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THE IMPACT OF DROPS ON LIQUID SURFACES AND THE UNDERWATER NOISE OF RAIN

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INTRODUCTION

> There will be but few of my readers who have not, in some heavy shower of rain, beguiled the tedium of enforced waiting by watching, perhaps half-unconsciously, the thousand little crystal fountains that start from the surface of pool or river; noting now and then a surrounding coronet of lesser jets, or here and there a bubble that floats for a moment and then vanishes.
>
> It is to this apparently insignificant transaction, which always has been and always will be so familiar, and to others of a like nature, that I desire to call the attention of those who are interested in natural phenomena; hoping to share with them some of the delight that I have myself felt, in contemplating the exquisite forms that the camera has revealed, and in watching the progress of a multitude of events, compressed indeed within the limits of a few hundredths of a second, but none the less orderly and inevitable, and of which the sequence is in part easy to anticipate and understand, while in part it taxes the highest mathematical powers to elucidate.

Thus begins the book *A Study of Splashes* (1908) in which A. M. Worthington (1852–1916) presents "in a form acceptable to the general reader the outcome of an inquiry conducted by the aid of instantaneous photography, which was begun about fourteen years ago." Worthington's fascination with these phenomena actually went all the way back to 1875 (he was then 23 years old), when H. F. Newall, a student at the famous Rugby boys' public school, gave a report at the Rugby Natural History

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Society about the curious "marks of accidental splashes of ink drops that had fallen on some smoked glasses."[1]

After experimenting with drops splashing on solid surfaces from 1876 to 1894, Worthington turned to studying the impact of drops on liquid surfaces—which is the object of the present review. With considerable ingenuity he was able to take remarkably sharp photographs of which he reproduced abundant examples in his book (Figure 1). As a light source he used electric sparks and was able to achieve exposure times of a few μsec. Thirty years later Edgerton improved on this method by using his newly invented electronic flash and published some very high-quality photographs of drops falling on a liquid surface in his book illustrating "the unseen by ultra high-speed photography" (Figure 2).

While Worthington was motivated purely by intellectual curiosity,[2] more recent investigations of this problem have been prompted by the very specific puzzle posed by the underwater noise of rain. This and further applications will be described in what follows.

Occasional references to the falling of drops and solid objects on liquid surfaces can be found scattered in the scientific literature in the following fifty years. Mallock (1919), in a paper titled *Sounds Produced by Drops Falling on Water*, states on the first page that "the same class of sounds were produced whether the falling body was a liquid drop or a solid sphere. The experiments, therefore, were made with solid spheres." He never mentions drops again and proceeds in fact to propose an incorrect explanation of the sounds in question. Several studies of impacting solid spheres followed Mallock's, notably by the celebrated Indian physicist C. V. Raman (Raman & Dey 1920), and E. G. Richardson (1948, 1955). In a brief note, Jones (1920) reported on some earlier observations of airborne drop sounds. Minnaert, with remarkable insight, concluded his famous 1933 paper (in which he found for the natural frequency of oscillation f_0 of a bubble of radius R the formula

$$f_0 = \frac{1}{2\pi R}\sqrt{\frac{3\gamma P}{\rho}}, \tag{1}$$

[1] Worthington's book is "Dedicated to the Natural History Society of the Rugby School and its Former President Arthur Sidgwick in Remembrance of the Encouragement Given to the Early Observations Made in Boyhood by my Old School-Friend H. F. Newall from which this Study Sprang." Later (1885) Newall and J. J. Thomson published the paper "On the formation of vortex rings by drops falling into liquids, and some allied phenomena."

[2] In 1894, opening a lecture delivered in front of the Royal Society—of which he was later to become a member—he said "... it may seem to some that a man who proposes to discourse on the matter for an hour must have lost all sense of proportion."

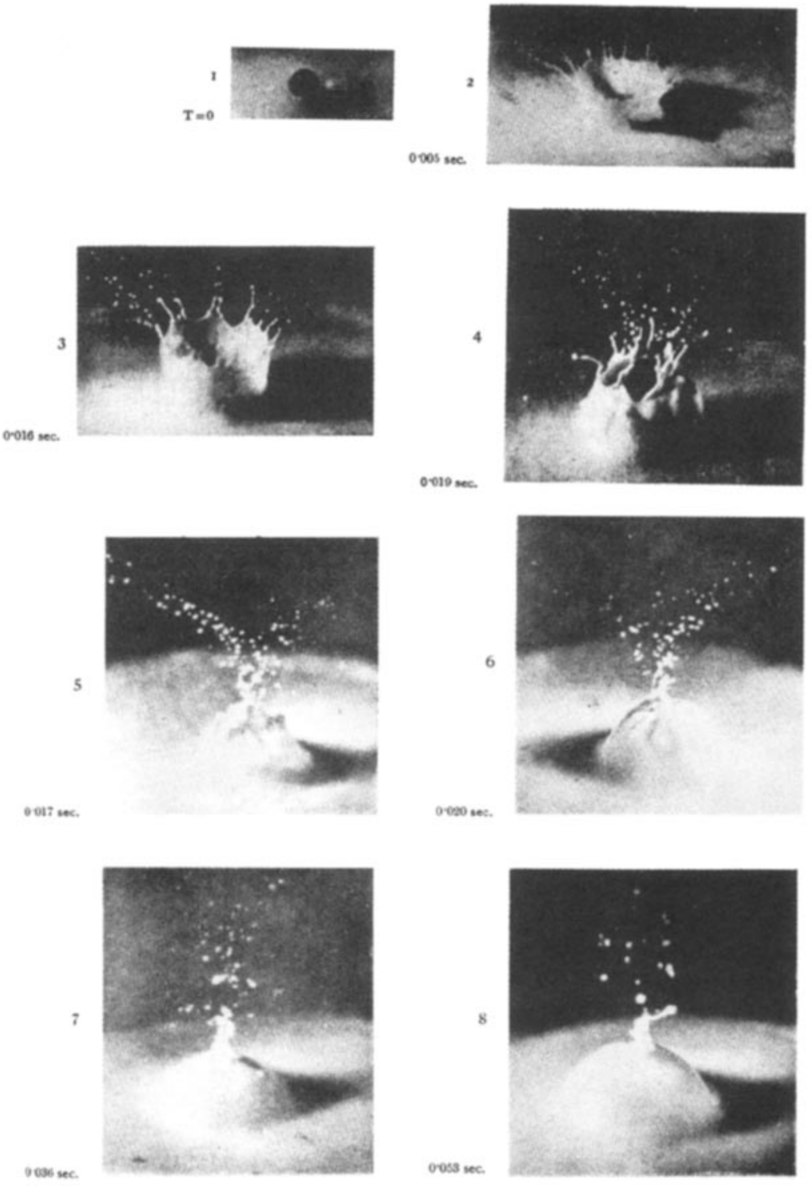


Figure 1 An example of Worthington's (1908) photos for a 9.1 mm diameter water drop impacting with a speed of approximately 5 m/sec (height of fall = 1.37 m). Each frame was obtained with a different drop released under nominally identical conditions. Note the thin splash that eventually closes with an upward jet originating at the point of closure. The photos are 5, 16, 16, 19, 17, 20, 36, and 53 msec after the first contact shown in the first image. [Reproduced with permission from Worthington (1963).]

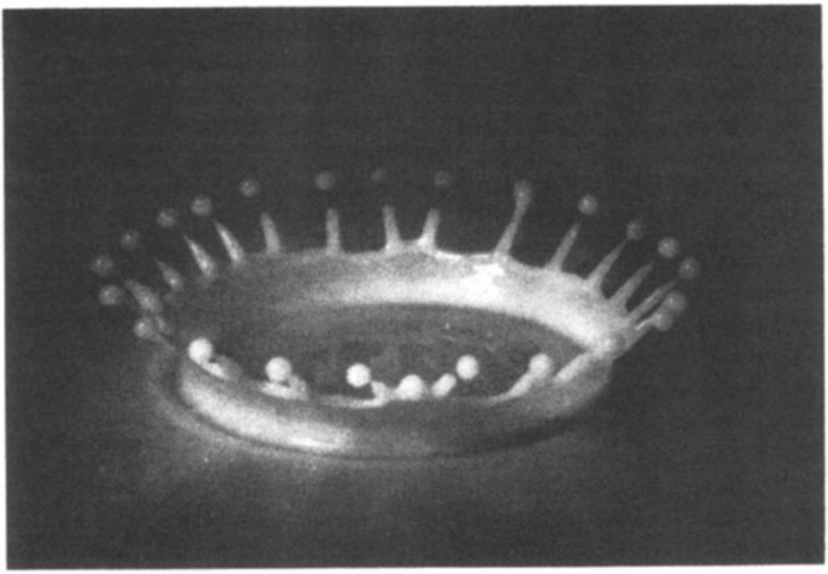


Figure 2 The coronet produced soon after the first contact of a drop of milk falling upon a plate covered with a thin layer of milk. [Reproduced with permission Edgerton & Killian (1987).]

where γ is the ratio of the gas specific heats, p the ambient pressure, and ρ the liquid density), with the words "It remains to investigate ... if the sounds of falling drops cannot have the same origin as the bubble sounds."

In 1959 Franz, in a paper titled *Splashes as Sources of Sound in Liquids*, discussed several mechanisms of noise production by impacting drops and solids and then describes an experiment devised "to substantiate some of the speculations about how splashes produce sound." He seems to have been the first one to make the crucial discovery that "Under some conditions, an air bubble can be entrained in the water ... The total sound energy radiated by an individual bubble was often greater than the sound energy radiated by the impact of the water droplet." However, since bubble entrainment seemed to be a random and infrequent event, he felt that, on average, bubbles had a relatively minor importance in sound generation. As we shall see, recent research has instead proven quite the opposite to be true.

The complexity of the fluid mechanic process uncovered by the photographs of Worthington and Edgerton was such as to discourage attempts at a theoretical interpretation of the observed flow. The first contribution of this nature was an early application of the MAC computational method of Harlow and Amsden (Harlow & Shannon 1967a,b). Later research, both theoretical and experimental, has proven however that the numerical method was too dissipative to give precise results.

THE RAIN NOISE PARADOX . . .

World War II research had shown that rain produces a substantial amount of underwater noise (Knudsen et al 1948). In a short note, Heindsman et al (1955) published the first rain noise spectra, but their inability to obtain reliable data beyond 10 kHz prevented them from discovering the unique acoustic signature of rain and they concluded that the spectral "characteristic was that of wide band white noise," a misconception reinforced by some aspects of Franz's work (1959) and reiterated in the well-known review paper of Wenz (1962). A few years later Bom (1969) took better data in an Italian lake but, due to his study of the same frequency range, he was only able to confirm that the noise level increases with rainfall rate and exhibits a slight decline with frequency.

In the mid-1970s, two factors were coming together to rekindle specific interest in the underwater noise of rain. One—as always with underwater noise studies—was the military need for a better understanding of the natural mechanisms of underwater noise generation brought about by improvements in sound detection, signal processing, and the silencing of vessels. The other factor was the beginning of the use of acoustical techniques to probe phenomena of geophysical significance. In a paper devoted to the acoustical measurement of wind speed and stress at the ocean surface, Shaw et al (1978) realized the possibility that the results could be contaminated by rain noise. By using acoustic monitoring at three different frequencies, Lemon et al (1984) were able to identify the rain periods of their time traces, but their attempt to infer the amount of rainfall was not successful.

Some time before the study of Lemon et al, Walter Munk had suggested to J. A. Nystuen—then a graduate student at Scripps—that he study the possibility of using underwater noise to infer rainfall rates. This is an important quantity because rain is an essential component of the atmospheric heat balance, but about 80% of it falls on the oceans where measurements with good spatial resolution and over long periods of time are next to impossible. In the fall of 1982 Nystuen obtained the first data at Clinton Lake in Illinois over a frequency range much broader than used until then and found an astonishingly pronounced peak around 15 kHz. Publication of this result was however delayed until 1986. In 1984 Scrimger and coworkers (who had been in contact with both Nystuen and Farmer) made similar measurements in Canada with analogous results and were the first to report the discovery in the open literature (Scrimger 1985; Scrimger et al 1987, 1989).

The remarkable results obtained by these investigators can be illustrated with reference to Figure 3 (from Scrimger et al 1987). The left-hand side of the figure shows drop size spectra obtained with a distrometer during two rain events, labeled (*a*) and (*b*), while the right-hand side shows the corresponding underwater acoustic spectra. While it is obvious from the distrometer records that the data correspond to different rainfall rates—0.4 mm/hr for (*a*) and 0.3 for (*b*)—and drop sizes, the noise spectra are remarkably similar above about 10 kHz. This similarity between the spectra of different rain events is quite common at light rainfall rates and low wind speeds and is so striking that one may refer to a *universality* of the underwater noise of rain above 10 kHz or so.

. . . AND ITS RESOLUTION

In a footnote to their paper mentioned before, Raman & Dey (1920) state that "the splash of a liquid droplet is practically soundless unless the height of fall exceeds a certain minimum." This remark prompted the publication in the same year of a one-page note in *Science* in which A. T. Jones mentioned some preliminary observations he had conducted in 1915 where he found "not only a single minimum height . . . , but also other greater heights of fall for which the drops enter the water without sound." Curiously, this absolutely key observation was overlooked and Jones's note completely forgotten.

For his undergraduate thesis at Cambridge University, under the guidance of A. J. Walton, H. C. Pumphrey in 1984 had started a systematic investigation of the impact of a single drop on a liquid surface (Pumphrey & Walton 1988). Upon graduation, he entered the physics PhD program at the University of Mississippi where L. A. Crum had recently become involved in oceanic ambient noise research. Pumphrey conducted a very careful and exhaustive series of experiments releasing drops of different sizes from various heights, measuring the sound and, at the same time, taking high-speed movies (Pumphrey 1989, Pumphrey et al 1989, Pumphrey & Crum 1988,1990). Examples of sequences of frames from these movies are shown in Figures 4 and 5. In the first case (Figure 4), as a result of the impact, a small bubble is entrapped in the liquid. Figure 6 shows a detailed view of the formation and detachment of such a bubble. In the case shown in Figure 5, on the other hand, the dynamics of the cavity is quite different and no bubble is produced. A decisive innovation with respect to earlier research was to photograph *on the same frame* both the physical event and the trace of an oscilloscope driven by a hydrophone

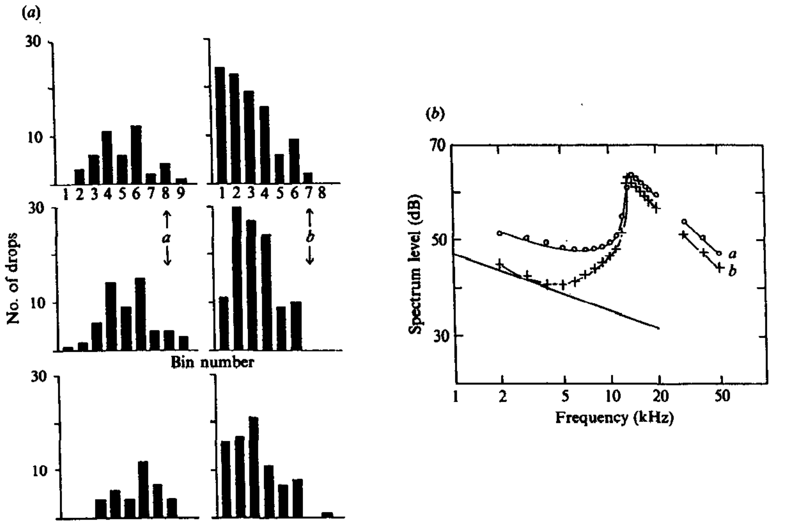
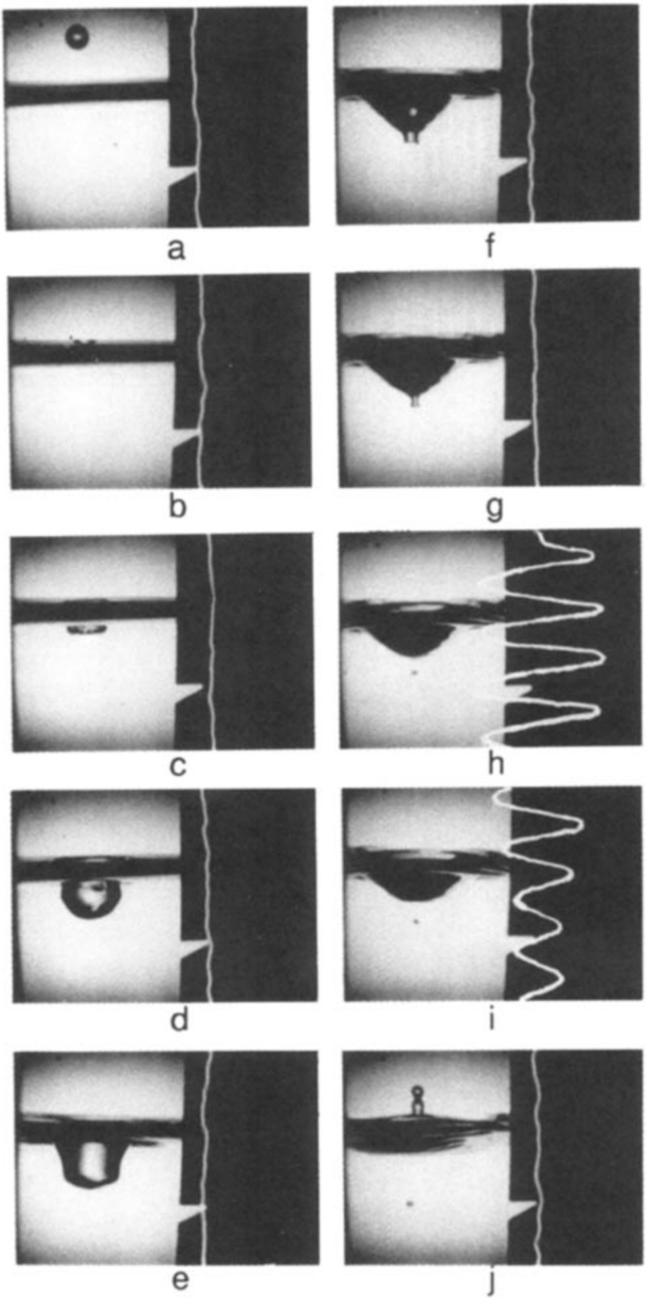


Figure 3 The left part of the figure shows three sets of 30 sec distrometer data corresponding to two different rain events (*a*) and (*b*). The right part shows the corresponding underwater sound spectra with the solid line indicating the background noise level. The wind speed was 0.9 m/sec and the rainfall rate was 0.4 mm/hr for event (*a*) and 0.3 mm/hr for event (*b*). Bin numbers 1 to 5 correspond to drop diameters between 0.3 and 0.8 mm in 0.1-mm intervals, larger bin numbers correspond to 0.2-mm intervals. [Reproduced with permission from Scrimger et al (1987).]

located near the point of impact. In this way it became apparent that, with the level of amplification used, the impact itself produced an acoustic signal indistinguishable from the electronic noise, while the bubble noise was very strong. While these observations confirmed earlier work, Pumphrey was able to determine that there was a rather limited and well defined region of impact velocities and drop diameters that resulted in bubble entrainment and, therefore, in the production of significant noise (Figure 7). At this point the resolution of the rain noise paradox was at hand (Prosperetti et al 1989). Although in the laboratory drop size and impact velocity are independent, in natural rain they are connected by the terminal velocity curve, which is the steeply rising line plotted on the left of Figure 7. If bubble-entraining drops are responsible for the underwater noise of rain above 10 kHz, it is clear that the only relevant size range is that where the terminal velocity curve crosses the bubble-entrainment region. Due to the steepness of the terminal velocity line, this range is quite narrow, extending from a diameter of about 0.82 mm to 1.1 mm, which happens to correspond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to sizes quite common in natural rain. To the extent that all raindrop spectra contain drops of this size, the acoustic signature of rain is therefore nearly universal.

In the presence of wind and waves both the velocity and the angle of impact of the drops on the water surface are affected and the bubble entrainment probability decreases (Medwin et al 1990). Furthermore, in these conditions, the bubble layer that forms at the ocean surface strongly attenuates the surface-generated sound (Farmer & Lemon 1984). Nevertheless, the 14 kHz peak remains a clearly recognizable feature of the acoustic spectra for wind velocities at least up to several m/sec (Medwin et al 1992).

This explanation of the nature of rain noise has received several confirmations. For example, a very simple and striking observation is that rain noise in an outdoor tank can be nearly completely shut off by the addition of a small amount of a surfactant that also inhibits bubble entrapment (Pumphrey & Crum 1988).

HIGHER-ENERGY IMPACTS

In the paper cited earlier, Jones (1920) mentions not one, but two ranges of impact velocity in which noise is associated with the drop impact. An investigation of high-kinetic-energy impacts along the terminal velocity curve has recently been carried out by Medwin et al (1992). The results are in agreement with Jones's in that bubble entrainment for impact velocities higher than those considered in the previous section is also encountered. By high-speed cinematography it was found that the mechanism of air entrainment is however completely different from the one described earlier. A reproduction of tracings from the high-speed film is shown in Figure 8, which may be compared with Figure 1.

The early stages of the process are qualitatively identical to those found upon the impact of a solid object and already noted and illustrated by Worthington (1908).[3] At first the drop creates a nearly vertical thin splash

Figure 4 Successive frames of a single 3 mm diameter water drop impacting on water at 2 m/sec. The right half of each frame shows the screen of an oscilloscope driven by a hydrophone placed in the vicinity of the impact point. Note the strong damped sinusoid that appears in correspondence with the detachment of a bubble from the nipple formed at the bottom of the crater. The times, in msec, are: (*a*) –3; (*b*) 0; (*c*) 2; (*d*) 6.5; (*e*) 14.5; (*f*) 20; (*g*) 20.5; (*h*) 21; (*i*) 21.5; (*j*) 35.5. (Courtesy of H.C. Pumphrey.)

[3] who, in this connection, observes: "I can recommend any reader who is not afraid of being late for breakfast to keep a bag of marbles in his bath-room."

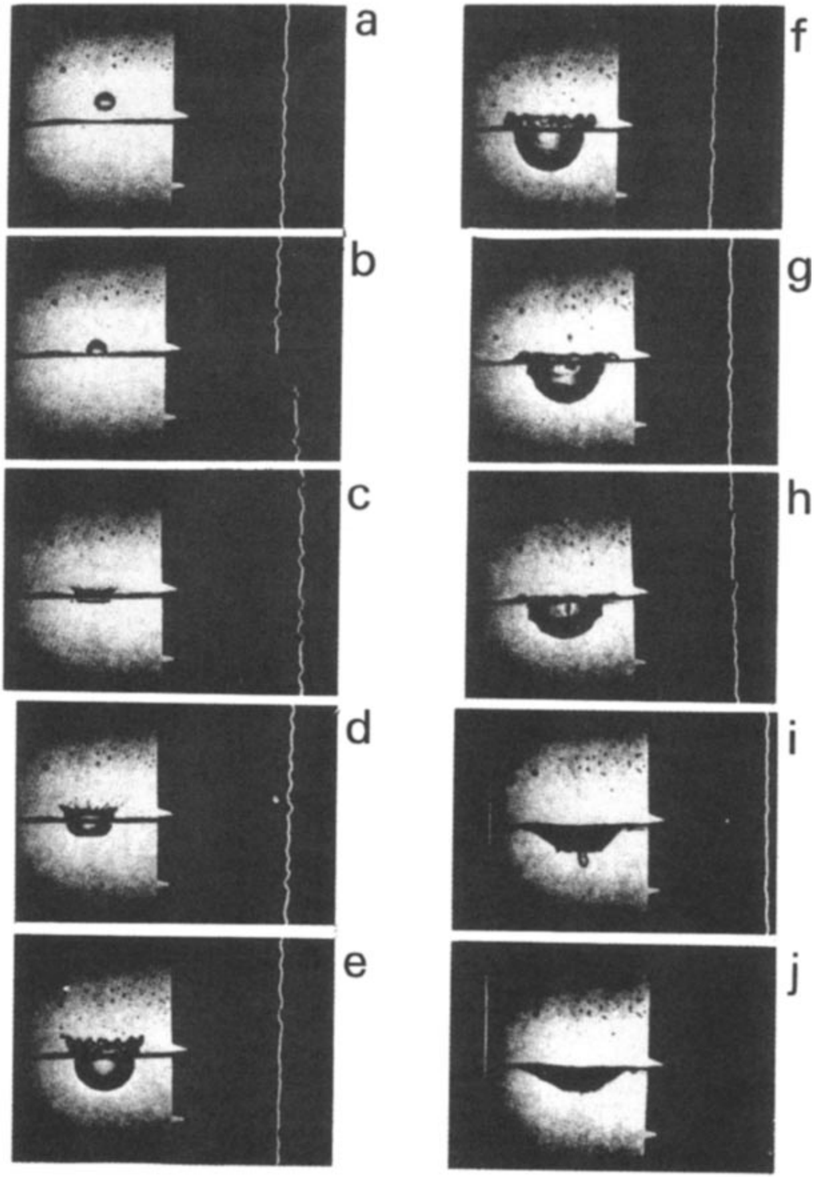

Figure 5 Successive frames of a single 2.9 mm diameter water drop impacting on water at 2.4 m/sec. The right half of each frame shows the screen of an oscilloscope driven by a hydrophone placed in the vicinity of the impact point. Although in these conditions no bubble is entrained by the primary drop, a tiny bubble is seen to be entrained in frame (*i*) by a Plateau's spherule [visible in frame (*g*)] that follows the main drop. The times, in msec, are: (*a*) –3; (*b*) 0; (*c*) 2; (*d*) 5; (*e*) 13; (*f*) 22; (*g*) 30 ; (*h*) 35; (*i*) 44; (*j*) 46. [Reproduced with permission from Pumphrey & Crum (1988).]

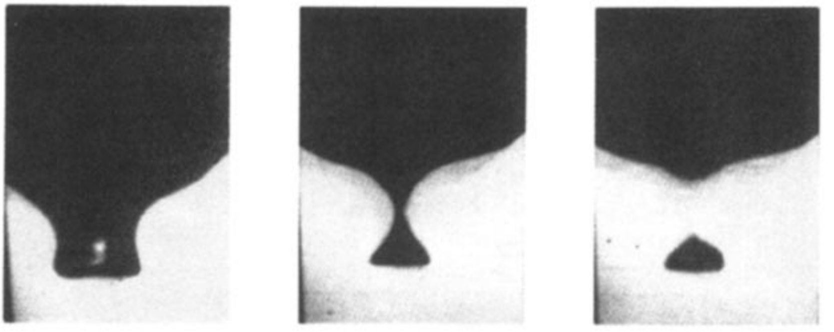

Figure 6 Detail of bubble formation and detachment for a 2.5 mm diameter water drop impacting at 2 m/sec. [Reproduced with permission from Chahine et al (1991).]

which has been shown by Engel (1967) to consist of receiving liquid rather than drop liquid. At a certain point, if the drop's energy is sufficiently large, the thin-walled liquid cylinder thus created closes at the top and forms "a bubble that floats for a moment and then vanishes." Worthington

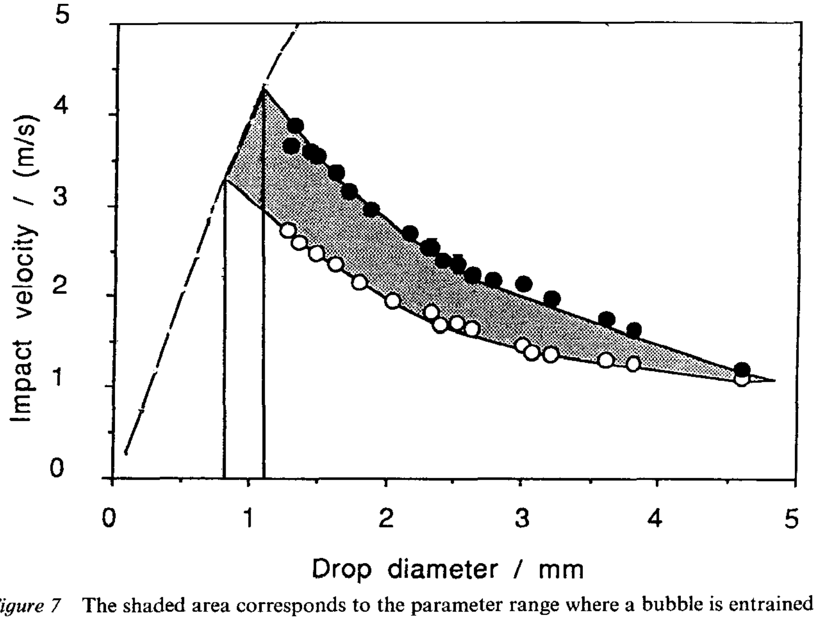

Figure 7 The shaded area corresponds to the parameter range where a bubble is entrained by every water drop. The dashed line on the left is the terminal velocity curve, and the two vertical parallel lines indicate the size range where drops impacting at their terminal velocity entrain bubbles. [Reproduced with permission from Pumphrey et al (1989).]

hypothesized that this process is governed by surface tension forces, and indeed this must be so since the duration of this phase is too short for gravity to have any effect. The precise mechanism, however, cannot be said to be clearly understood. As the canopy closes, the motion must stop abruptly and the inward-directed momentum generates a sharp pressure rise that results in two nearly symmetrical jets, one directed upward and the other one downward into the cavity. According to the observations of Medwin et al (1992) it is this second jet that entrains an air bubble upon striking the water surface (last frame of Figure 8). Understandably, in view of the higher energy of this process and the precise timing required, about half the drops entrain bubbles in this way as opposed to the previous mechanism for which the entrainment probability was essentially 100%. The size of the entrained bubble also exhibits a much greater variation from drop to drop.

These results have a substantial bearing on rain noise at frequencies below the peak discussed in the previous sections. As already noted by

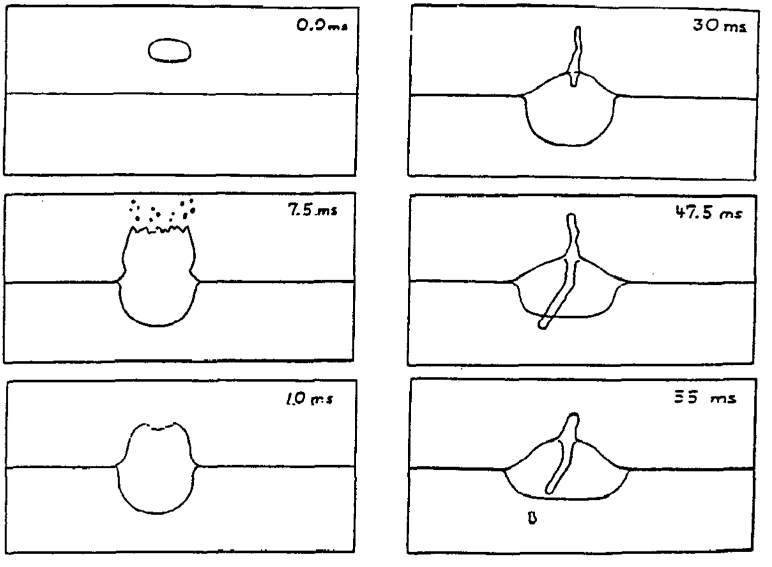

Figure 8 Tracings from a high-speed movie of a 4.57 mm diameter drop impacting at 9.3 m/sec; times in msec. This sequence is comparable to that shown in Figure 1. Note the bubble entrained by the impact of the downward jet formed by the closing of the canopy. [Reproduced with permission from Medwin et al (1992).]

Knudsen et al (1948), and later confirmed by the work of Heindsman et al (1955) and Bom (1969), there is a clear correlation between rainfall rate and underwater noise below 10 kHz. Medwin et al (1992) present convincing arguments that the noise in this range is the product of the alternative entrainment mechanism they describe, which becomes more and more important with increasing rainfall rates due to the presence of greater numbers of larger drops.

A further mechanism, first reported by Franz (1959) and confirmed by Pumphrey & Elmore (1990), can entrain bubbles in the case of fairly large and energetic drops. In this mechanism, the liquid column collapses and fills the crater produced by the impact.

An extreme case is that of huge "drops" (in fact, several liters of water) released above a liquid pool (Kolaini et al 1991). Here again large bubbles are entrained with acoustic emissions at frequencies of the order of a few tens of Hz.

NUMERICAL SIMULATIONS

Harlow & Shannon's (1967a,b) work already mentioned, although remarkable at the time, employed too crude a numerical method to disentangle the subtleties of the physical process found by Pumphrey in his experiments. This was confirmed by calculations by Nystuen (1986) who used the same MAC code without being able to improve on those results. We decided therefore to use a potential-flow boundary-integral method that had been found to work remarkably well in other simulations of free-surface flows. Of course this implied doing away with viscous effects and therefore could only be expected to be useful for times t much shorter than a^2/ν, where a is the drop radius and ν the liquid's kinematic viscosity. For water drops of $a = 1$ mm, however, $a^2/\nu \simeq 1$ sec, so that the restriction is not too stringent.

The early attempts were plagued by severe numerical instabilities. It was necessary to develop a new boundary-integral formulation resulting in a Fredholm integral equation of the second, rather than the first, kind, and novel time stepping techniques and surface parametrizations which are described in detail in the original paper (Oğuz & Prosperetti 1990). We were finally successful and some typical results of the computations are shown in Figure 9, to be compared with the experiment of Figure 4. Other examples are given in Oğuz & Prosperetti (1990). In spite of some differences, the important features of both bubble-entrapping and non-entrapping events are well reproduced numerically.

Given this early success, we decided to attempt a completely synthetic—i.e. numerical—calculation of underwater rain noise. The only exper-

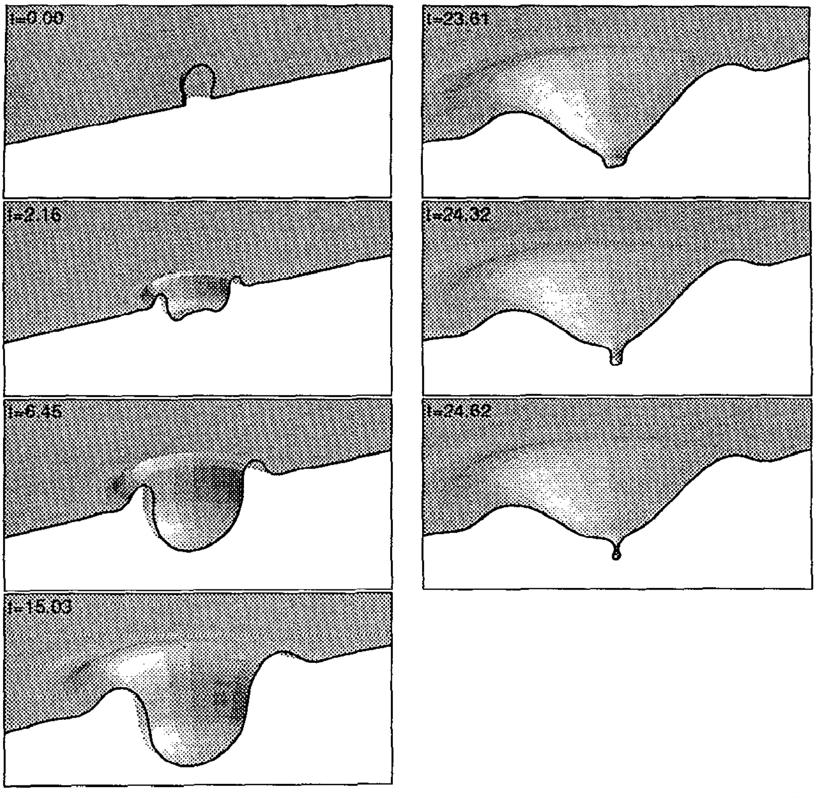

Figure 9 Numerical simulation of a 2.86 mm diameter drop impacting at 2 m/sec (Fr = 285.4, We = 79.4). The times are in msec.

imental input was to be the background noise and the total drop count in the bubble-entrapping size range. The procedure was as follows. A number of drop radii were chosen along the terminal velocity line and the corresponding simulations were run to determine the size of the entrained bubble and its initial energy. After closure, the bubble was allowed to oscillate with the natural frequency and damping rate corresponding to its size, and the acoustic emissions from all the bubbles were added incoherently with the assumption of dipole radiation due to the presence of the neighboring free surface.

The plan was rather ambitious. It was known from Pumphrey's laboratory study that the reproducibility of the bubble is not very good in the neighborhood of the boundaries. For instance, in the case of a 1 mm-diameter drop, the experimental standard deviation of the frequency of bubble emission is of the order of 0.3% near the center of the entrainment

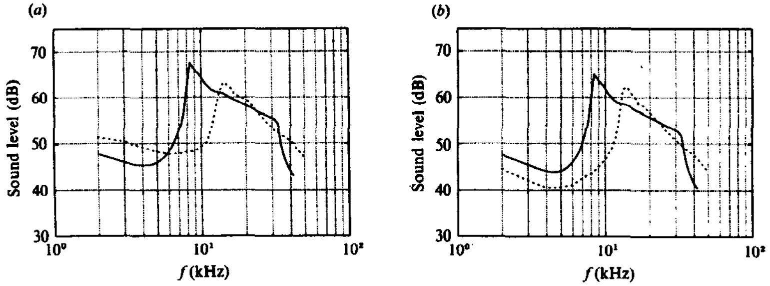

Figure 10 Comparison between measured (*dotted lines*) and computed (*solid lines*) underwater noise spectra for the two rain events (*a*) and (*b*) of Figure 3. [Reproduced from Oğuz & Prosperetti (1991).]

region, but becomes of the order of 100% near the upper and lower velocity boundaries. Furthermore, it can be appreciated from Figure 9 that the bubble region is only a small fraction of the flow domain, so that its good numerical resolution is not trivial. Nevertheless our attempt was moderately successful, as shown by the comparison between measured and calculated results given in Figure 10. The general shape of the peak is in good agreement with the measured one, and the noise level is also very close. The biggest discrepancy is in the position of the peak, which the calculations put at around 9 kHz rather than 14, thus indicating—from Equation 1—that the size of the "numerical" bubbles is somewhat greater than that of the physical ones. We have carried out a detailed analysis of the origin of this discrepancy (Oğuz & Prosperetti 1992) but the results are rather ambiguous. The indications are that it is probably a consequence of the neglect of viscous effects in the air as well as in the water.

The computations do however afford an interesting insight into the fluid dynamics of the process.[4] For example, we show in Figure 11 the last computed configuration of the very bottom of the drop crater for several drop diameters and impact velocities along the terminal velocity curve. It is seen that, for small drops, the crater is not deep enough to "pinch" an air bubble. As the drop's energy increases, the crater becomes deeper and

[4]This would have been of some solace to Worthington who ruefully observes: "But even were the photographic record complete, what does it amount to? All that we have done has been merely to follow the rapid changes of form that take place in the bounding surface of the liquid. The interior particles of the liquid itself have remained invisible to us. But it is precisely the motion of these interior particles that the student of hydrodynamics desires to be able to trace."

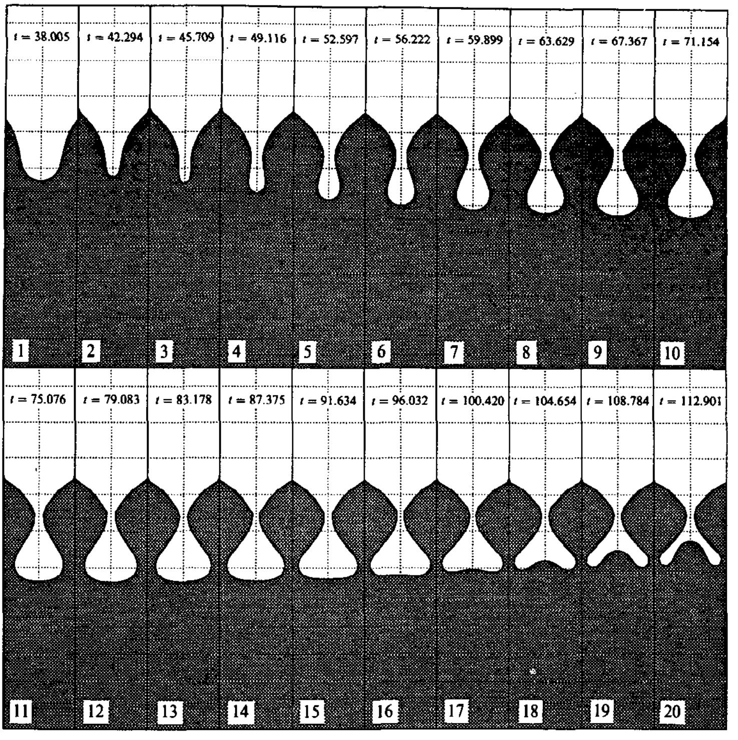

Figure 11 Last computed frame for the impact of water drops along the terminal velocity curve and corresponding dimensionless times Vt/a. The first panel is for a drop diameter of 0.82 mm, the second one for 0.84 mm, and so on; the last one is for 1.2 mm. The terminal velocity for a 0.80 mm drop is 3.28 m/sec, for 0.90 mm—3.66 m/sec, for 1 mm—4.01 m/sec, and for 1.2 mm—4.65 m/sec. The spacing of the dashed lines equals one drop radius. [Reproduced from Oğuz & Prosperetti (1991).]

the sideways motion reverses faster than the downward one, so that a bubble remains entrapped. At still higher energies, the directionality of the drop's momentum becomes less important and the crater grows more nearly spherically (compare Figures 4 and 5). Now the bottom of the crater is the highest-energy point and the downward motion tends to reverse earlier and earlier, until eventually the inward lateral velocity is not sufficient to close the cavity before the bottom jet shoots out of it and prevents the formation of the bubble.

THEORETICAL ASPECTS

The sequence of events uncovered by experiment and computation is indeed so complex as to "tax the highest mathematical powers to elucidate." So much so, as a matter of fact, that not much progress has been made along this line.

If the drops are assumed to strike as spheres (a point to which we shall return below), there are three basic dimensionless groups on which the process depends, the Froude Fr, Weber We, and Reynolds Re numbers defined by

$$Fr = V^2/ga, \qquad We = \rho V^2 a/\sigma, \qquad Re = aV/\nu. \tag{2}$$

Here V is the impact velocity, σ the surface tension coefficient, and g the acceleration of gravity. With the neglect of viscous effects, the controlling

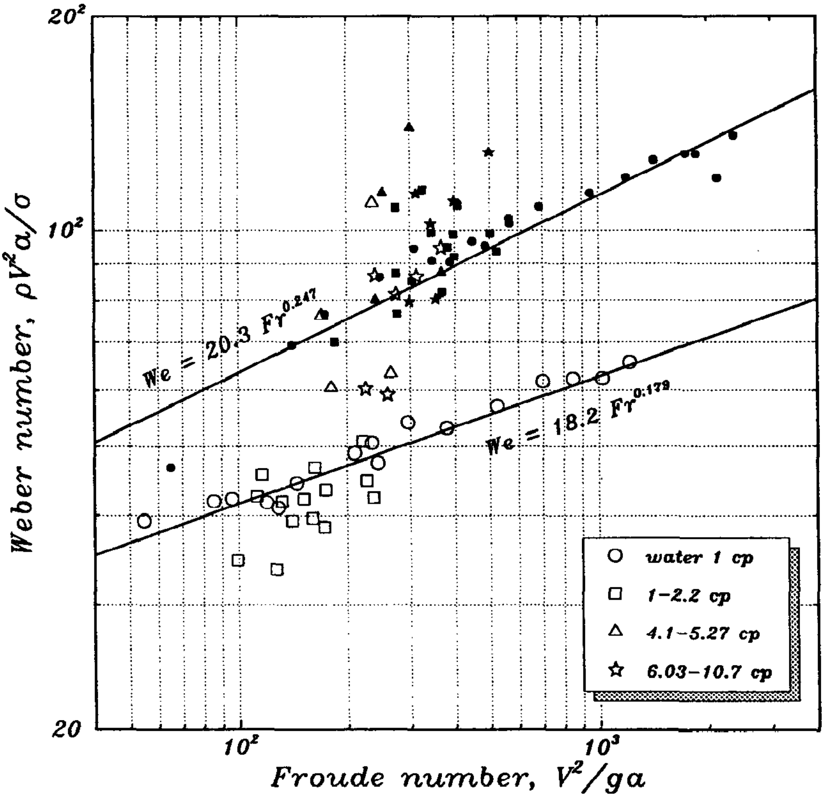

Figure 12 Nondimensional plot of the regular entrainment region. The circles are the water data of Figure 7. The other data points are for different liquids of various viscosities as specified in the legend. The straight lines are fits to the curves of Figure 7.

parameters are *Fr* and *We* and one can replot the boundaries of the entrainment region of Figure 7 in the (*Fr*, *We*) plane. We show such a plot in Figure 12 where the data of Figure 7 are supplemented by others (indicated by squares, triangles, and stars), due to Detsch & Harris (1992), and taken with other liquids. The black symbols are for the upper entrainment boundary and the open ones for the lower boundary. Circles are for water, squares for other low-viscosity liquids ($\mu = 1$ to 2.2 cp), triangles for intermediate viscosities ($\mu = 4.1$ to 5.27 cp), and stars for high-viscosity liquids ($\mu = 6.03$ to 10.7 cp). A general consistency of the data for water and other low-viscosity liquids is present, but the amount of scatter is significant. This is not surprising in view of the results previously mentioned concerning the large variance of the data near the entrainment boundaries. The lines shown in the figure are the same as those shown in Figure 7. They are power laws of the form $We = kFr^{\alpha}$, with k a constant and $\alpha = 0.247$ for the upper line and $\alpha = 0.179$ for the lower line. Simple arguments given in Oğuz & Prosperetti (1990) lead to the values $\frac{1}{4}$ and $\frac{1}{5}$, respectively.

It may be noted that if, rather than by using a different liquid, surface tension is varied by the addition of a surfactant, the previous scaling is found to be incorrect (Pumphrey & Elmore 1990). The spreading of the surfactant on the newly formed liquid surface is evidently an important phenomenon under these conditions.

The maximum size of the crater can be estimated by equating the potential energy stored into it to the initial energy of the drop. For this argument to work, it is necessary that the velocity field vanish nearly simultaneously in the liquid, which is only approximately true. If the additional assumption is made that the crater is a hemisphere (which could be corrected by the use of a dimensionless shape factor of order 1), one finds the crater radius R_c to be

$$\frac{R_c}{a} = \left[2\left(\tfrac{2}{3}Fr + 4\frac{Fr}{We} + \frac{Fr^2}{We^2}\right)^{1/2} - 2\frac{Fr}{We}\right]^{1/2}. \tag{3}$$

Numerically Fr/We is much smaller than Fr and the equation can be simplified to

$$\frac{R_c}{a} = (\tfrac{8}{3}Fr)^{1/4} \simeq 1.278\,Fr^{1/4}. \tag{4}$$

This expression is found to be in very good agreement with the data of Pumphrey & Elmore. At the higher energies of Engel's (1967) experiments, the numerical constant is somewhat lower—about 1.05.

The dynamics of the crater after it reaches the conical shape visible e.g.

in the 6th and 7th frames of Figures 4 and 9 has been investigated by Longuet-Higgins (1990). By assuming a self-similar potential flow solution of the form

$$\phi = \tfrac{1}{2}A(t)r^2(2\cos^2\theta - 1), \tag{5}$$

and neglecting the effect of gravity and surface tension, he found that a limit angle exists past which the pressure gradient is unable to further increase the aperture of the cone. This limit angle is 109.5°, and compares very well with the numerical calculations of Oğuz & Prosperetti (1990). On the basis of his solution, Longuet-Higgins was able to estimate the acoustic dipole moment of the entrained bubble in good agreement with experiment.

ACOUSTICS OF IMPACT

Insofar as they originate from the entrapped bubble, the acoustic emissions considered so far may be regarded as a byproduct of the fluid flow generated by the impacting drop. The impact itself, however, is also a source of sound. The two acoustic emissions can clearly be differentiated as seen in Figure 13, where the initial signal is the impact and the subsequent damped sinusoid is the bubble noise. Although it is now known that the acoustic energy directly due to the impact represents an insignificant fraction of the total radiated energy, it is interesting to briefly consider its mechanics.

As described by Lesser & Field (1983), at the very early stages after contact, a compression wave propagates supersonically in the two liquids following the progress of the geometric circle of contact between them. Guo & Ffowcs Williams (1991) find that this supersonic stage lasts until a time t_c given approximately by

$$t_c \simeq \tfrac{1}{2}M^2\frac{a}{V}, \tag{6}$$

where $M = V/c$ is the Mach number of the impact. For $V = 2$ m/sec and $a = 1$ mm, this gives $t_c \simeq 0.4$ nsec. After this time the liquid can expand and the initial thin splash is formed. A calculation of the acoustic energy E radiated during this initial stage gives

$$E \simeq \frac{3}{16}M^3T, \tag{7}$$

where $T = 2/3\pi\rho a^3V^2$ is the drop's kinetic energy.

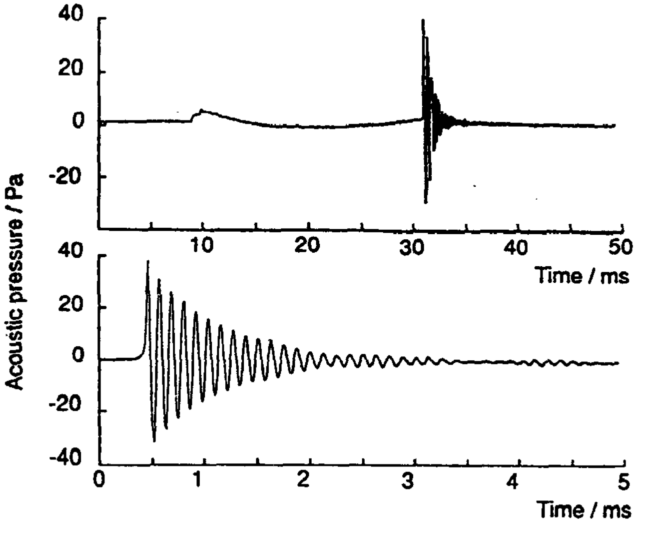

Figure 13 Near-field pressure disturbance produced by an impacting drop. The upper trace shows the initial impact and the bubble emission about 20 msec later. The lower part of the figure is a magnification of the bubble noise. (Note: Origin has been shifted for clarity.) [Reproduced with permission from Pumphrey et al (1989).]

The analysis beyond t_c is far from simple, but a dimensional argument can be constructed as follows (Oğuz & Prosperetti 1991). As expected, and as confirmed by Pumphrey & Elmore (1990), the radiated pressure p_r has the character of a dipole. The source/sink pair that constitutes it is separated by a distance of order a and must each have a strength of the order of the mass injection rate $\rho V a^2$ times the inverse characteristic time a/V. Thus

$$p_r \sim \frac{\cos\theta}{r}\,\frac{a}{ac/V}\,\frac{V}{a}\,\rho V a^2 \Phi(Vt/a, M). \tag{8}$$

Here θ and r are polar coordinates measured from the point of impact and the factor $a/(ac/V) \sim a/\lambda$, with λ the acoustic wavelength, converts the monopole to dipole radiation. The dimensionless function Φ, taken to depend on the only available dimensionless variables, accounts for the details of the process. Since $M \ll 1$, for times that are not too small, we can approximately take $\Phi(Vt/a,M) \simeq \Phi(Vt/a,0) \equiv \Phi(Vt/a)$. The total energy radiated can readily be calculated from the form (8) of the pressure field and is

$$E = \frac{2\pi a}{3\rho V c}\left(\frac{\rho a V^3}{c}\right)^2 \int_0^\infty \Phi^2(t_*)\, dt_*, \tag{9}$$

where $t_* = Vt/a$. By using Parseval's equality, the integral over time can be converted into an integral over the dimensionless frequency $f_* = fa/V$, from which the radiated energy in the frequency band between f and $f+df$ can be read off directly to find

$$\hat{E}(f) = \frac{4\pi a}{V} TM^3 |\hat{\Phi}(f_*)|^2, \tag{10}$$

where $\hat{\Phi}(f_*)$ is the Fourier transform of $\Phi(t_*)$. This argument leads to the expectation that the dimensionless function

$$\hat{E}_*(f_*) \equiv 4\pi|\hat{\Phi}|^2 = \frac{V\hat{E}(f)}{aTM^3} \tag{11}$$

is a universal function, as indeed was found by Franz (1959), whose data are plotted in the form suggested by this equation in Figure 14.

ALLIED PHENOMENA

However complex and intriguing the fluid mechanics processes touched upon in the previous sections, by far they do not exhaust the astonishingly rich range of phenomena related to the impact of drops on liquid surfaces.

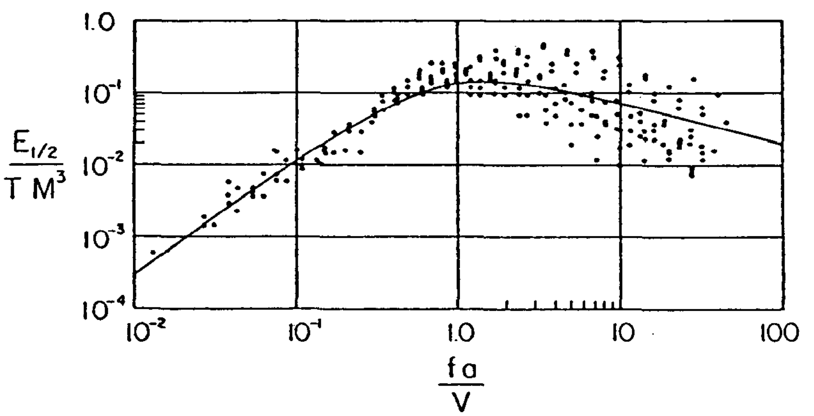

Figure 14 Impact noise data plotted according to the scaling implied by Equation (11). $E_{1/2}$ is the energy per half-octave band, T the drop's kinetic energy, M the Mach number at impact, f the frequency, a the drop radius, and V the impact velocity. [Reproduced with permission from Franz (1959).]

Still very little understood, for instance, is the formation and penetration of vortex rings into the receiving liquid. This process was first reported in 1858 by William Barton Rogers,[5] and later (1885) studied by J. J. Thomson & H. F. Newall. They observed that certain heights produce the most penetrating vortex rings, and that these heights are spaced so as to permit one drop oscillation between them. A beautiful photo by Peck and Sigurdson of the very early stages of vortex ring formation has recently been published by Reed (1991), and one corresponding to a few instants later, taken by Okabe & Inoue (1961), is reproduced in Batchelor's (1967) *An Introduction to Fluid Mechanics*. According to a study by Rodriguez & Mesler (1988), there is an effect of the drop shape upon impact which, rather than being direct as hypothesized by Chapman & Critchlow (1967), is mediated by the crater dynamics. After its formation, the vortex ring expands and eventually becomes unstable—a regime studied by Kojima et al (1984) in the low Reynolds number limit.

Vortex ring formation is a phenomenon typical of relatively low-velocity impacts (Hsiao et al 1988). Most characteristically, at higher velocities the drop fluid mixes only negligibly with the receiving fluid and collects at the tip of the jet that ultimately results from the filling up of the crater. This fact was known to Worthington, who observes:

> The reappearance of the original drop at the head of the rebounding column ... is easily verified by naked-eye observation. Let the reader when he next receives a cup of tea or coffee to which no milk has yet been added, make the simple experiment of dropping into it from a spoon at the height of fifteen or sixteen inches above the surface, a single drop of milk. He will have no difficulty in recognizing that the column which emerges carries the white-milk drop at the top only slightly stained by the liquid into which it has fallen.

An intriguing phenomenon is found when the depth of the receiving liquid is so small that bottom effects become important. As first reported by Hobbs & Osheroff (1967), and recently studied by Shin & McMahon (1990), the height of the rebounding jet reaches a maximum for a certain pool depth. This is the effect of two competing mechanisms. For very shallow pools, the jet's height is limited by the fact that not enough liquid is readily available to feed it. For deep pools, on the other hand, a substantial amount of liquid flows into the jet which is therefore thicker and limited by gravity in its ascent. The jet velocity can be as large as four times the velocity of the drop that originates it. Mori et al (1987) have also studied this phenomenon and reported that the size of the bubble entrained grows at the same time as the pool depth and the jet height.

[5] Although at the time he was actively engaged in efforts that ultimately led to the establishment of M.I.T. seven year later, evidently he still had time for science.

For high-velocity impacts, the drops that form the coronet along the rim of the initial splash (Figure 2) detach. Hobbs & Kezweeny (1967) studied their number and the process of electric charge separation that takes place in these circumstances. Hashimoto & Sudo (1980) gave a detailed account of bubble entrainment by splashing drops in a vertically vibrated liquid column, and Sudo et al (1991) studied the impact of drops of a magnetic liquid with and without a magnetic field.

A point of fundamental interest that has not been addressed yet is the behavior of the two fluids at the very initial instant when contact is established. From a macroscopic point of view, geometry forces the free surface to possess a cusp, which is incompatible with the standard requirement of classical fluid mechanics that the jump in the normal stresses be compensated by surface tension times the local curvature. This boundary condition is a consequence of the assumption of local thermodynamic equilibrium, and therefore the presence of a cusp would indicate the prevalence of nonequilibrium conditions. Such a situation, therefore, must relax over molecular time scales to an equilibrium one with the sharp edge of the cusp softened into a region of very high local curvature. The subsequent behavior of the free surface has been studied in Oğuz & Prosperetti (1989) with the striking conclusion that further closing of the surface separating the two contacting liquid masses occurs in jumps that incorporate tiny air bubbles. Striking photos of small bubbles (diameter of the order of a few tens of μm) along the surface of the entering drop have been published by Sigler & Mesler (1990) at low impact velocities. It is difficult to say whether these are the bubbles predicted in Oğuz & Prosperetti, the effect of the breakup of a lubrication-type air film that remains entrapped between the two approaching liquid surfaces, or a combination of the two.

CONCLUSIONS

"I have some hope that ... I may have succeeded in producing in the mind of my reader some sympathy with the state of perplexity of Mr. Cole and myself," says Worthington. While nearly a century of progress has helped to dissipate some of that "perplexity," it is clear that we are still a long way from an understanding of drop impact phenomena in all their subtleties. The range of "very difficult hydro-dynamical questions involved" encompasses some fundamental problems, such as the role of surface tension when two liquid surfaces come together. A host of other absolutely nontrivial fluid mechanical phenomena has been mentioned in the previous pages.

While "One can almost regret that so beautiful a process should have been so long unwatched," in addition to aesthetic, scientific reasons for its investigation are readily found. A case in point is rain noise, the explanation of which has been the most notable success of recent progress in this area. In addition, bubbles in the upper layers of water bodies exert a strong influence on the exchange of atmospheric gases, with implications that range from the trophism of planktonic organisms at the beginning of the food chain to the absorption of CO_2 by the ocean. The tiny drops produced by a variety of mechanisms by the impact of drops and splashes are responsible for seeding the atmosphere with the salt grains that act as condensation nuclei for rain (Blanchard & Woodcock 1957). In plant physiology one talks of the splash-cup dispersal mechanism (Brodie 1951), according to which the tiny drops produced by raindrops hitting the splash cup are responsible for the dispersal of spores and gemmae of certain plants.

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