* 1. **Fourier transform and Fourier Series**

We have already seen that the Fourier transform is important. For an LTI system, , then the complex number determining the output is given by the Fourier transform of the impulse response:

Well what if we could write arbitrary inputs as superpositions of complex exponentials, i.e. via sums or integrals of the following kind:

Then notice, outputs of LTI systems y(t) will always take the form

This is the root of the Fourier series.

**Proposition 1.1.** *Let x(t) be period with period T, so that the frequencies , and*

 *- SYNTHESIS EQUATION*

*Then, , and*

*- ANALYSIS EQUATION*

**Proof:** Use the property that

Then we have

OK, so how do we use this. Well, for periodic signals with period T, then we just have to evaluate the Fourier series coefficients .

**Example 1.1.**

1. x(t)=constant, then =constant and for any period T.

2. , then , , .

3. , then , , , .

4. , then , , , , .

**1.2 Relationship of Fourier Series and Fourier Transform**

So, Fourier series is for periodic signals. Fourier transform is for non-periodic signals. Let’s examine and construct the Fourier transform by allowing the period of the periodic signals go to , see what we get.

Let’s define to be the periodic version of x(t), where x(t) has finite support , . Thus,

**Definition 1.1.** *Define the Fourier transform of x(t) to be*

Then we have the relationship between FT and FS.

**Proposition 1.2.**

 where

*where*

, where

**Example 1.2.** *Let , , and 0 otherwise. Then*

Let , . Then,

 where

OK, so we see that the Fourier transform can be used to define the Fourier series. Now what we would like to do is understand how to represent the periodic signals when the period goes to infinity , so that we can have a synthesis pair. Let’s remind ourselves that is the periodic version of x(t), where x(t) has finite support .

**Proposition 1.3.** *Let be periodic with period T, and . Then*

*To see this,*