results show that $\Delta > 0$ in every scenario we have tried. Also note that if the opposite situation $\Delta \leq 0$ holds, it indicates that all three players (developer, mortgage lender and construction loan lender) in the model are worse off when the mortgage downpayment is larger. This is not very likely to happen in the real world.
However, while it might not be obvious, our results also demonstrate that construction loan lenders will also suffer from the introduction of mortgages into the presale system. Developers, on the other hand, should be the winners from an increase in the mortgage downpayment.

### 4.2 The effects of the first-phase construction loan ratio $\gamma$

We now turn our attention to the effects of the first-phase construction loan ratio $\gamma$ on the equilibrium outcomes. Again, we start with Case 1, the situation where the developer will not abandon the project. In this environment, an increase in the first-phase construction loan ratio $\gamma$, by assumption, should not affect the developer’s abandonment risk $P_{ab}$. Since a buyer’s default decision does not involve the remaining construction loan, $\gamma$ does not affect the buyer’s default probability $P_{def}$ either. Given that the first-phase construction loan ratio $\gamma$ has no impacts on the default and abandonment decisions, it does not affect the presale price $p_p^*$. Without changes in option values and $p_p^*$, there will be no changes on the commitments of the buyer and the developer for paying back their loans. Consequently, the mortgage loan return $E(Rp)$, the construction loan return $E(Bp)$ and the developer’s expected presale profit $E(\pi)$ will not be affected by $\gamma$. In other words, an increase in $\gamma$ does not affect any equilibrium outcome when the developer will for sure not exercise the abandonment option.

We now examine Case 2, the situation where the abandonment option has a positive value while the default option value is zero. Under this circumstance, an increase in the first-phase construction loan ratio $\gamma$ enlarges the construction loan lender’s downpayment loss to the developer when the project is abandoned. It will also increase the probability that the developer abandons the project $P_{ab}$ by reducing the cash inflow for the developer to complete the remaining construction (a higher $\gamma$ means that the developer will receive less from the construction loan lender to finance the remaining construction costs). Given this, an increase in the first-phase construction loan ratio will shrink
the construction loan return $E(Bp)$. The buyer will demand a lower presale price $p_p^*$ when the developer’s abandonment risk is higher. Correspondingly, an increase in $\gamma$ will indirectly reduce the buyer’s default probability $Pdef$, as her remaining equity payment for fulfilling the presale contract will be reduced with a lower $p_p^*$. We also know that the buyer has to default when the developer abandons the project. Given this, an increase in $\gamma$ will increase the buyer’s total default risk $TPdef$. This will, in turn, reduce the mortgage loan return $E(Rp)$. Again, under a zero sum game, when both the construction loan lender and the mortgage lender lose, the developer must be the player who enjoys the benefits (an increase in developer’s profit $E(\pi)$) when the buyer is indifferent about the outcome. Mathematically, this will happen if $\Delta > 0$.

Our analyses indicate that when the interest rate of a construction loan is not risk-adjusted according to the total financing package used (which also includes the presale contract and mortgage), the developer (or the buyer under alternative arrangements) can take advantage of this situation. The ones who will suffer from this construction loan downpayment increase include the mortgage lender. When the developer abandons a project, the buyer must also default on the mortgage. Given this, an increase in the first-phase construction loan ratio will hurt not only the construction loan lender but also the mortgage lender. In other words, the mortgage lender’s return reduces because of the actions of the developer and the construction loan lender. As a result, it might be prudent for the mortgage lender to be involved in the developer’s discussion with the construction loan lender on the construction loan arrangement.

4.3 The effects of the buyer’s downpayment ratio $d$

The effects of the buyer’s downpayment ratio on her default decision and the developer’s abandonment decision have been discussed extensively in Chan, Wang and Yang (2012). Given this, we will not analyze the developer’s expected profit $E(\pi)$ here. In this subsection, we will only exam-
ine the relationships among the developer, the construction loan lender and the mortgage lender when the buyer increases her presale downpayment ratio \( d \). It should be noted that the use of a construction loan together with a presale method is a common practice in most places around the world (including the U.S.A.).

When the buyer’s default option value is positive and the developer’s abandonment option has no value (Case 1), an increase in the buyer’s presale downpayment \( d \) will reduce the buyer’s probability of default \( P_{def} \) and, therefore, also decrease her total probability of default \( TP_{def} \). This is true because the buyer will default voluntarily if the remaining equity payment is larger than the realized spot price. Since an increase in the downpayment will reduce the remaining equity payment, it is less likely that the buyer will default voluntarily. This will increase the mortgage lender’s return \( E(R_p) \) as the mortgage lender is less likely to suffer a loss due to the buyer’s default. To compensate for a lower default option value, the developer will reduce the presale price \( p_p^* \) to attract the buyer to accept the presale. Under this special circumstance where the developer will not abandon the project, the buyer’s presale downpayment \( d \) will not affect the abandonment probability \( P_{ab} \) and correspondingly will not affect the construction loan expected return \( E(Bp) \).

When the developer’s abandonment option has a positive value while the buyer’s default option value is zero (Case 2), an increase in the buyer’s presale downpayment \( d \) will increase the probability that the developer abandons the project \( P_{ab} \). This is true because the developer will abandon a project if his remaining construction cost exceeds the net cash inflow (which includes the remaining equity payment from the buyer) upon the continuation of the project. Since a higher equity downpayment ratio \( d \) implies a lower remaining equity payment, the likelihood for the developer to abandon the project increases. This higher abandonment probability implies that the expected return of the construction loan \( E(Bp) \) is reduced. It also implies a higher buyer’s total probability
to default $TP_{def}$ (as the buyer will have to default when the developer abandons the project). Given this, the mortgage loan expected return $E(Rp)$ decreases. To compensate the buyer for a higher abandonment probability, the developer will lower the equilibrium price $p^*_o$. This will further increase the developer’s abandonment risk (reducing $E(Bp)$) and the corresponding buyer’s total default risk (reducing $E(Rp)$).

The market we describe in this subsection is the type of market that begins to gain momentum in the U.S.A. In the past, a developer in the U.S.A. normally relies on a construction loan (which sometimes includes a land loan) to complete a project. After the project is completed, the developer will sell the units to buyers (buyers can pay for the units with both equity and mortgage) and use the sales proceeds to pay back the construction loan. However, with the use of the presale method gradually gaining momentum for some high-rise buildings in large cities, construction loan lenders will have to understand the effect of the presale method on the construction loan expected return. Our analyses clearly demonstrate that the introduction of the presale method into the system is problematic for the construction loan lender if the interest rate of the construction loan is not adjusted according to the developer’s presale strategies. The buyer’s downpayment from the presale will increase the probability that the developer defaults on the construction loan. Given this, the construction loan lender must take the developer’s presale decisions into consideration to make prudent lending decisions. It is also advisable for the construction loan lender to get involved in the developer’s presale decision.

### 4.4 Information asymmetry and agency issues

It is important to note that our model results could change with different institutional environments. In our current model setup, an important assumption is that the buyer can bargain with the developer on the presale price based on the default risk and abandonment risk, while the mortgage
lender and construction lender are passive players who will accept the market interest rates (that are unrelated to the developer’s presale decision). There are at least two necessary conditions for this assumption to hold. First, the buyer has sufficient information on the mortgage and construction loan to accurately estimate the default risk and abandonment risk. Second, the buyer has control over the developer on his leverage decision after the presale stage. This means that the developer cannot increase the leverage after the presale. However, at least in some countries (such as China) where presale is the dominant method for selling newly constructed real estate projects, buyers normally have neither knowledge nor control on the developer’s leverage decision before or after the presale.

As shown in our model, the important determinants of the default risk and abandonment risk are the downpayment ratios from the buyer, \( d \), the mortgage lender, \( m \), and the construction loan lender, \( \gamma \). As we have demonstrated in our model, holding the terms of the presale constant, an increase in the first-phase construction loan \( \gamma \) will increase the developer’s profit \( E(\pi) \). Under this situation, there is no incentive for the developer to disclose information on the construction loan at the presale stage. It is also beneficial for the developer to increase the construction loan ratio after the presale stage if it is agreed by the construction loan lender. Under this circumstance, when the bargaining is based on the currently available information, the buyer may not be able to bargain for a full compensation since there might be a change in the abandonment risk at a later stage. Consequently, an increase in the developer’s profit \( E(\pi) \) through an increase in the first-phase construction loan \( \gamma \), should be at the buyer’s cost. It should be noted that an increase in the first-phase construction loan \( \gamma \) will also increase the buyer’s default probability (due to a higher abandonment probability). If the mortgage lender does not know the detailed developer’s arrangement with the construction loan lender, the mortgage lender will not be able to estimate
correctly the risk exposures with the presale contract. Since the developer’s profit $E(\pi)$ increases in the mortgage downpayment ratio $m$, if the construction loan lender does not know the detailed developer’s arrangement with the mortgage lender, the construction loan lender will not be able to estimate correctly the risk exposures with the presale contract either. Given these, in a presale market (such as in China) where buyers do not have detailed information about the developer’s financing arrangements, it is possible that developers can use presale contracts to shift an even more significant amount of risk to buyers and lenders through the use of leverage.

5 Conclusions

In this paper we establish a game-theoretic model to explore the dynamics among four players (developer, buyer, mortgage lender and construction loan lender) in a presale market. The developer (or the leader of the game) is the person who has all the information and makes presale decisions. The buyer, mortgage lender and construction loan lender are passive players (or the followers of the game) who make acceptance decisions based on the developer’s offer. In our model, we assume that there are standard packages for the construction loan lender and the mortgage lender that are irrelevant to the developer’s presale decision. Under this model framework, the developer’s decision is to select a presale price that is acceptable to the buyer.

However, it should be noted that the proceeds from the presale, the first-phase construction loan and mortgage downpayment can all be viewed as financing methods for the developer. Intuitively, we know that the lower the equity amount the developer has in a project, the higher the probability that the developer will abandon the project when market conditions change. Given this, any increase in either the buyer’s presale downpayment, the first-phase construction loan or the mortgage downpayment will increase the developer’s abandonment probability. This, in turn, will
reduce the payoffs of the buyer, mortgage lender and construction loan lender if the three players do not communicate with each other. In other words, an increase in any of the three downpayments can shift the development risk from the developer to the three passive players.

It should be noted that the risk-shifting effect of debt reported in the corporate finance literature (such as in Jensen and Meckling (1976) and Brender and Lewis (1986)) is through an increase in the convexity of the equity holders’ payoff curve or a substitution of high risk assets. The risk-shifting effect discussed in our paper is through an increase in the value of the abandonment option or the default option. Compared to the risk shifting phenomenon reported in the corporate finance literature, the risk-shifting mechanism reported in our model seems to be more complicated and involves more players. Given the complexity of the mechanism, it is fair to say that more studies on the presale market are needed before we can fully understand the operations of the system.

As a first step, this paper points out two areas that might need the attention of researchers and practitioners when they study a presale market involving multiple lenders. First, in order to address the risk-shifting issue among players, it is the best to have one bank to handle all the proceeds from the buyer, mortgage lender, and construction loan lender. This will eliminate the information asymmetry problem and provide full information for all the players to make informed decisions. Second, our model clearly points out a need for risk adjusted (or option based) pricing for the multiple players involved in the process (buyer, mortgage lender and construction loan lender). In this paper, we derive a pricing model in which the buyer is always indifferent between either buying now at the presale market or purchasing in the future spot market, while the interest rates of the mortgage and construction loans are held constant (and are made independent of the developer’s presale decision). More work is needed to figure out a model in which the optimal presale price and interest rates of the construction loan and mortgage are all endogenously determined.
While the main purpose of our model is to link the presales method to the use of financing in the real estate development market, the concept of our model can be applied to many other areas as well. For example, in a joint venture the general partner needs to value the equity interest of the limited partners under different debt arrangements. A firm might want to buy a partial interest in an existing development (normally with debt in place) with an exit option. Broadly speaking, our model framework can also be applied to solving the pricing issue of other economic activities, such as the pricing of R&D projects. When an R&D project involves two stages of development (the result of the first stage determines if the second stage will continue) and uses both debt and venture capital, the pricing of the venture capital, debt interest and equity position will face the similar issues discussed in this paper.
APPENDIX

A  Proof for Lemma 1

We start our proof with the buyer’s default decision at \( t = 1 \). Since the construction cost \( c \) and future price \( p \) are known to all players from \( t = 1 \), the buyer’s default decision is based on a comparison of the incremental cost from committing, \( \Delta C_{\text{commit}} \), with that from defaulting, \( \Delta C_{\text{default}} \), where

\[
\Delta C_{\text{commit}} = (1 - d)(1 - l)p_p + l(p_p - \bar{p}) + lp(1 + r), \quad \Delta C_{\text{default}} = p. \tag{18}
\]

Here \( (1 - d)(1 - l)p_p \) is the buyer’s second equity payment, \( l(p_p - \bar{p}) \) is her payment to the balance of the presale contract, and \( lp(1 + r) \) is the projected total payment to the mortgage lender at \( t = 2 \). The necessary and sufficient condition to commit the remaining presale payment is \( \Delta C_{\text{default}} \geq \Delta C_{\text{commit}} \), or, \( p \geq \hat{p} = [1 - d(1 - l)]p_p + rl\bar{p} \), which is Equation (1).

Now we can analyze the developer’s abandonment decision at \( t = 1 \). At this moment, from Equation (1), the developer can clearly see whether or not a buyer will default. Given this, we need to consider two situations. First, the developer knows that the buyer will not default on the remaining payment. Second, the developer knows that the buyer will default. In the first situation, if the developer continues the construction at \( t = 1 \), he will incur an additional construction cost \( (1 - h)c \), and also receive the remaining construction loan \( (1 - \gamma)\beta c \). Meanwhile, he will also receive the buyer’s second equity payment \( (1 - d)(1 - l)p_p \) and balance payment \( l(p_p - \bar{p}) \), and the mortgage lender’s remaining mortgage payment \( (1 - m)lp \). However, at \( t = 2 \), the developer also needs to pay back the construction loan that totals \( \beta c(1 + i) \). If the developer decides to abandon the project, his incremental payoff will be zero. The developer will compare his incremental payoff from continuing the project \( \Delta \pi'_{\text{continue}} \) with that from abandoning the project \( \Delta \pi'_{\text{abandon}} \), where

\[
\Delta \pi'_{\text{continue}} = -(1 - h)c + (1 - \gamma)\beta c + (1 - d)(1 - l)p_p + l(p_p - \bar{p}) + (1 - m)lp - \beta c(1 + i),
\]

\[
\Delta \pi'_{\text{abandon}} = 0. \tag{20}
\]

The necessary and sufficient condition for the developer to continue the project when he knows the buyer will not default is \( \Delta \pi'_{\text{continue}} \geq \Delta \pi'_{\text{abandon}} \). This can be specified as

\[
c \leq \hat{c} = \frac{1}{1 - h} \{[1 - d(1 - l)]p_p - lm\bar{p} - \beta c(\gamma + i)\},
\]

which is Equation (2).

For the second situation in which the buyer defaults on the presale payment, the developer will still need to decide if he wants to continue the project. At \( t = 1 \), if he chooses to continue the construction, he will incur an additional construction cost \( (1 - h)c \) and receive the remaining construction loan \( (1 - \gamma)\beta c \). At \( t = 2 \), he will receive the spot sale price \( p \) from the completed property but also needs to pay off the total construction loan \( \beta c(1 + i) \). Consequently, at \( t = 1 \), the developer will compare the incremental payoff from continuing the project \( \Delta \pi'_{\text{continue}} \) with that from abandoning the project \( \Delta \pi'_{\text{abandon}} \), where

\[
\Delta \pi'_{\text{continue}} = -(1 - h)c + (1 - \gamma)\beta c + p - \beta c(1 + i), \tag{21}
\]

\[
\Delta \pi'_{\text{abandon}} = 0. \tag{22}
\]
The necessary and sufficient condition for a developer to continue the project when he knows the buyer will default is \( \Delta \pi|_{\text{continue}} \geq \Delta \pi|_{\text{abandon}} \). This can be specified as
\[
c \leq \tilde{c} = \frac{1}{(1-h)} [p - \beta \pi(\gamma + i)],
\]
which is Equation (3).

**B Proof for Proposition 1**

To prove Proposition 1, we need to solve for the equilibrium presale price \( p^*_p \) first. Once we get \( p^*_p \), we can derive solutions for the buyer’s default probability \( P_{\text{def}} \) (or \( TP_{\text{def}} \)), developer’s abandonment probability \( P_{\text{ab}} \), developer’s expected profit \( E(\pi) \), mortgage loan return \( E(Rp) \) and construction loan return \( E(Rp) \) as they all depend on \( p^*_p \). To solve for \( p^*_p \), we will first prove that both the developer’s expected profit \( E(\pi) \) and the buyer’s expected cost \( E(C(p_p)) \) are increasing in the presale price \( p_p \). Given this, the developer will select the highest presale price \( p_p \) that is acceptable to the buyer. On the other hand, the buyer will not accept a presale price that is higher than the expected future spot price \( \bar{p} \). Consequently, the equilibrium presale price \( p^*_p \) is the price that satisfies the condition \( E(C(p^*_p)) = \bar{p} \).

Since the solution for \( p^*_p \) is very lengthy and does not show clear intuitions, we follow the method established by Chan, Wang and Yang (2012) that does not report the full solution and draws implications from two special cases. In Case 1, we set the developer’s abandonment option value to zero and the buyer’s default option value as positive. In Case 2, we set the buyer’s default option value to zero and the developer’s abandonment option value as positive.

**B.1 Case 1: only with a positive default option value**

To prove Case 1, we first set \( P_{\text{ab}} = 0 \). Equations (4), (7), (8) and (9) can now be reduced to
\[
E(C(p_p)) = \int_{\pi-\theta}^{\pi+\theta} \left\{ \int_{\bar{p}}^{\bar{p}+\delta} [p_p + rl\bar{p}] d\Omega(\bar{p}) + \int_{\bar{p}-\delta}^{\bar{p}} [d(1-l)p_p + \bar{p}] d\Omega(\bar{p}) \right\} d\Phi(\bar{c}),
\]
\[
E(\pi) = \int_{\pi-\theta}^{\pi+\theta} \left\{ \int_{\bar{p}}^{\bar{p}+\delta} (p_p - \bar{c} - i \beta \pi) d\Omega(\bar{p}) + \int_{\bar{p}-\delta}^{\bar{p}} (d(1-l)p_p + mlp + \bar{p} - \bar{c} - i \beta \pi) \right\} d\Phi(\bar{c}),
\]
\[
E(Rp) = \int_{\pi-\theta}^{\pi+\theta} \left\{ \int_{\bar{p}}^{\bar{p}+\delta} rl\bar{p} d\Omega(\bar{p}) + \int_{\bar{p}-\delta}^{\bar{p}} (-mlp) d\Omega(\bar{p}) \right\} d\Phi(\bar{c}),
\]
\[
E(Bp) = i \beta \pi,
\]
and the buyer’s default probability is
\[
P_{\text{def}} = TP_{\text{def}} = \int_{\pi-\theta}^{\pi+\theta} \int_{\bar{p}-\delta}^{\bar{p}} d\Omega(\bar{p}) d\Phi(\bar{c}).
\]
To ensure that the buyer will have a chance to commit on the presale contract, we have
\[
(\bar{p} + \delta) - \hat{p} = (\bar{p} + \delta) - p_p T - rl\bar{p} > 0,
\]
where \( T \) is defined in Proposition 1.
We are now ready to examine if both the developer’s expected presale profit \( E(C(p_p)) \) and the buyer’s expected cost \( E(\pi) \) (see Equations (23) and (24)) are increasing in the presale price \( p_p \). We derive

\[
\frac{\partial E(C(p_p))}{\partial p_p} = \frac{T [ (\bar{p} + \delta) - \bar{p} ]}{2\delta} + d(1-l) > 0. 
\]

(29)

\[
\frac{\partial E(\pi)}{\partial p_p} = \frac{T \{ (\bar{p} + \delta) - \bar{p} \} + l(m + r)p}{2\delta} + d(1-l) > 0. 
\]

(30)

Given that both \( \frac{\partial E(C(p_p))}{\partial p_p} \) and \( \frac{\partial E(\pi)}{\partial p_p} \) are positive, the maximum acceptable presale price to the buyer must satisfy

\[ E(C(p_p^*)) = E(C)_{\text{reject}} = \bar{p}. \]

Given this, the equilibrium presale price is

\[
p_p^* = \frac{(1-r\ell)\bar{p} + Y\delta - 2A}{T^2},
\]

(31)

where \( Y \) and \( A \) are defined in Proposition 1. Substituting \( p_p \) with \( p_p^* \) in Equations (24), (25), (26) and (27), we derive the results of Case 1 in Proposition 1.

\[ \square \text{Q.E.D.} \]

B.2 Case 2: only with a positive abandonment option value

To prove Case 2, we first set \( P_{\text{def}} = 0 \). Equations (4), (7), (8) and (9) can now be reduced to

\[
E(C(p_p)) = \int_{\bar{p} - \delta}^{\bar{p} + \delta} \left\{ \int_{\bar{c} - \theta}^{\bar{c}} (p_p + r\bar{p}) d\Phi(\bar{c}) + \int_{\bar{c}}^{\bar{c} + \theta} (d(1-l)p_p + \bar{p})d\Phi(\bar{c}) \right\} d\Omega(\bar{p}), 
\]

(32)

\[
E(\pi) = \int_{\bar{p} - \delta}^{\bar{p} + \delta} \left\{ \int_{\bar{c} - \theta}^{\bar{c}} (p_p - \bar{c} - i\beta \bar{c}) d\Phi(\bar{c}) + \int_{\bar{c}}^{\bar{c} + \theta} (d(1-l)p_p + m\bar{p} + \gamma\beta \bar{c} - h\bar{c}) d\Phi(\bar{c}) \right\} d\Omega(\bar{p}), 
\]

(33)

\[
E(Rp) = \int_{\bar{p} - \delta}^{\bar{p} + \delta} \left\{ \int_{\bar{c} - \theta}^{\bar{c}} r\bar{p} d\Phi(\bar{c}) + \int_{\bar{c}}^{\bar{c} + \theta} (\lambda\bar{c} + \gamma\beta \bar{c} - h\bar{c}) d\Phi(\bar{c}) \right\} d\Omega(\bar{p}), 
\]

(34)

\[
E(Bp) = \int_{\bar{p} - \delta}^{\bar{p} + \delta} \left\{ \int_{\bar{c} - \theta}^{\bar{c}} i\beta \bar{c} d\Phi(\bar{c}) + \int_{\bar{c}}^{\bar{c} + \theta} (\lambda\beta \bar{c} - h\beta \bar{c}) d\Phi(\bar{c}) \right\} d\Omega(\bar{p}), 
\]

(35)

and the developer’s abandonment probability is

\[
P_{\text{ab}} = TP_{\text{def}} = \int_{\bar{p} - \delta}^{\bar{p} + \delta} \int_{\bar{c} - \theta}^{\bar{c} + \theta} d\Phi(\bar{c}) d\Omega(\bar{p}). 
\]

(36)

To ensure that the developer will have a chance to continue the construction, we have

\[
\bar{c} - (\bar{c} - \theta) > 0. 
\]

(37)

From Equation (32), we know that the buyer’s incremental cost upon an abandonment \( \Delta C_{ab} \) and that without an abandonment \( \Delta C_{nab} \) is

\[
\Delta C_{ab} = d(1-l)p_p + \bar{p}, \quad \Delta C_{nab} = p_p + r\bar{p}. 
\]

(38)
It is reasonable to assume that an abandonment will increase the buyer’s average incremental net cost, or
\[
\Delta C_{ab}\big|_{\bar{p}=\bar{p}} - \Delta C_{nab} = [d(1-l)p_p + \bar{p}] - (p_p + rl\bar{p}) = \bar{p} - (p_pT + rl\bar{p}) = \bar{p} - \bar{p} > 0. \tag{39}
\]

We are now ready to examine if the developer’s expected presale profit \(E(\pi)\) (in Equation (33)) and the buyer’s expected cost \(E(C(p_p))\) (Equation (32)) are both increasing in the presale price \(p_p\). First, we derive
\[
\frac{\partial E(\pi)}{\partial p_p} = \frac{T[\bar{c} - (\bar{c} - \theta)] + 2d\theta(1-l)}{2\theta} > 0. \tag{40}
\]
Regarding \(E(C(p_p))\), we derive
\[
E(C(p_p)) = \frac{1}{2\theta(1-h)}\{T^2p_p^2 + p_p[\theta(1-h)Y - T(F\bar{c} + Z\bar{p})] + \bar{p}\theta(1-h)(1+rl) + (1-rl)(F\bar{c} + m[l\bar{p}])\}, \tag{41}
\]
where \(F\) and \(Z\) are defined in Proposition 1. Given \(\frac{\partial^2 E(C(p_p))}{\partial p_p^2} = \frac{\tau^2}{\theta(1-h)} > 0\), Equation (41) shows that \(E(C(p_p))\) is a convex quadratic function of \(p_p\), and it will be less or equal to \(E(C)|_{\text{reject}}\) when
\[
p_p \in [p_p^L, p_p^U], \quad \text{where } p_p^L \text{ and } p_p^U \text{ are two roots for } E(C(p_p)) = E(C)|_{\text{reject}}, \tag{42}
\]
and
\[
p_p^L = \frac{\tau T F - (1-h)\theta Y + \bar{p}TZ - W}{2T^2}, \quad p_p^U = \frac{\tau T F - (1-h)\theta Y + \bar{p}TZ + W}{2T^2}, \tag{43}
\]
where \(W\) is defined in Proposition 1. Equation (40) shows that \(\frac{\partial E(\pi)}{\partial p_p} > 0\), which indicates that the developer has an incentive to offer a presale price that is as high as possible. Given the constraint \(E(C(p_p)) \leq E(C)|_{\text{reject}}\), and the fact that \(p_p^U\) is the highest presale price that meets this constraint, the optimal price (that can be accepted by the buyer) for the developer must be \(p_p^* = p_p^U\). As an alternative interpretation, the developer will only choose a presale price that falls on the right half of the convex function curve of \(E(C(p_p))\), where
\[
\frac{\partial E(C(p_p))}{\partial p_p} > 0, \tag{44}
\]
leading to a price \(p_p^* = p_p^U\) that satisfies \(E(C(p_p^*)) = E(C)|_{\text{reject}} = \bar{p}\).

Q.E.D.

C Proof for Proposition 2

This section analyzes the impacts of mortgage downpayment \(m\), first-phase construction loan \(\gamma\), and buyer’s presale downpayment \(d\) on the optimal presale price \(p_p^*\), buyer’s default probability \(Pdef\) (and \(TP\text{def}\)), developer’s abandonment probability \(Pab\), developer’s expected profit \(E(\pi)\), mortgage loan return \(E(Rp)\) and construction loan return \(E(Bp)\). For each of the three downpayment ratios (\(m, \gamma\) and \(d\)), we will start with Case 1 (when only the default option has a positive value) and then discuss Case 2 (when only the abandonment option has a positive value).
C.1 Effects of \( m \)

We start our proof with Case 1 (when only the buyer’s default option has value). Under this circumstance, the results in Proposition 1 show that \( m \) does not affect \( p_e^* \), \( P_{def} \) (or \( TP_{def} \)), \( P_{ab} \) or \( E(Bp) \). Given \( \frac{\partial p_e^*}{\partial m} = 0 \), taking a derivative of \( E(\pi) \) and \( E(Rp) \) with respect to \( m \), we derive

\[
\frac{dE(\pi)}{dm} = \frac{\partial E(\pi)}{\partial m} + \frac{\partial E(\pi)}{\partial p_e^*} \cdot \frac{\partial p_e^*}{\partial m} = \frac{l_p[\hat{p} - (\bar{p} - \delta)]}{2\delta} + \frac{\partial E(\pi)}{\partial p_e^*} \cdot 0 > 0 \tag{45}
\]

(See Equation (12));

\[
\frac{dE(Rp)}{dm} = \frac{\partial E(Rp)}{\partial m} + \frac{\partial E(Rp)}{\partial p_e^*} \cdot \frac{\partial p_e^*}{\partial m} = -\frac{l_p[\hat{p} - (\bar{p} - \delta)]}{2\delta} + \frac{\partial E(Rp)}{\partial p_e^*} \cdot 0 < 0. \tag{46}
\]

We now analyze Case 2 (when only the developer’s abandonment option has value). \( P_{def} \) is zero so it is not affected by \( m \). We define

\[
f = - \left[ E(C(p_e^*)) - E(C) \right]_{\text{reject}} = 0, \tag{48}
\]

and we will use conditions, \( \bar{p} - \hat{p} > 0 \) and \( \bar{c} + \theta > \hat{c} \), derived from Equations (39) and (16) to help our proof. Taking a partial derivative of \( f \) with respect to \( p_e^*, d, m \) and \( \gamma \) by treating \( p_e^* \) as an exogenous variable, we derive

\[
\frac{\partial f}{\partial p_e^*} = -\frac{\partial E(C(p_e^*))}{\partial p_e^*} = -\frac{W}{2\theta(1-h)} < 0,
\]

\[
\frac{\partial f}{\partial d} = -\frac{\partial E(C(p_e^*))}{\partial d} = -\frac{(1-l)p_e^*}{2\theta(1-h)} [(1-h)(\bar{c} + \theta - \hat{c}) + (\bar{p} - \hat{p})] < 0,
\]

\[
\frac{\partial f}{\partial m} = -\frac{\partial E(C(p_e^*))}{\partial m} = -\frac{l_p}{2\theta(1-h)} (\bar{p} - \hat{p}) < 0,
\]

\[
\frac{\partial f}{\partial \gamma} = -\frac{\partial E(C(p_e^*))}{\partial \gamma} = -\frac{\beta \bar{c}}{2\theta(1-h)} (\bar{p} - \hat{p}) < 0.
\]

Applying Roy’s identity, we have

\[
\frac{\partial p_e^*}{\partial d} = -\frac{\partial f}{\partial d} < 0, \quad \frac{\partial p_e^*}{\partial m} = -\frac{\partial f}{\partial m} < 0, \quad \frac{\partial p_e^*}{\partial \gamma} = -\frac{\partial f}{\partial \gamma} < 0. \tag{49}
\]
From Equations (33), (34), (35) and (36), given \( \frac{\partial p_b^*}{\partial m} < 0 \) (from Equation (49)), we can derive

\[
\frac{dP_{ab}}{dm} = \frac{dT P_{def}}{dm} = \frac{\partial P_{ab}}{\partial m} + \frac{\partial P_{ab}}{\partial p_b^*} \cdot \frac{\partial p_b^*}{\partial m} = \frac{l \bar{p}}{2(1-h)\theta} - \frac{T}{2(1-h)\theta} \cdot \frac{\partial p_b^*}{\partial m} > 0; \tag{50}
\]

\[
\frac{dE(R_p)}{dm} = \frac{\partial E(R_p)}{\partial m} + \frac{\partial E(R_p)}{\partial p_b^*} \cdot \frac{\partial p_b^*}{\partial m} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} < 0; \tag{51}
\]

\[
\frac{\partial E(R_p)}{\partial p_b^*} = \frac{l \bar{p}(m + r)T}{2(1-h)\theta} > 0; \tag{52}
\]

\[
\frac{dE(B_p)}{dm} = \frac{\partial E(B_p)}{\partial m} + \frac{\partial E(B_p)}{\partial p_b^*} \cdot \frac{\partial p_b^*}{\partial m} = -\frac{\beta c(\gamma + i)T}{2(1-h)\theta} + \frac{\partial E(B_p)}{\partial p_b^*} \cdot \frac{\partial p_b^*}{\partial m} < 0; \tag{53}
\]

\[
\frac{\partial E(B_p)}{\partial p_b^*} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} > 0; \tag{54}
\]

\[
\frac{dE(\pi)}{dm} = \frac{\partial E(\pi)}{\partial m} + \frac{\partial E(\pi)}{\partial p_b^*} \cdot \frac{\partial p_b^*}{\partial m} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} + \frac{\partial E(\pi)}{\partial p_b^*} \cdot \frac{\partial p_b^*}{\partial m} < 0; \tag{55}
\]

\[
\frac{dE(\pi)}{\partial p_b^*} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} > 0; \tag{56}
\]

\[
\frac{\partial E(\pi)}{\partial \gamma} > 0 \quad \text{if } \Delta > 0,
\]

where \( \Delta = \frac{(\bar{c} + \theta - \bar{c}) - [\bar{p} - \bar{p}][(\bar{c} - (\bar{c} - \theta)T + 2d\theta(1-l)]}{WT} \), with \( \bar{c} \) and \( \bar{p} \) defined in Equations (16) and (17).

Note that in Equation (56), given \( \frac{\partial E(\pi)}{\partial p_b^*} > 0 \) and \( \frac{\partial p_b^*}{\partial m} < 0 \), \( \frac{dE(\pi)}{dm} > 0 \) if \( \Delta > 0 \).

### C.2 Effects of \( \gamma \)

The effects of \( \gamma \) under Case 1 (when the developer will not abandon a project) can be derived directly from the results reported in Case 1 of Proposition 1. From Equation (31), we can see that \( \gamma \) does not have any effect on any variable examined, as the developer will for sure not abandon the project.

The effects of \( \gamma \) under Case 2 (when the buyer will not default on the payment) can be derived using Equations (33), (34), (35) and (36), or

\[
\frac{\partial E(\pi)}{\partial \gamma} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} > 0; \tag{57}
\]

\[
\frac{\partial E(R_p)}{\partial \gamma} = \frac{-\beta c[(1-h)(\bar{c} + \theta - \bar{c}) + \beta c(\gamma + i)]}{2(1-h)\theta} < 0; \tag{58}
\]

\[
\frac{\partial E(B_p)}{\partial \gamma} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} < 0; \tag{59}
\]

\[
\frac{\partial E(\pi)}{\partial \gamma} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} < 0; \tag{60}
\]

\[
\frac{\partial E(\pi)}{\partial \gamma} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} > 0; \tag{61}
\]

\[
\frac{\partial E(\pi)}{\partial \gamma} = \frac{\beta c(\gamma + i)T}{2(1-h)\theta} > 0; \tag{62}
\]

33
In these derivations, we use properties $\frac{\partial p_p^*}{\partial \gamma} < 0$, $\frac{\partial E(Rp)}{\partial p_p^*} > 0$ and $\frac{\partial E(Bp)}{\partial p_p^*} > 0$ (from Equations (49), (53) and (55)). From Proposition 1, we know that $\gamma$ does not affect $Pdef$. Similarly, $\frac{dE(\pi)}{d\gamma} > 0$ when $\Delta > 0$.

C.3 Effects of $d$

In Case 1, since the developer will not abandon the project, $Pab$ is zero and not affected by $d$. From Proposition 1, we can see that $d$ does not affect $Bp$ either. We define

$$f = - [E(C(p_p^*)) - E(C)]_{\text{reject}} = 0,$$  

(63)

Taking partial derivatives of $f$ with respect to $p_p^*$ and $d$ by treating $p_p^*$ as an exogenous variable, we derive

$$\frac{\partial f}{\partial p_p^*} = - \frac{\partial E(C(p_p^*))}{\partial p_p^*} < 0 \text{ (from Equation (29))},$$

$$\frac{\partial f}{\partial d} = - \frac{(1 - l)p_p^*}{2\delta} [\bar{p} - (\bar{p} - \delta)] < 0 \text{ (from Equation (12))}.$$

Applying Roy’s identity, we have

$$\frac{\partial p_p^*}{\partial d} = - \frac{\partial f}{\partial p_p^*} < 0.$$  

(64)

From Proposition 1, taking a derivative of $Pdef$ and $E(Rp)$ with respect to $d$, and given $\frac{\partial p_p^*}{\partial d} < 0$ (from Equation (64)), we obtain

$$\frac{dPdef}{dd} = \frac{dT Pdef}{dd} + \frac{dPdef}{dd} \cdot \frac{\partial p_p^*}{\partial d} = - \frac{(1 - l)p_p^*}{2\delta} + \frac{T}{2\delta} \cdot \frac{\partial p_p^*}{\partial d} < 0;$$  

(65)

$$\frac{dE(Rp)}{dd} = \frac{\partial E(Rp)}{dd} + \frac{\partial E(Rp)}{dd} \cdot \frac{\partial p_p^*}{\partial d} = \frac{(1 - l)\bar{p} p_p^*(m + r)}{2\delta} + \frac{\partial E(Rp)}{\partial p_p^*} \cdot \frac{\partial p_p^*}{\partial d} > 0, \text{ given (66)}$$

$$\frac{\partial E(Rp)}{\partial p_p^*} = - \frac{T \bar{p} (m + r)}{2\delta} < 0.$$  

(67)

In Case 2, since the buyer will not default on the payment, $Pdef$ is zero and unaffected by $d$. Using Equations (36), (34) and (35), and properties $\frac{\partial p_p^*}{\partial d} < 0$, $\frac{\partial E(Rp)}{\partial p_p^*} > 0$ and $\frac{\partial E(Bp)}{\partial p_p^*} > 0$ (from Equations (49), (53) and (55)), we derive

$$\frac{dPab}{dd} = \frac{dPdef}{dd} + \frac{\partial Pab}{\partial d} \cdot \frac{\partial p_p^*}{\partial d} = \frac{(1 - l)p_p^*}{2(1 - h)\theta} - \frac{T}{2(1 - h)\theta} \cdot \frac{\partial p_p^*}{\partial d} > 0;$$  

(68)

$$\frac{dE(Rp)}{dd} = \frac{\partial E(Rp)}{dd} + \frac{\partial E(Rp)}{dd} \cdot \frac{\partial p_p^*}{\partial d} = - \frac{(m + r)(1 - l)\bar{p} p_p^*(m + r)}{2(1 - h)\theta} + \frac{\partial E(Rp)}{\partial p_p^*} \cdot \frac{\partial p_p^*}{\partial d} < 0;$$  

(69)

$$\frac{dE(Bp)}{dd} = \frac{\partial E(Bp)}{dd} + \frac{\partial E(Bp)}{dd} \cdot \frac{\partial p_p^*}{\partial d} = - \frac{\gamma + i(1 - l)\bar{p} p_p^*(\gamma + i)}{2(1 - h)\theta} + \frac{\partial E(Bp)}{\partial p_p^*} \cdot \frac{\partial p_p^*}{\partial d} < 0.$$  

(70)

$$\blacksquare$$

Q.E.D.
References


Exhibit 1  Decision Flows for the Developer

- **Spot price**: $p_{\bar{p}} \sim \Omega[p_{\bar{p}} - \delta, p_{\bar{p}} + \delta]$;
- **Total construction cost**: $c \sim \Phi[c - \theta, c + \theta]$

$p_{\bar{p}}$

**1st-phase development**

- **Buyer downpayment**: $d(1-l)p_{\bar{p}}$
- **Mortgage downpayment**: $m\bar{p}$
- **Cons. loan downpayment**: $\gamma\bar{c}$

$t=0$

- **$\tilde{p}$ is realized at value $p$;**
- **$\tilde{c}$ is realized at value $c$**

**t=1**

- **Buyer commits**
  - **Developer continues**
    - Development is completed
      - Buyer pays $l(p_{\bar{p}} - \bar{p})$
  - **Developer abandons**

- **Buyer defaults**
  - **Cons. loan pays**
    - $(1-\gamma)/\bar{c}$

**t=2**

- **Both loans are paid off**
  - **PACC**
- **Developer continues**
  - Development is completed
  - Sell property at market price $p$
  - **PACQ**
- **Developer abandons**
  - **Construction loan is paid off**
  - **PADQ**
### Exhibit 2 Information Content and Cash Flows at Each Decision Point under a Presale

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>Net cash flows:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information status:</td>
<td>$\tilde{\pi} \sim \Omega[\bar{p} - \delta, \bar{p} + \delta]$,</td>
<td>$\bar{p} \rightarrow p$ and $\bar{c} \rightarrow c$</td>
<td>Components in Eqn. (7)</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\sigma} \sim \Phi[\bar{v} - \theta, \bar{v} + \theta]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Developer's cash flows when:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No default or abandon (PACC)</td>
<td>$d(1-l)p_r + m\bar{p} + \gamma \beta \bar{c} - h\tilde{c}$</td>
<td>$-l\bar{p}(1+r)$</td>
<td>Components in Eqn. (4)</td>
</tr>
<tr>
<td>Abandon only (PACQ)</td>
<td>$d(1-l)p_r + m\bar{p} + \gamma \beta \bar{c} - h\tilde{c}$</td>
<td>$-p$</td>
<td></td>
</tr>
<tr>
<td>Defaults only (PADC)</td>
<td>$d(1-l)p_r + m\bar{p} + \gamma \beta \bar{c} - h\tilde{c}$</td>
<td>$-p$</td>
<td></td>
</tr>
<tr>
<td>Default and abandon (PADQ)</td>
<td>$d(1-l)p_r + m\bar{p} + \gamma \beta \bar{c} - h\tilde{c}$</td>
<td>$-p$</td>
<td></td>
</tr>
<tr>
<td>Buyer's cash flows when:</td>
<td></td>
<td></td>
<td>Components in Eqn. (8)</td>
</tr>
<tr>
<td>No default or abandon (PACC)</td>
<td>$-d(1-l)p_r$</td>
<td>$-(1-d)(1-l)p_r - l(p_r - \bar{p})$</td>
<td>$\bar{p}(1+r)$</td>
</tr>
<tr>
<td>Abandon only (PACQ)</td>
<td>$-d(1-l)p_r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Defaults only (PADC)</td>
<td>$-d(1-l)p_r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default and abandon (PADQ)</td>
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<tr>
<td>Mortgage lender's cash flows when:</td>
<td></td>
<td></td>
<td>Components in Eqn. (9)</td>
</tr>
<tr>
<td>No default or abandon (PACC)</td>
<td>$-m\bar{p}$</td>
<td>$-(1-m)\bar{p}$</td>
<td>$\bar{p}(1+r)$</td>
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<tr>
<td>Abandon only (PACQ)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Defaults only (PADC)</td>
<td>$-m\bar{p}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default and abandon (PADQ)</td>
<td>$-m\bar{p}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction loan lender's cash flows when:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No default or abandon (PACC)</td>
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<td>$-(1-\gamma)\beta \bar{c}$</td>
<td>$\beta \bar{c}(1+i)$</td>
</tr>
<tr>
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</tr>
<tr>
<td>Defaults only (PADC)</td>
<td>$-\gamma \beta \bar{c}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default and abandon (PADQ)</td>
<td>$-\gamma \beta \bar{c}$</td>
<td></td>
<td></td>
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</tbody>
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