2.0, Appendix A - Notes on Performance of Block Codes

Definition 1 *The* **binary symmetric channel** *with crossover probability* p *(BSC(p)) is a* **memoryless** *binary channel for which*

$$\begin{aligned} \Pr(0|1) &= \Pr(1|0) = p \\ \Pr(0|0) &= \Pr(1|1) = 1 - p \end{aligned}$$

□

Consider a *block code* with

- word length n
- minimum distance $d_{min} = 2t + 1$

$$\begin{aligned} P_{CD} &\triangleq \Pr[correct\ decoding] \\ &= \Pr(|e| \leq t) \end{aligned}$$

So, if $\mathbf{y} = \mathbf{c} + \mathbf{e}$, then

$$\Pr[w_H(\mathbf{e}) = j] = \binom{n}{j} p^j (1-p)^{n-j},$$

and

$$\begin{aligned} P_{CD} &= \Pr(|\mathbf{e}| \leq t) \\ &= \sum_{j=0}^{t} \binom{n}{j} p^j (1-p)^{n-j} \end{aligned}$$

which is plotted below.

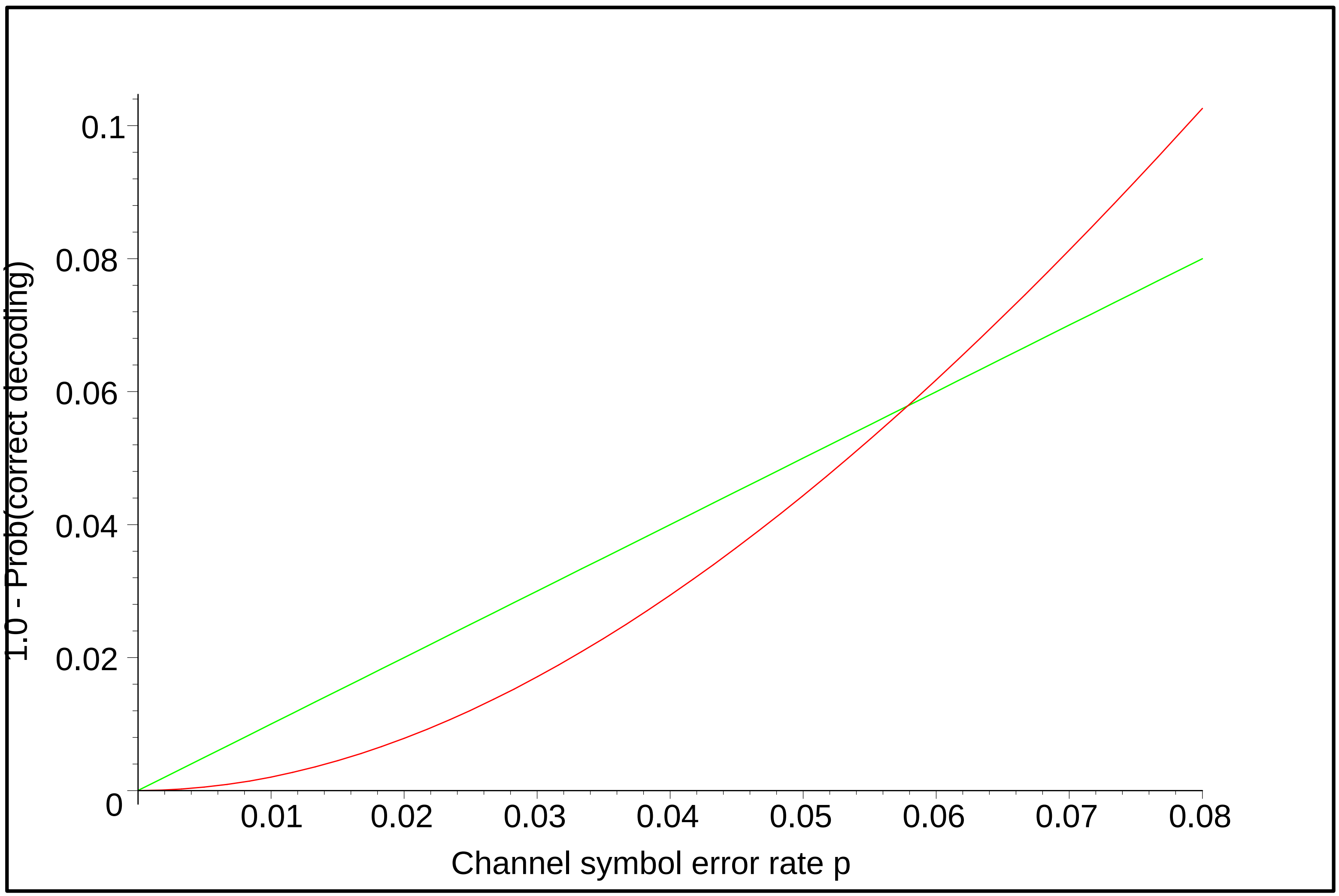

Figure 1: Upper bound to $\Pr[e]$ for the BSC.

Three possible events:

- correct decoding (CD)
- decoding error (DE)
- decoding failure (DF)

For this code, we will see that $\mathrm{Pr}_{DF} = 0$, *i.e.*, that the decoder always outputs a code word.

So, preceding plot shows

- an upper bound to the decoding error as a function of the BSC symbol error probability
- the line $P_{CD} = p$, for comparison.