Dispersion and loss limitations on the performance of optical delay lines based on coupled resonant structures

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The relative importance of group-velocity dispersion and loss-induced distortion in limiting the performance of optical delay lines based on coupled resonator structures is investigated. It is shown that for the current state of the quality of fabrication both factors play roughly comparable roles for bit rates of 2.5–40 Gbits/s and that as the storage capacity grows, the relative weight of loss-imposed limitation increases. © 2006 Optical Society of America

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Recent years have seen rapid progress in understanding and development of linear and nonlinear optical devices characterized by light propagation with low group velocity.[1] These so-called slow light (SL) devices offer advantages of compactness and tunability in a wide range of applications, for instance, in optical communication as optical buffers, in microwave photonics, as variable delay lines, and in laser radar and gyroscopes.[1] Assorted SL media, ranging from low-pressure metal vapors[2] to semiconductor,[3] Raman,[4] or Brillouin[5] amplifiers to the various coupled resonator structures[6,7] (CRSs), are all characterized by the presence of sharp resonance associated with the atomic transition, pump light, or a photonic structure. These sharp resonances predictably result in strong dispersion of absorption (or gain if the medium is amplifying) and group velocity. For this reason the operating bandwidth of SL devices is inherently limited,[8–11] and when it comes to wideband operation the CRS-based SL delay lines are expected to outperform the schemes based on atomic resonances, from the point of view of both dispersion[8–10] and power dissipation.[12,13] The critical distinction between CRS and the atomic schemes can be traced to one simple fact. In the atomic medium the loss/gain ratio is fundamentally closely coupled to the group-velocity dispersion (GVD), usually via Kramers–Kronig relations); thus large slow-down factors are unthinkable without a corresponding increase in loss/gain. In the CRS, on the other hand, the nature of loss is anything but fundamental; being typically of a residual absorptive or scattering nature, it can be manipulated separately from the GVD. Therefore, hopefully, progress in fabrication of low-loss CRS will eventually lead to high-performance delay lines.

The presence of loss in a CRS affects its performance via three mechanisms. First, if the loss per individual ring approaches the coupling between the resonators κ, the dispersion characteristics of the CRS become affected. Below I shall show that as it turns out this mechanism is not important for delay lines with reasonably large storage capacity requiring large κ's. Second, the absolute value of loss causes deterioration of the signal. This loss can be effectively compensated by the embedded gain, and, at least as long as the signal-to-noise ratio is not depleted by amplified spontaneous emission,[13] the delay capacity is then limited only by the higher-order GVD. The third and more intractable effect is the frequency dependence (dispersion) of loss that results in suppression of higher-frequency components of the signal, corresponding to broadening in the time domain and inter-symbol interference. Depending on whether the GVD or loss dispersion or their combination is the limiting factor, the bandwidth dependence of the SL delay line performance has different characters. Therefore, to develop efficient CRS delay lines, it is important to establish where (as a function of the bandwidth) the boundary lies between the loss-limited and GVD-limited regions. This is precisely what the present work attempts and achieves.

As has been shown in numerous works, various CRSs, based on, say photonic crystal cavities, ring resonators, or Moire Bragg gratings,[14] all have essentially the same dispersion characteristics—hence we can use an example of the more mature technology, ring resonators that in addition have the advantage of the relative ease with which the coupling coefficient can be manipulated. The results will hold for all CRSs. Consider the CRS in Fig. 1(a), consisting of N_r coupled rings of radius r and effective index n. According to Ref. 15 the dispersive relationship between the phase and frequency, shown in Fig. 1(b), is

$$\sin(\omega\tau) = \kappa\cos(\Phi/N_r), \tag{1}$$

where $\tau = \pi rn/c$ is the one-way delay in each ring. The group delay, also shown in Fig. 1(b), can then be estimated as

$$T_d(\omega) = \frac{\partial\Phi}{\partial\omega} = N_r\frac{\tau}{\kappa} + \frac{1}{2}N_r\left(\frac{\tau}{\kappa}\right)^3(1-\kappa^2)(\omega-\omega_0)^2 + \cdots , \tag{2}$$

where $\omega_0 = \mathrm{m}\pi\tau$ is the resonance frequency.

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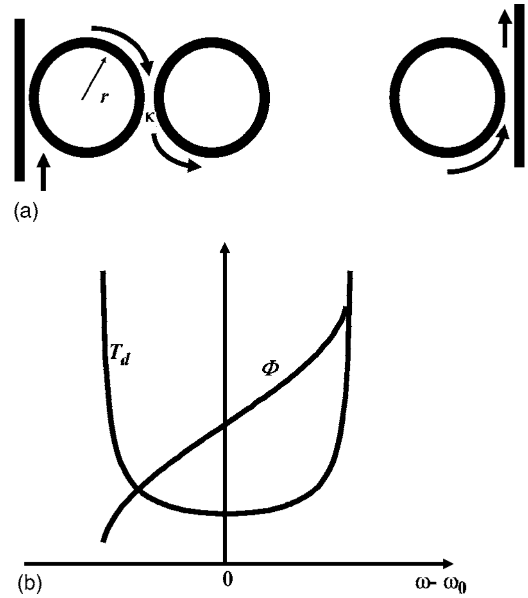

Fig. 1. (a), Implementation of CRS delay line with ring resonators. (b), Dispersion and group delay (in arbitrary units) in the CRS of (a).

Consider now a return-to-zero signal of bit rate B consisting of the Gaussian pulses with FWHM equal to one half of the bit interval $\Delta T_{\text{sig}} = B^{-1}$, $P(t) = \exp(-16\ln(2)B^2t^2)$, whose spectrum, centered on ω_0, is also Gaussian with FWHM $\Delta\omega_{\text{sig}} = 8\ln(2)B$. The delay at the central frequency should be adequate to store N bits, i.e.,

$$N_r(\tau/\kappa)B = N, \tag{3}$$

and the maximum differential time delay within the FWHM should not exceed roughly one quarter of the bit interval,

$$\Delta T_d(\omega_0 \pm \omega_{1/2}/2) = 8\ln^2(2)N_r(\tau/\kappa)^3(1-\kappa^2)B^2 \leq B^{-1}/4. \tag{4}$$

At the same time, the spectrum of the signal will be distorted, since it takes different times for different spectral components to propagate; i.e., the loss dispersion completely mimics the delay dispersion of Fig. 1(b),

$$P_{\text{out}}(\omega) = P(\omega)\exp\left[-\alpha\frac{c}{n}T_d(\omega)\right] \sim \exp\left[-\frac{4\ln 2(\omega-\omega_0)^2}{(\Delta\omega_{\text{sig}})^2} - \frac{\alpha c}{2n}N_r\left(\frac{\tau}{\kappa}\right)^3(1-\kappa^2)(\omega-\omega_0)^2\right]. \tag{5}$$

The temporal FWHM of the output pulse then becomes

$$\Delta T_{\text{out}}^2 = \Delta T_{\text{sig}}^2\left[1 + \frac{\Delta\omega_{\text{sig}}^2}{4\ln 2}\frac{\alpha c}{2n}N_r\left(\frac{\tau}{\kappa}\right)^3(1-\kappa^2)\right], \tag{6}$$

and, limiting the temporal broadening to a factor of $2^{1/2}$, we obtain

$$16\ln(2)\frac{\alpha c}{2n}N_r\left(\frac{\tau}{\kappa}\right)^3(1-\kappa^2)B^2 \leq 1. \tag{7}$$

The two independent GVD and loss distortion limitations (4) and (7) can be combined into one general condition as

$$32\ln^2(2)N_r\left(\frac{\tau}{\kappa}\right)^3(1-\kappa^2)B^3\sqrt{1+\frac{B_{\text{loss}}^2}{B^2}} \leq 1, \tag{8}$$

where we have introduced $B_{\text{loss}} = \alpha c/4n\ln 2$, the bit rate at which the signal will experience 3 dB loss per one quarter of the bit interval. Clearly, at bit rates less than B_{loss} the loss distortion dominates, while at higher rates GVD becomes the overriding factor. Combining relations (6) and (8), we obtain the key equation determining the number of resonators N_r necessary to store N bits without distortion,

$$N_r(B,N) = 4\sqrt{2}\ln 2N^{3/2}\frac{\sqrt[4]{1+B_{\text{loss}}^2/B^2}}{\sqrt{1+B^2/B_{\max}^2}}, \tag{9}$$

where we have introduced $B_{\max}^{(N)} = 1/(4\sqrt{2}\ln 2\tau N^{1/2})$, which can be understood as the maximum bit rate at which the CRS offers any advantage over a straight waveguide of length c/nB^{-1}. The dependence, Eq. (9), is plotted in Fig. 2 for two different values of N by using the example of coupled ring resonators from Ref. 7 with a ring radius of 6 μm ($\tau \sim 0.22$ ps) and a 0.05 dB combined (scattering and bending) round-trip loss per ring ($\alpha_R = 2\alpha c\tau/n \sim 0.012$) that yields $B_{\text{loss}} \sim 9$ Gbits/s. This value indicates that with present day technology both GVD and loss distortion play roughly equal roles for 10 Gbits/s signals, while at 40 Gbits/s and higher rates GVD is expected to dominate. Clearly the plot can be split into three distinct regions,

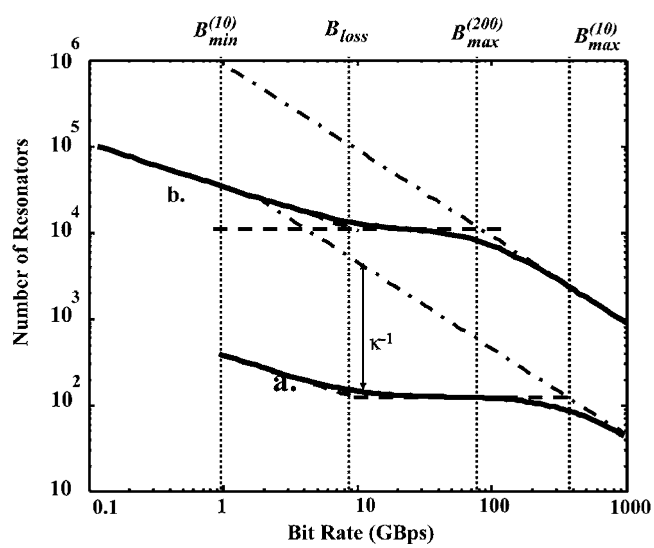

Fig. 2. Required number of resonators in the CRS delay line versus bit rate. a, Storage capacity $N = 10$ bits; b, Storage capacity $N = 200$ bits.

$$N_r \approx \begin{cases} 2(\alpha c/nB)^{1/2}N^{3/2} & B \ll B_{\text{loss}} \\ N_r \approx 4N^{3/2} & B_{\text{loss}} \ll B \ll B_{\max}^{(N)} \\ N/B\tau & B \gg B_{\max}^{(N)} \end{cases} . \quad (10)$$

In the first region the loss-induced distortion dominates, and the required number of resonators is inversely proportional to the square root of the bit rate. In the second region the required number of resonators is determined by dispersion and remarkably does not depend on the bit rate, as first shown in Ref. 8. Finally, at high bit rates the coupling coefficient κ (note that the slow-down factor κ^{-1} in Fig. 2 is represented by the distance between the solid curve and the dashed–dotted curve $N/B\tau$) becomes so close to unity that the CRS delay line becomes essentially equivalent to a straight waveguide. It should be noted that the condition for the existence of the second, GVD-limited, region is $B_{\text{loss}} \ll B_{\max}$, or $\alpha_R \ll (2/N)^{1/2}$; hence for the present structure the region should exist for the delay lines with a few hundred bits of storage capacity. As one can see, the flat region is clearly present in Fig. 2(a) for $N=10$ bits but is far less prominent in Fig. 2(b) for $N=200$ bits.

It is now important to return to the issue of the influence of ring loss on the dispersion characteristics by imposing the condition that round-trip loss α_R should be much smaller than coupling coefficient κ. Solving relations (4) and (8) for $B \ll B_{\text{loss}}$, we obtain

$$\kappa = 2\sqrt{\ln 2N\alpha_R B\tau} = \alpha_R\sqrt{B/B_{\min}}, \quad (11)$$

where $B_{\min} = \alpha c/4nN \ln 2 = B_{\text{loss}}/N$; thus for reasonably large storage capacity the coupling coefficient determined from the loss-induced distortion considerations is always substantially larger than the loss per ring, and one can neglect the influence of loss on the dispersion itself. It should be noted that as α_R approaches κ the group-delay curve in Fig. 1(b) may actually flatten and even change the sign of curvature, but with more than 4 dB loss its dispersion will negate the improvement.

If we now turn our attention to the magnitude of the slow-down factor κ^{-1} it is easy to observe from Fig. 2 that for bit rates higher than 10 Gbits/s the slow-down factor does not exceed 30 for $N=10$ stored bits and is actually less than 10 for $N=200$ stored bits. Unless the CRS delay line is made tunable, this small slow-down factor might actually call into question the worthiness of fabricating this elaborate structure. According to Eq. (11) the only way to improve the performance is to reduce the size of the resonator (τ) without incurring additional loss. With ring resonators further reduction in radius may result in excessive bending loss, but with a photonic crystal CRS[16] one can realistically expect reduction of τ to less than 50 fs, and thus slow-down factors as high as 100 can be attained.

Having seen the importance of loss-induced distortion, one can conclude that the best way to compensate for it would be by embedding the gain medium in the rings themselves rather than using lumped amplifiers that only compensate for the insertion loss but not the loss-induced distortion. This can be best achieved, in my view, by using Raman amplification,[17] provided the ASE noise does not cause signal deterioration as analyzed in Ref. 13.

In summary, I have considered the relative importance of group-velocity dispersion and loss-induced distortion on the performance of a coupled resonator structure slow-wave devices by using the most up to date example of high-quality coupled ring resonators. My conclusion is that for the bit rates of interest (2.5–40 Gbits/s) both factors play important roles, with the relative weight of loss-induced dispersion increasing with the increase in storage capacity.

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