Finance

**Amount:** A principal $P$ placed at an annual rate of interest $r$ for $n$ years accumulates to an amount $A_n$ as follows:

Simple interest: \[ A_n = P(1 + rn) \]

Compound interest: \[ A_n = P(1 + r)^n \]

Interest compounded q times per year: \[ A_n = P \left( 1 + \frac{r}{q} \right)^{nq} \]

**Nominal and Effective Interest Rates:** The rate of interest quoted in describing a given compound interest is called the “nominal rate.” The rate per year at which interest is earned during the year is called the “effective rate.” The effective rate $i$ corresponding to the nominal rate $r$, is \[ i = \left( 1 + \frac{r}{q} \right)^q - 1. \] Note that $i \geq r$. If interest is paid only once per year, i.e., $q = 1$, then the nominal and effective rates are the same. Compounding interest more than once per year, i.e., $q > 1$, yields a larger effective rate. APR (annual percentage rate) is the nominal annual interest rate plus one-time fees and additional charges. Although APR is intended to represent the total cost of credit to the consumer, it understates the true (effective) rate. An announced APR of 12.99% compounded monthly (as with credit card debt) is effectively a rate of 13.78%.

**Present or Discounted Value of a Future Amount:** The present quantity $P$ which in $n$ years will accumulate to the amount $A_n$ at the rate of interest $r$ is:

Simple interest: \[ P = \frac{A_n}{(1 + nr)} \]

Compound interest: \[ P = \frac{A_n}{(1 + r)^n} \]

Interest compounded q times per year: \[ P = \frac{A_n}{\left( 1 + \frac{r}{q} \right)^{nq}} \]

**Amount of an Annuity:** If an annuity $P$ is deposited at the end of each year, and interested, compounded annually, is paid on the accumulated deposit at the end of each year, the total amount $N$ accumulated at the end of $n$ years is
\[ N = P \frac{(1+r)^n - 1}{r} \]

N is called the amount of an annuity P.

**Present Value of an Annuity:** The total present amount \( P \) which will supply an annuity \( N \) at the end of each year for \( n \) years, beginning one year hence, (assuming that in successive years the amount not yet paid out earns interest at rate \( r \), compounded annually) is

\[ P = N \frac{(1+r)^n - 1}{r(1+r)^n} = N \frac{1-(1+r)^{-n}}{r} \]

**Amount of a Sinking Fund:** If a fixed investment \( N \) is made at the end of each successive year (beginning at the end of the first year), and interest paid at rate \( r \), compounded annually, is paid on the accumulated amount of the investments at the end of each year, the total amount \( S \) accumulated at the end of \( n \) years is:

\[ S = N \frac{(1+r)^n - 1}{r} \]

S is called the amount of the sinking fund.

Note: These descriptions and formulas have been taken from Burington, R., “Handbook of Mathematical Tables and Formulas”