Consistency Between Visual Acuity Scores Obtained at Different Test Distances

Theory vs Observations in Multiple Studies

Li Ming Dong, PhD; Barbara S. Hawkins, PhD; Marta J. Marsh, MS

APPENDIX

Figure 6 illustrates visual angles of lines on the test charts in a visual acuity test at two distances. We use the following notations to calculate the exact visual angle of each line at different test distances. For convenience, all sizes and distances are in centimeters.

\[ l_i = \text{letter size at line } i, \]
\[ s_i = \text{stoke width at line } i, \]
\[ h_i = \text{distance from the midpoint of line } i \text{ to the midpoint of the chart}, \]
\[ H_i = \text{distance from the top of line } 1 \text{ to bottom of line } i, \]
\[ \alpha_i(d) = \text{visual angle of letters on line } i \text{ at test distance } d, \]
\[ d = \text{test distance}. \]

From the design of the ETDRS chart, we have

\[ \rho = \text{progression of letter size down the chart} = 10^{-0.1} = 0.7943, \]
\[ l_i = l_1 \rho^{i-1}, \]
\[ H_i = l_1 + 2l_2 + \cdots + 2l_i, \]
\[ l_1 = 5.818 \text{ cm}, s_1 = 1.2 \text{ cm}, \frac{s_i}{l_i} = 0.2062564, \]
\[ H_{14} = \text{length of the chart} = 58.5 \text{ cm}. \]

Using the relationship

\[ \tan \alpha_i(d) = \tan (\beta_1 - \beta_2) = \frac{\tan \beta_1 - \tan \beta_2}{1 + \tan \beta_1 \tan \beta_2}. \]
where
\[ \tan \beta_1 = \frac{h_i + 0.5l_i}{d}, \quad \text{and} \quad \tan \beta_2 = \frac{h_i - 0.5l_i}{d}, \]
we have, for \( \alpha_i(d) \) visual angle of letters on line \( i \) at test distance \( d \),
\[ \tan[\alpha_i(d)] = \frac{dl_i}{d^2 + (h_i + 0.5l_i)(h_i - 0.5l_i)}. \]

Therefore, the exact visual angle for letters in line \( i \) at test distance \( d \) is
\[ \alpha_i(d) = \tan^{-1}\left( \frac{dl_i}{d^2 + (h_i + 0.5l_i)(h_i - 0.5l_i)} \right). \]

Since the proportion of stroke width to letter size is constant, \( s_i/l_i = 0.2062564 \), the logMAR for line \( i \) at test distance \( d \) is
\[ \log_{10} \left[ \alpha_i(d) \times \frac{360 \times 60}{2\pi} \times 0.2062564 \right], \]
where the factor \( \frac{360 \times 60}{2\pi} \) is to rescale the visual angle to minutes of arc.