Expanding the bandwidth of slow-light photonic devices based on coupled resonators

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It is shown theoretically that canceling third-order dispersion can substantially increase the useful bandwidths of linear and nonlinear optical devices based on slow propagation of light. Cancellation on both global and local scales can be achieved by combination of the ring-based coupled resonator lines and all-pass optical filters. © 2005 Optical Society of America
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Recent years have seen remarkable gains in the fabrication of high-quality resonant photonic structures such as fiber gratings,[1,2] microring and microdisk cavities,[3–6] and photonic crystal structures.[7–9] These advances open the possibility of fabricating photonic circuits that incorporate large numbers of these structures for a variety of functions. One of the most exciting opportunities exists in developing guides to incorporate coupled resonator structures (CRSs) of various types whose salient feature is slow group velocity v_g of propagating light. Obvious uses of slow-light structures are not only in optical delay lines[10] but also in all kinds of nonlinear optical devices, as the reduction of group velocity by a factor of $n_g = c/v_g$, increases the effective nonlinearity by a factor of $n_g{}^2$. Regrettably, a feature that is inherent in all the slow-light devices, namely, large group-velocity dispersion, severely limits their effective bandwidth.[5,11] When a CRS is operated at resonance, only the odd orders of the dispersion are present, and third-order dispersion $\beta_3 = \partial^3\beta/\partial\omega^3$ is the largest order. As a result, for a given length L, bandwidth B is limited roughly as $B < B_{\max,3} \sim \gamma_3(\beta_3 L)^{-1/3}$,[12] where γ_3 is a parameter of the order of 1/3 that depends on an exact modulation format (for Gaussian pulses it is 0.324). As a result the maximum delay time for high bandwidth (or a high bit rate for digital signals) becomes small, which is disadvantageous for linear as well as nonlinear applications. One way to flatten the group-delay dispersion characteristics would be to take the path followed in microwave technology and use resonators with slightly detuned resonant frequencies. Such a method, however, requires precise control of minute size differences, is difficult to implement, and is still plagued by a residual ripple in the dispersion curve. Here a simple method for cancellation of the third-order dispersion by use of two types of CRS with identical resonators is proposed. Although here we use the example of ring resonators, the proposed structures can also be implemented by use of other types of resonator, particularly those in photonic crystals.[7]

It is well known that there are two ways for microring resonator structures to be coupled to each other—either directly [Fig. 1(a)] or by use of a common bus [Fig. 1(b)]. The first structure, originally proposed in Ref. 3, is usually referred to as a coupled resonator waveguide (CRW), and its distinct feature is the presence of both forward- and backward-propagating waves manifested by the appearance of high reflectance regions—photonic bandgaps on both sides of the resonance. The dispersion of a CRW, shown in Fig. 1(c), is determined by the equation $\sin \omega\tau_1 = \kappa_1 \cos \beta_1 d_1$, where τ_1 is the round-trip delay in the ring, κ_1 is the interring coupling coefficient, d is the spatial period of the structure, and β is the propagation constant. The resonance takes place at $\omega_0 = m_1\pi\tau_1{}^{-1}$, and the width of the passband between two bandgaps is $\Delta\omega = 2\tau_1{}^{-1} \sin^{-1} \kappa_1$. Expanding dispersion into a Taylor series, we can find the accumulated phase delay after N_1 resonators:

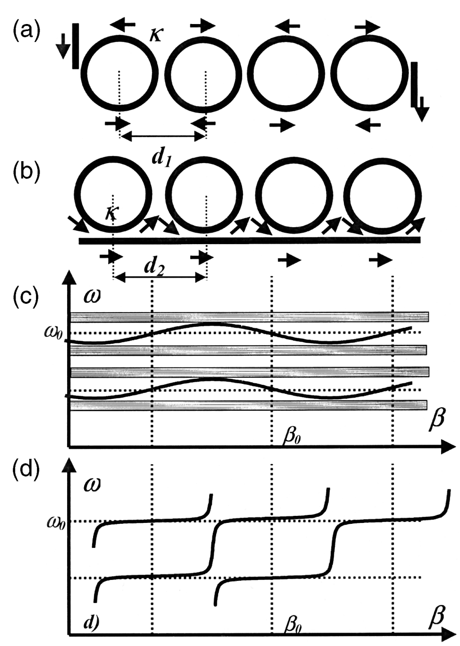

Fig. 1. Two different slow-wave structures and their dispersion: (a), (c) CRW; (b), (d) SCISSOR.

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$$\Phi_1(\omega) = N_1\beta d = (2m-1)N_1\frac{\pi}{2} + N_1\frac{\tau_1}{\kappa_1}(\omega-\omega_0) + \frac{N_1}{6}\frac{\tau_1^3}{\kappa_1^3}(1-\kappa_1^2)(\omega-\omega_0)^3 + \frac{N_1}{120}\frac{\tau_1^5}{\kappa_1^5}(1-\kappa_1^2)(9-\kappa_1^2)(\omega-\omega_0)^5 + \ldots. \quad (1)$$

The dispersion characteristic of a CRW is thus superlinear, with $\beta_3 > 0$.

The bus-coupled CRS of Fig. 1(b), usually referred to as a side-coupled integrated spaced sequence of resonators[5,6] (SCISSOR), is also a periodic structure, but it supports only a forward-propagating wave; thus it can also be construed as an array of all-pass filters (APFs).[10] The SCISSOR dispersion curve [Fig. 1(d)] is periodic, but it does not exhibit a bandgap:

$$\frac{\kappa_2^2 \sin \omega\tau_2}{(1+\rho_2^2)\cos \omega\tau_2 - 2\rho_2} = \tan(\beta_2 d_2 - \varphi_0). \quad (2)$$

Here τ_2 is the round-trip delay in one ring, κ_2 is the coupling coefficient, $\rho_2^2 = 1-\kappa_2^2$, and $\varphi_0 \sim \omega c^{-1} n d_2$ is the additional phase delay in the bus. Expanding Eq. (2) about $\omega_0 = 2m_2\pi\tau_2^{-1}$, we obtain the expression for the phase delay of SCISSOR with N_2 rings:

$$\Phi_2(\omega) = N_2(\pi+\varphi_0) + N_2\tau_2\frac{1+\rho_2}{1-\rho_2}(\omega-\omega_0) - \frac{1}{3}N_2\tau_2^3\frac{(1+\rho_2)\rho_2}{(1-\rho_2)^3}(\omega-\omega_0)^3 + \frac{1}{60}N_2\tau_2^5\frac{(1+\rho_2)\rho_2(1+10\rho_2+\rho_2^2)}{(1-\rho_2)^5}(\omega-\omega_0)^5. \quad (3)$$

The dispersion characteristic of a SCISSOR is thus sublinear, with $\beta_3 < 0$. Therefore, one can rather easily cancel the third-order dispersion effects by combining a CRW and a SCISSOR sequentially [Fig. 2(a)] into a tandem CRS (TCRS). Because the fifth-order terms in the CRW and the SCISSOR are positive, they would remain uncompensated for and would thus define the operational bandwidth of the TCRS.

Let us now make an estimate of the TCRS's performance. To facilitate fabrication, the ring sizes in CRW and SCISSOR sections should be equal; thus $\tau_2 = 2\tau_1 = 2\tau$, leading to common resonance frequency $\omega_0 = m\pi\tau^{-1}$ for the two sections. Furthermore, to achieve a large reduction of group velocity we assume that coupling coefficients are small, leading to the compensation condition $2\kappa/(1-\rho_2) = 2^{-2/3}(N_1/N_2)^{1/3}$ and to the expression for the total group delay:

$$T_d(\omega_0) = \left.\frac{\partial\Phi}{\partial\omega}\right|_{\omega_0} = \frac{\tau}{\kappa_1}\left(N_1 + 2N_2\frac{2\kappa_1}{1+\rho_2}\right) = \frac{\tau}{\kappa_1}[N_1 + 2^{1/3}(N-N_1)^{2/3}N_1^{1/3}], \quad (4)$$

where N is the total number of rings in the TCRS. Maximizing T_d yields $N_1 \approx 3N_2$ and $2\kappa_1/(1-\rho_2) \sim 0.9$, an indication that the group velocities in the two sections should be roughly equal.

In Fig. 2(b) we show the group delays (in units of $N\tau$) as a function of frequency (in units of τ^{-1}) for the CRW, the SCISSOR, and the TCRS for the example of $\kappa_1 = 0.05$ and $\rho_2 = 0.89$. One can see that the flat region of constant group delay is greatly extended in the TCRS. The maximum operating bandwidth $B_{\max}$ of a given CRS can be determined from simple considerations. For a given bandwidth B the differential group delay per resonator is $T_{\text{diff}}'(B) = N^{-1}|T_d(\omega_0+\pi B) - T_d(\omega_0)|$. This delay, plotted in Fig. 2(c) for the same three CRSs, must not exceed $1/N$th of the half-period of the highest frequency in the spectrum, i.e., $T_{\max}^{-1}(B) = N^{-1}B^{-1}$, also plotted in Fig. 2(c) for $N = 1000$ as a straight line (on a logarithmic scale). Intersection with this line defines $B_{\max}$ of a given CRS. According to Fig. 2(c), in the TCRS the maximum operating bandwidth is increased by roughly a factor of 2.6.

For digital signals one can obtain analytical expressions for the maximum bit rate by assuming return-to-zero Gaussian signaling[12] and only the lowest-order dispersions (i.e., β_3 for the CRW or the SCISSORS and the order of β_5 for the TCRS):

$$BR_{\max,\text{CRW}} \sim 0.32N^{-1/3}(\kappa/\tau), \quad BR_{\max,\text{TCRS}} \sim 0.36N^{-1/5}(\kappa/\tau). \quad (5)$$

These results are actually very close to those obtained from Fig. 3(c), indicating that the bandwidth (or the bit-rate) enhancement attained by compensating for third-order dispersion is roughly $N^{2/15}$; thus this enhancement is best manifested in the CRSs with large numbers of resonators. (Of course, depending on the application, rather than expanding the bandwidth one

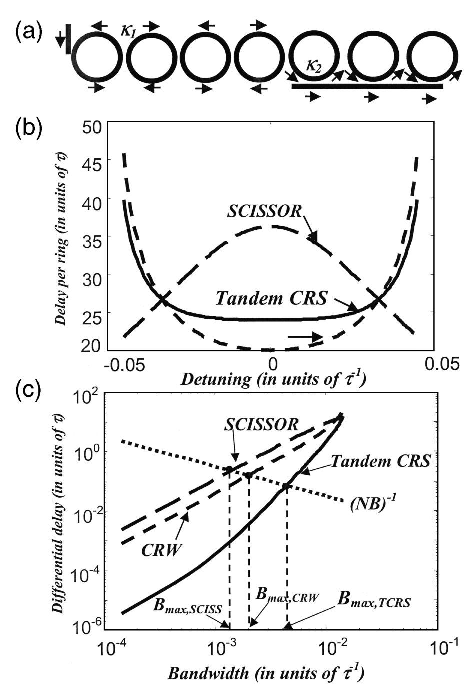

Fig. 2. (a) TCRS with global dispersion compensation. (b) Group-delay dispersion for different CRSs. (c) Different group delay as a function of bandwidth for different CRSs.

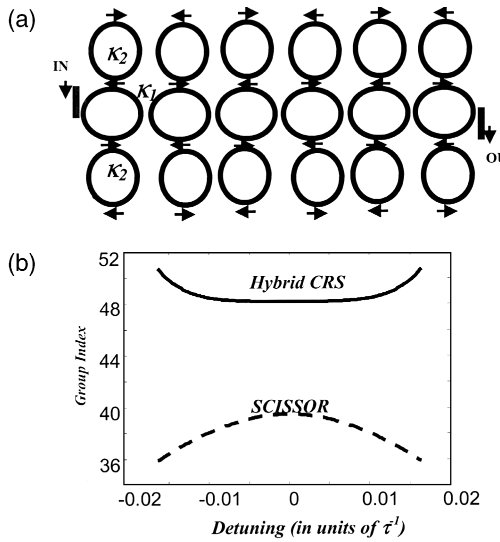

Fig. 3. (a) Hybrid CRS (HCRS) with local dispersion compensation. (b) Group-delay dispersion for SCISSOR and HCRS.

can choose to keep it unchanged and use the compensated CRS to increase total delay time.)

The dispersion compensation in the TCRS is attained on a global scale, i.e, after the light passes through both sections. Such sequential compensation is adequate for the linear devices (delay lines), but for the nonlinear (including electro-optic) devices the compensation must be local, which calls for the HCRS shown in Fig. 3(a), which combines CRW and APF characteristics by augmenting each CRW element with a pair of APFs. The axial rings are coupled both to each other with coupling coefficient κ_1 and to the side rings with coupling coefficient κ_2. The side rings are coupled only to the axial rings. The dispersion equation for the HCRS is

$$\beta = \beta_0 + \frac{1}{d}\sin^{-1}\left\{{\kappa_1}^{-1}\sin\left[\omega\tau_1 + \pi + \tan^{-1}\frac{{\kappa_2}^2\sin\omega\tau_2}{(1+{\rho_2}^2)\cos\omega\tau_2 - 2\rho_2}\right]\right\}. \tag{6}$$

As in the case of TCRS, we assume that the rings have equal circumference. Expanding Eq. (6) near the resonant frequency $\omega_0 = m_1\pi\tau^{-1}$ (where $\tau = \tau_2 = 2\tau_1$), we obtain

$$\beta = \beta_0 + \frac{\tau}{d\kappa_1}\left(\frac{1}{2} + \frac{1+\rho_2}{1-\rho_2}\right)(\omega - \omega_0) + \frac{\tau^3}{6d{\kappa_1}^3}\left[(1-{\kappa_1}^2)\left(\frac{1}{2} + \frac{1+\rho_2}{1-\rho_2}\right)^3 - 2{\kappa_1}^2\rho_2\frac{1+\rho_2}{(1-\rho_2)^3}\right](\omega-\omega_0)^3 + \ldots. \tag{7}$$

Thus the third-order dispersion is canceled when

$${\kappa_1}^2 = \left(\frac{1+\rho_2}{1-\rho_2} + \frac{1}{2}\right)^3 \Bigg/ \left[\left(\frac{1+\rho_2}{1-\rho_2} + \frac{1}{2}\right)^3 + 2\rho_2\frac{\rho+1}{(1-\rho)^3}\right]. \tag{8}$$

For weak coupling between the central line and the side rings ($\rho_2 \sim 1$), we obtain $\kappa_1 \approx \sqrt{2/(2+\rho_2)} \approx \sqrt{2/3}$, a rather strong coupling that indicates that the slow-wave effect is due to the APF, and the CRW coupling between the axial rings only serves to compensate for the dispersion. The dispersion curves of the group index for the SCISSOR and the HCRS of the same ring size and $\rho_2 = 0.95$ are shown in Fig. 3(b). As one can see, the HCRS offers both a higher group index and a wider effective bandwidth.

In conclusion, I have shown that the bandwidths of the slow-wave photonic structures can be significantly enhanced by use of global or local dispersion compensation achieved by combining two different kinds of CRS with identical resonators. One can also use this technique to further decrease the group velocity of light for a given bandwidth to facilitate development of compact and efficient linear and nonlinear optical devices. These results show that if and when the fabrication issues of slow-wave structures are resolved the performance of the structures can be enhanced beyond standard dispersion limitations.

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