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10/10

Physical Chemistry Laboratory 030.305

# EXPERIMENT 6: DETERMINATION OF THE MEAN AGGREGATION NUMBER OF A MICELLAR SYSTEM


## ABSTRACT

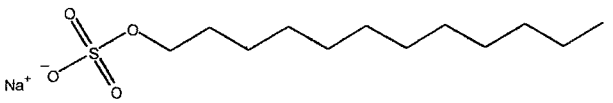
The addition of amphiphilic molecules to water produces several phenomena that have been studied and reported throughout the literature. When a certain concentration of an amphiphilic molecule such as sodium dodecyl sulfate is reached, micelles begin to form. This concentration is known as the critical micelle concentration. Both this concentration and the average number of molecules in a micelle have been studied and values have been reported using a variety of techniques including membrane osmometry and light scattering. Using a luminescence quenching technique, the mean aggregation number was calculated in this experiment. Values of 50 ± 7 and 40 ± 5 molecules were obtained. The critical micelle concentration was also calculated to be (13 ± 4) x $10^{-3}$ M. In this report, the technique used, results obtained, and possible sources of error are discussed.


## INTRODUCTION

A micelle is an organized cluster, or aggregate, consisting of a number of monomer units of molecules possessing both hydrophilic and hydrophobic properties. Examples of these molecules, termed amphiphilic, are long chain carboxylic acids or sulfates. These compounds contain long hydrocarbon chain "tail groups," insoluble in water, and polar "head groups," which are soluble in water. When the concentration of monomer in water is sufficiently high, these molecules will cluster in an approximately spherical shape. The hydrocarbon chains are directed toward the inside of the clusters, protected from the water by the surface of the sphere which consists of the polar head groups. This is the lowest energy configuration, as the smallest possible number of hydrogen bonds between water molecules are disturbed and the nonpolar hydrocarbon groups are not exposed to the polar water.

As an amphiphilic molecule is added to water, it first exists in the form of monomers. Certain interesting information, such as the thickness of a monolayer, can be determined during this stage. After the concentration becomes sufficiently high, several bulk properties of the solution change as the network of hydrogen bonds in the water is disrupted. The surface tension decreases and the solubility of hydrocarbons increase. The concentration at which these properties change is known as the critical micelle concentration (*CMC*). This term is appropriate because at the *CMC*, micelles begin to form.

Micelles can be thought of as constructed by a finite number of monomer units. In reality, the number of monomers in a micelle is not distinct, but consists of a distribution of monomers. The average number of monomers in a micelle is termed the mean aggregation number. Through a fluorimetric technique, the mean aggregation number of sodium dodecyl sulfate (SDS) was determined in this experiment. The structure of SDS is shown below. As can be seen, it is a surfactant that possesses amphiphilic properties.




The concentration of micelles in a solution of surfactant with a total concentration of $[S]_o$ can be seen in equation (1). This equation assumes that $[S]_o$ is above the *CMC*. The mean aggregation number is given by $N$ and the micelle concentration is given by $[M]$.

$$[M] = \frac{[S]_o - CMC}{N} \quad (1)$$

There are two unknowns in equation (1). In this experiment, a fluorimetric technique was used to obtain the desired information. The micelles formed were labeled with a luminescent probe molecule. The concentration of the probe was less than $[M]$, and the properties of the probe were such that every probe molecule was associated with a micelle. A luminescence quencher was added to the solution which also

See the back of your pre-lab.

associated only with micelles. Both the probe and the quencher associated randomly with micelles. Neither had an affect on the other. A micelle could be occupied by one or both. If a probe shared a micelle with a quencher, the probe did not luminesce.

Because of the random nature of the associations and the large number of quenchers, $q$, and micelles, $m$, Poisson statistics are used for analysis. Equation (2) gives the probability of finding $n$ quenchers in a micelle, where $<q>$ is the probability that a micelle contains at least one quencher ($<q>=q/m$).

$$P_n = <q>^n \frac{e^{-<q>}}{n!} \quad (2)$$

A micelle will luminesce only if it contains no quenchers. Equation (3) shows the probability of finding no quenchers in a micelle. It is simply the case of equation (2) where $n$ equals zero. Macroscopically, $<q>=[Q]/[M]$ where $[Q]$ is the bulk quencher concentration and $[M]$ is the mean micelle concentration. Equation (4) relates the molecules given by equation (3) to the luminescence intensity, $I/I^o$. Combining equation (1) and (4), equation (5), which can be used to determine $N$ and $CMC$, is obtained.

$$P_o = e^{-<q>} = e^{\frac{-[Q]}{[M]}} \quad (3)$$

$$\frac{I}{I^o} = e^{\frac{-[Q]}{[M]}} \quad (4)$$

$$ln\left(\frac{I^o}{I}\right) = \frac{[Q]N}{[S]_o - CMC} \quad (5)$$

In this experiment, ruthenium trisbipyridine served as the probe and 9-methylanthracene as the quencher. The ruthenium species is an excellent probe because the lifetime of the luminescent state is approximately 0.5 μs, which is swift relative to the rate at which the monomers of the micelle exchange with the solvent. This is a necessity. Because the micelle is continuously reorganizing, a "snapshot" must be taken quickly before this reorganization occurs or the mean aggregation number could not be determined. It can be seen from equation (5) that if $[S]_o$ is held constant and $[Q]$ varied, a regression line can be used to determine $N$. As expected, increasing $[Q]$ will cause decreased luminescence as fewer micelles occupied only by probes are present. If $[Q]$ is held constant and $[S]_o$ varied, the regression line from equation (6), which is a simple variation of equation (5), can be used to determine both $N$ and $CMC$. As $[S]_o$ is increased, there is a smaller probability that a probe and quencher will be present in the same micelle and therefore the luminescence will increase. In this experiment, both methods were used to determine the desired information, which was compared to the accepted values[1] and sources of uncertainty were analyzed.

$$\left[ln\left(\frac{I^o}{I}\right)\right]^{-1} = \frac{[S]_o}{[Q]N} - \frac{CMC}{[Q]N} \quad (6)$$

## EXPERIMENTAL

The procedure outlined in the packet was followed. A Perkin-Elmer LS-5B Fluorimeter was used to take readings. The experiment consisted of two parts. In the first, the surfactant concentration was fixed and quencher concentration varied. In the second, the quencher concentration was held constant and the surfactant concentration varied. In all cases, the probe concentration was held constant.

To an empty volumetric flask was added 9-methylanthracene (144.2 mg, 0.75 mmol). Absolute ethanol (5 ml) was added and the solid allowed to dissolve to provide a 0.15 M solution of quencher. To

[1] N. J. Turro and A. Yekta, "Luminescent Probes for Detergent Solutions: A Simple Procedure for Determination of the Mean Aggregation Number of Micelles," *J. Am. Chem. Soc.* **100**, 5951 (1978).

two volumetric flasks filled with water (45 ml) was added sodium dodecyl sulfate (648.9 mg, 2.25 mmol). To one of these flasks was added 9-methylanthracene solution (200 µl, 30 µmol), and both filled to the line with water. Overall, both solutions were 45 mM in SDS. One also had a concentration of 600 µM of quencher. These solutions were combined in volumetric flasks (10 ml) as shown in **Table I** to provide solutions of desired quencher concentrations. To each volumetric flask was added 100 µl of a 7 mM ruthenium trisbipyridine (probe) solution. The overall probe concentration was 70 µM. The probe concentration was checked by measuring the solution absorbance. On the spectrometer, the excitation and wavelengths were set to 450 and 625 nm, respectively, and luminescence recorded.

| Solution | Solution with quencher (ml) | Solution without quencher (ml) | Final [SDS] (mM) | Final [Quencher] (µM) |
|---|---|---|---|---|
| 1 | 0 | 10 | 45 | 0 |
| 2 | 2 | 8 | 45 | 120 |
| 3 | 4 | 6 | 45 | 240 |
| 4 | 6 | 4 | 45 | 360 |
| 5 | 8 | 2 | 45 | 480 |
| 6 | 10 | 0 | 45 | 600 |

**Table I: Fixed surfactant concentration solutions**

To an empty volumetric flask was then added 9-methylanthracene (48.1 mg, 0.25 mmol). Absolute ethanol (5 ml) was added and the solid allowed to dissolve to provide a 0.05 M solution of quencher. To a volumetric flask was added sodium dodecyl sulfate (144.2 mg, 0.5 mmol) and water (50 ml) to provide a 0.01 M solution. To another flask was added SDS (720.9 mg, 2.5 mmol) and water (50 ml) to provide a 0.05 M solution. Quencher solution (100 µl, 5 µmol) was added to each flask to yield an overall concentration of 100 µM. The solutions were combined in volumetric flasks (10 ml) as shown in **Table II** to provide solutions of desired concentrations. To each volumetric flask was added 100 µl of a 7 mM ruthenium trisbipyridine (probe) solution. The overall probe concentration was 70 µM. The probe concentration was again checked by measuring the solution absorbance. On the spectrometer, the excitation and wavelengths were set to 450 and 625 nm, respectively, and luminescence recorded.

| Solution | Amount 0.01M SDS solution (ml) | Amount 0.05M SDS solution (ml) | Final [SDS] (mM) | Final [Quencher] (µM) |
|---|---|---|---|---|
| 1 | 1 | 9 | 46 | 100 |
| 2 | 2 | 8 | 42 | 100 |
| 3 | 4 | 6 | 34 | 100 |
| 4 | 6 | 4 | 26 | 100 |
| 5 | 8 | 2 | 18 | 100 |

**Table II: Fixed quencher concentration solutions**

## *RESULTS & ANALYSIS*

The values of luminescence intensity obtained in both parts of the experiment are shown in **Table III**. Before the fixed surfactant data could be analyzed, it was necessary to analyze the fixed quencher data. This is because the fixed quencher data enabled the calculation of both *CMC* and *N*. The *CMC* from the second part of the experiment is necessary for the calculation of *N* from the first part. Equation **(6)** shown above was used to analyze the data and a graph of $(\ln [I^o/I])^{-1}$ vs. $[S]$ generated. The graph is shown in **Figure II** in the appendix (to keep numbering consistent between parts of the experiment). The slope and intercept were calculated using Microsoft Excel and confirmed using equations found in Taylor.[2] Data are also shown in the appendix. $I^o$ simply represents the luminescence intensity with no quencher present. Because the probe concentration was consistent throughout the experiment, $I^o$ was taken to be the luminescence intensity of the first solution in the fixed surfactant trials, which contained no quencher.

[2] Taylor, John R. *An Introduction to Error Analysis.* Sausalito, California. 1997. Pages 184-188.

| Fixed Surfactant | | Fixed Quencher | |
|---|---|---|---|
| Solution | Luminescence Intensity | Solution | Luminescence Intensity |
| 1 | 62.4 (=I°) | 1 | 55.6 |
| 2 | 49.8 | 2 | 54.5 |
| 3 | 42.1 | 3 | 49.7 |
| 4 | 34.9 | 4 | 46.6 |
| 5 | 29.9 | 5 | 34.0 |
| 6 | 23.9 | | |

**Table III: Luminescence intensity**

Equation (6) shows that the mean aggregation number and critical micelle concentration are given by equations (7) and (8) where $m$ is the slope and $b$ is the y-intercept of **Figure II**. **Table IV** shows the values obtained for $CMC$ and $N$ from the plot with associated errors. The fractional error in [Q] was taken to be 5%, which arises from the uncertainty associated with the addition of quencher during preparation of the solution. This uncertainty is due to the precision of the pipette used to make the transfer and could have been reduced through use of a syringe instead of a graduated pipette. Other uncertainties, such as precision of the balance, are negligible as they are less than 0.1%. It is easily seen that the fractional uncertainty in $N$ is simply the sum in quadrature of the fractional uncertainties associated with $[Q]$ and $m$, as two values are independent of each other.

$$m = \frac{1}{[Q]N} \rightarrow N = \frac{1}{[Q]m} \quad (7)$$

$$b = \frac{-CMC}{[Q]N} \rightarrow CMC = -b[Q]N = -b/m \quad (8)$$

It can be seen that there are two possible methods that can be used to calculate the error in the $CMC$. One involves propagation of the error computed for the slope, $N$ and $[Q]$, and the other involves propagation of the uncertainty of slope and intercept. Both methods were used and it was seen that the error from the first method was slightly greater (see calculations). This value was used. Uncertainties in slope and intercept were calculated according to the standard equations given in Taylor. They are not shown here to save space but can be seen in the calculations in the appendix.

| | Value |
|---|---|
| $N$ | $40 \pm 5$ |
| $CMC$ | $(13 \pm 4) \times 10^{-3}$ M |

**Table IV: Fixed quencher data**

After obtaining the value shown for the critical micelle concentration, the fixed surfactant concentration data could be analyzed according to equation (5). **Figure I** shows a plot of ln (I°/I) vs. $[Q]$. Clearly, the intercept of this graph should be the origin as luminescence intensity without quencher is the definition of I°. This was in fact the case; the intercept as seen in the regression line was very close to zero. The error found to be associated with the intercept was extremely large, nearly 90%. This is easily understood as the intercept is close to zero, so even the small error seen appears large as a fraction. This value does not factor into any calculations so no adjustments in error analysis techniques needed to be made.

Equation (9) shows the manner in which the mean aggregation number was calculated from the slope of **Figure I**. The uncertainty propagation was straightforward but slightly complicated. The fractional uncertainty in $[S]_0$ was taken to be 2%, or an absolute uncertainty of $9 \times 10^{-4}$ M. The uncertainty from measurements by the balance, taken to be half the value of the most precise figure, was less than 0.01%. This estimated error takes into account material lost during transfer. As shown in the calculations,

the overall uncertainty is first the sum in quadrature of the *absolute* uncertainties of surfactant concentration and *CMC* (obtained above), followed by the sum in quadrature of the *fractional* uncertainties of this value and slope. **Table V** shows the value obtained for this critical micelle concentration. **Table VI** summarizes the results from both parts and compares them to the values obtained by Turro and Yekta.

$$m = \frac{N}{[S]_o - CMC} \rightarrow N = m([S]_o - CMC) \quad (9)$$

| | Value |
|---|---|
| $N$ | $50 \pm 7$ |

**Table V: Fixed surfactant data**

| | Fixed Surfactant | Fixed Quencher | Accepted |
|---|---|---|---|
| $N$ | $50 \pm 7$ | $40 \pm 5$ | $55 \pm 5, 60 \pm 2$ |
| $CMC$ | | $(13 \pm 4) \times 10^{-3}$ M | $7.5 \times 10^{-3}$ M |

**Table VI: Data summary and comparison**

## DISCUSSION AND CONCLUSIONS

Considering the nature of this experiment and the many possible sources of random and systematic error, the values obtained for both *N* and *CMC* were very acceptable. The mean aggregation number value obtained during the fixed surfactant trials included the results found by Turro and Yekta in the range and the value obtained during the fixed quencher trials was acceptably close. The value obtained for the critical micelle concentration also coincided nicely with the reported value.

The largest source of error in this experiment arose from inaccuracies in concentration values. When preparing large solutions, concentration error was not large, although a small mistake may have caused incorrect concentration in a single solution, which could have systematically altered all results from one half of the experiment. However, when transferring small quantities of solutions, namely probe and quencher, concentration error became very significant. The methods used in transferring small amounts of solution involved the use of disposable pipettes. This was not an efficient method of making these measurements when concentration was extremely important. The error is high due to improper calibration of the pipettes and inability to add the quantity efficiently. Due to the wide opening at the end of the pipette, 100 μL of solution consisted of only three drops. Fractions of drops were very difficult to add, likely due to the hydrogen bonding solvent. Prior to taking luminescence intensity readings, absorbance was measured to be certain of fairly constant probe concentration. These results are shown in **Table VII**. In solution five in the fixed quencher trials, slightly over 100 μL was added and four drops entered the solution. As can be seen, the absorption rose dramatically. For other trials, it can be seen that the concentration (proportional to absorption by Beer's law) was not as constant as would be desired. The absorptions are fairly similar, aside from the trial mentioned above. Likely due to this method of addition, the concentrations were systematically inaccurate. A far better method of addition would involve the use of a syringe and high gauge needle.

The mean aggregation number, *N*, is sensitive to temperature as well as concentration. Temperature variations may have caused random uncertainty in determination of the value of the mean aggregation number, as it can change with temperature. The same cuvette was used for each trial. However, fingerprints placed on the cuvette while washing it between trials could have caused error, as oil from these marks would alter transmission values. Incorrect calibration of machinery or glassware could have been a source of error. Old chemicals or solutions may not have been pure or concentrated as labeled may have caused uncertainty. Finally, impurity electrolytes in the solvent could have caused error. Turro and Yekta, as well as other researchers, have shown that the addition of electrolyte to detergent solutions causes a large increase in the size of micellar aggregates. Deionized water was used to attempt to avoid this phenomenon.

Overall, however, the results obtained for this lab were informative and accurate. This lab provided an interesting and educational physical explanation of a phenomenon seen throughout organic and biological chemistry.

| Fixed Surfactant | | Fixed Quencher | |
|---|---|---|---|
| Solution | Absorption | Solution | Absorption |
| 1 | .6472 | 1 | .5707 |
| 2 | .6835 | 2 | .5798 |
| 3 | .7077 | 3 | .6022 |
| 4 | .6447 | 4 | .6543 |
| 5 | .6448 | 5 | .8441 |
| 6 | .6936 | | |

**Table VII: Absorption**

## QUESTIONS

(1) Assume that the hydrocarbon-like interior cavity of the SDS micelle has a diameter of ~3 nm and that it contains three solubilized benzene molecules. Calculate the "local" concentration of benzene in the micelle.

$$Concentration = moles/liter$$

$$d = 3\,nm,\ r = 1.5\,nm = 1.5x10^{-9}\,m$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.5x10^{-9})^3$$

$$V = 1.4136x10^{-26}\,m^3 = 1.4136x10^{-23}\,L$$

$$3\,molecules\,x\,\frac{1\,mole}{6.02x10^{23}\,molecules} = 4.9834x10^{-24}\,moles$$

$$C = \frac{4.9834x10^{-24}}{1.4136x10^{-23}} = 0.35\,M$$

(2) Assuming that at 298 K, the *CMC* for a certain surfactant is 6 x $10^{-3}$ M and that its mean aggregation number is 70, estimate the concentration of micelles if the bulk surfactant concentration is 0.08 M.

$$[M] = \frac{[S]_o - CMC}{N}$$

$$[M] = \frac{0.08 - 0.006}{70}$$

$$[M] = 1.06x10^{-3}\,M = 1.06\,mM$$

(3) A quantity (390 mg) of *n*-hexane is solubilized in 100 ml of the surfactant in question 2. Assuming that all the solubilized *n*-hexane molecules are in the micelles, what is the mean occupation number of *n*-hexane in the micelles?

$$390\,mg\,x\,\frac{1\,mmol}{84.16\,mg} = 4.634\,mmol hexane$$

$$1.06\,\frac{mmol}{L}\,x\,0.1\,L = 0.106\,mmol micelle$$

These values could be converted to number of molecules, but the conversion is unnecessary.

$$\frac{4.634}{0.106} = 43.7$$

There are approximately ***44 molecules*** of *n*-hexane per micelle.

## APPENDIX

I. Calculations
II. Data
III. Figures

# Experiment 6 Calculations

Solution concentrations:

## Part I:

$.045 \frac{mol}{L} \cdot .05 L = .00225 mol\ SDS \cdot 288.377 \frac{g}{mol} = \boxed{.64885 g\ SDS}$

$.15 \frac{mol}{L} \cdot .005 L = 7.5 \times 10^{-4} mol \cdot 192.2598 \frac{g}{mol} = \boxed{.14419 g\ 9\text{-methyl-anthracene}}$

$.15 \frac{mol}{L} \cdot 2 \times 10^{-4} L = \underline{3 \times 10^{-5} mol}$ 9-methylanthracene

$\frac{3 \times 10^{-5} mol}{.05 L} = \boxed{6 \times 10^{-4} M}$ → Final quencher concentration in A

$6 \times 10^{-4} \frac{mol}{L} \cdot .002 L = \frac{1.2 \times 10^{-6} mol}{.01 L} = \boxed{120 \mu M}$ → quencher concentration in solution containing 2mL A, 8mL B (other examples not shown)

$7 \times 10^{-3} \frac{mol}{L} \cdot .0001 L = \underline{7 \times 10^{-7}} mol\ probe$

$\frac{7 \times 10^{-7} mol}{.01 L} = \boxed{7 \times 10^{-5} M}$ → final probe concentration in all solutions

## Part II:

$.01 \frac{mol}{L} \cdot .05 L = 5 \times 10^{-4} mol\ SDS \cdot 288.377 \frac{g}{mol} = \boxed{.14419 g\ SDS \text{ in solution A}}$

$.05 \frac{mol}{L} \cdot .05 L = .0025 mol\ SDS \cdot 288.377 \frac{g}{mol} = \boxed{.72094 g\ SDS \text{ in solution B}}$

$0.05 \frac{mol}{L} \cdot .005 L = 2.5 \times 10^{-4} mol \cdot 192.2598 g = \boxed{0.04806 g\ 9\text{-methylanthracene}}$

$\left( .01 \frac{mol}{L} \cdot .001 L + .05 \frac{mol}{L} \cdot .009 L \right) / 0.01 L = \underline{0.046 M}$

Concentration of SDS in soln containing 1mL A, 9mL B - only example with

Probe concentration - same as part I.

$.05 \frac{mol}{L} \cdot 1 \times 10^{-4} L = \frac{5 \times 10^{-6} mol}{0.05 L} = \boxed{100 \mu M}$ → final quencher concentration

Error calculations

$$\sigma_y = \sqrt{\frac{1}{N-2}\underbrace{\sum_{i=1}^{N}(y_i - A - Bx_i)^2}_{\text{see Excel data in appendix for this - too long to write out}}} \qquad \sigma_b = \sigma_y\sqrt{\frac{\Sigma x^2}{\Delta}} \qquad \sigma_m = \sigma_y\sqrt{\frac{N}{\Delta}}$$

Fixed surfactant:

$$\sigma_y = \sqrt{\frac{1}{4}(0.001653)} = 0.02032855$$

$$\sigma_b = 0.02032855\sqrt{\frac{7.92\times10^{-7}}{1.51\times10^{-6}}} = 0.0147 = \delta_b$$

$$\sigma_m = 0.02032855\sqrt{\frac{6}{1.51\times10^{-6}}} = 40.49419 = \delta_m$$

Fixed quencher:

$$\sigma_y = \sqrt{\frac{1}{3}(1.094255)} = 0.6039467$$

$$\sigma_b = 0.6039467\sqrt{\frac{0.006036}{0.00264}} = 0.9159913 = \delta_b$$

$$\sigma_m = 0.6039467\sqrt{\frac{5}{0.00264}} = 26.36341\,88 = \delta_m$$

Fixed quencher:

$$N = \frac{1}{[Q]m}$$

$$\frac{\delta N}{N} = \sqrt{\left(\frac{\delta[Q]}{[Q]}\right)^2 + \left(\frac{\delta m}{m}\right)^2} = \sqrt{(.05)^2 + \left(\frac{26.3634}{247.2696}\right)^2} = .11776 \quad (\delta[Q]\text{ estimate})$$

$$\delta N = (.11776)(40.44) = 4.76$$

$$CMC = -b[Q]N = -b/m$$

$$\frac{\delta CMC}{CMC} = \sqrt{\left|\left(\frac{\delta b}{b}\right)\right|^2 + \left(\frac{\delta[Q]}{[Q]}\right)^2 + \left(\frac{\delta N}{N}\right)^2} = \sqrt{\left(\frac{.91599}{3.10521}\right)^2 + (.05)^2 + (.11776)^2} = .32$$

or

$$\frac{\delta CMC}{CMC} = \sqrt{\left|\left(\frac{\delta b}{b}\right)\right|^2 + \left(\frac{\delta m}{m}\right)^2} = \sqrt{\left(\frac{.91599}{3.10521}\right)^2 + \left(\frac{26.3634}{247.2696}\right)^2} = .31366$$

Use greater uncertainty: $\delta CMC = (.32153)(.01256) = .00404$

Fixed Quencher Concentration

| [S] | I | ln (Io/I) | [ln (Io/I)]^-1 | xy | | (y-b-mx)^2 |
|---|---|---|---|---|---|---|
| 0.046 | 55.6 | 0.115382 | 8.666858 | 0.398675 | | 0.158126 |
| 0.042 | 54.5 | 0.135365 | 7.387457 | 0.310273 | | 0.011521 |
| 0.034 | 49.7 | 0.22756 | 4.394439 | 0.149411 | | 0.82359 |
| 0.026 | 46.6 | 0.291965 | 3.425071 | 0.089052 | | 0.010258 |
| 0.018 | 34 | 0.607205 | 1.646891 | 0.029644 | | 0.090762 |
| | | | | | sum | 1.094255 |

| | |
|---|---|
| Sigma(x) | 0.166 |
| Sigma(x^2) | 0.006036 |
| Sigma(y) | 25.52072 |
| Sigma(y^2) | 163.4434 |
| Sigma(xy) | 0.977055 |

| | |
|---|---|
| delta | 0.002624 |
| slope | 247.2708 |
| intercept | -3.10525 |
| deviation | 0.603947 |
| delta(b) | 0.915991 |
| delta(m) | 26.36342 |

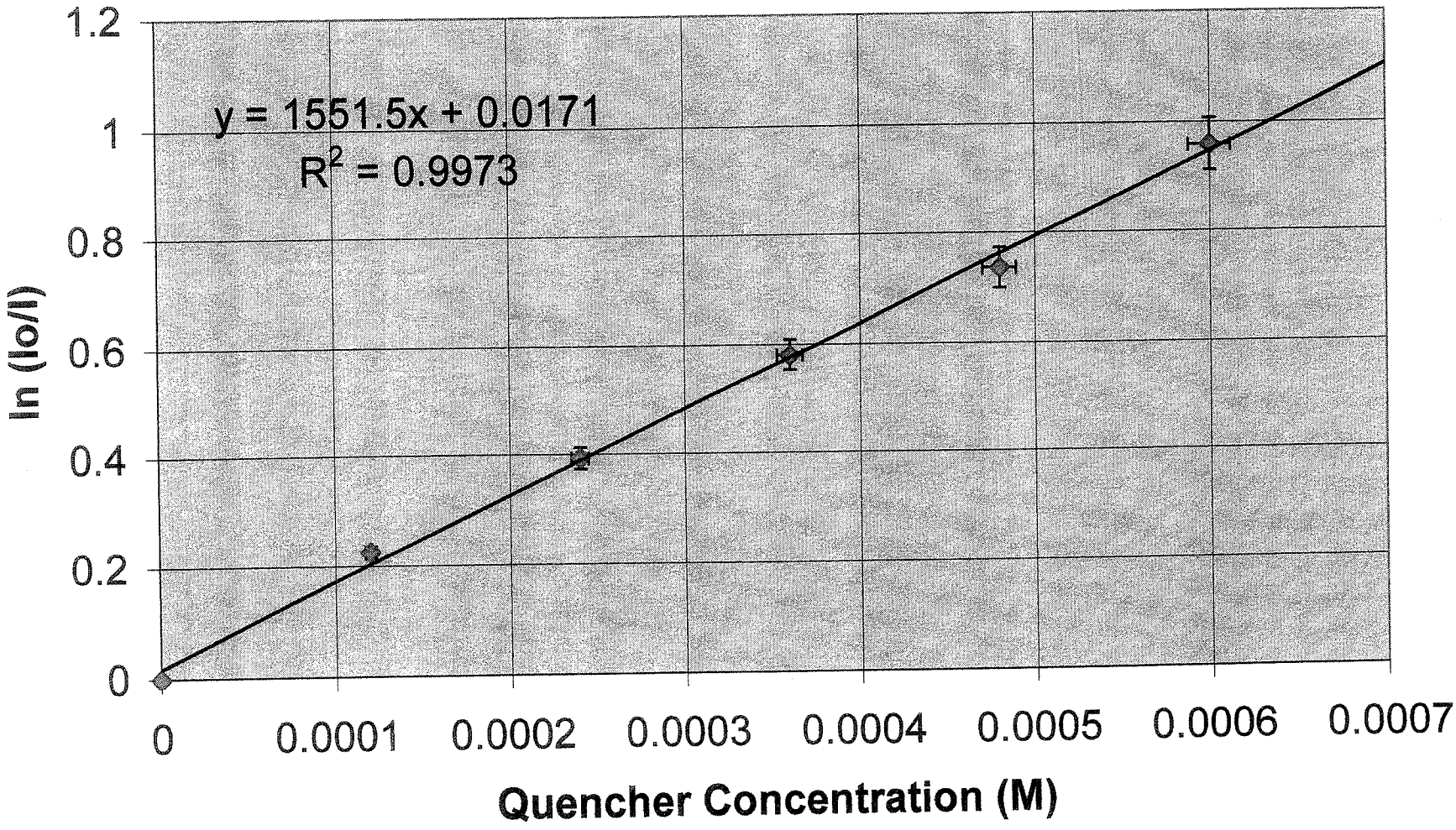
Fixed Surfactant Concentration

| [Q] | I | ln (Io/I) | xy | | (y-b-mx)^2 |
|---|---|---|---|---|---|
| 0 | 62.4 | 0 | 0 | | 0.000292 |
| 0.00012 | 49.8 | 0.22555 | 2.71E-05 | | 0.000496 |
| 0.00024 | 42.1 | 0.393518 | 9.44E-05 | | 1.65E-05 |
| 0.00036 | 34.9 | 0.581078 | 0.000209 | | 2.96E-05 |
| 0.00048 | 29.9 | 0.735707 | 0.000353 | | 0.000682 |
| 0.0006 | 23.9 | 0.959687 | 0.000576 | | 0.000137 |
| | | | | sum | 0.001653 |

| | |
|---|---|
| Sigma(x) | 0.0018 |
| Sigma(x^2) | 7.92E-07 |
| Sigma(y) | 2.89554 |
| Sigma(y^2) | 2.005644 |
| Sigma(xy) | 0.00126 |

| | |
|---|---|
| delta | 1.51E-06 |
| slope | 1551.539 |
| intercept | 0.017128 |
| deviation | 0.020328 |
| delta(b) | 0.014712 |
| delta(m) | 40.49419 |

## Figure I - Fixed Surfactant Concentration



## Figure II - Fixed Quencher Concentration