Cascaded Coupler Mach–Zehnder Channel Dropping Filters for Wavelength-Division-Multiplexed Optical Systems

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***Abstract*— We describe the design and characteristics of cascaded coupler Mach–Zehnder (CMZ) channel adding/dropping filters that select every *n*th channel ($n = 2, 3, 4, \cdots$) from the wavelength-division-multiplexed (WDM) optical signal. Using the truncated binomial coupling weight distribution, which appears to be optimal, such filters achieve low (< −30 dB) sidelobe levels and good finesse with just a modest number of stages. We also show that the number of filter stages grows only linearly with the required filter finesse and thus the number of selectable channels. Such WDM filters can be conveniently implemented using the silicon or InP integrated optical technology.**

I. Introduction

Wavelength-division-multiplexed (WDM) optical systems [1] require filters that select individual wavelength channels from the WDM signal stream [2]. Full multiplexers/demultiplexers [3], [4] direct all individual channels of the WDM signal into physically separate outputs. In contrast, channel adding/dropping filters [5]–[10] select certain (one or more) wavelength channels in one physical output, while leaving the other channels undisturbed in the second output. Optical channel dropping filters have been implemented using a number of different approaches: Mach–Zehnder interferometer with an output directional coupler [5]; grating-resonator-coupled waveguide filter [6]; grating-assisted co-directional coupler filter [7], [8]; or the meander coupler [9], [10]. In this paper, we describe the design and characteristics of cascaded coupler Mach–Zehnder (CMZ) filters [11] with weighted coupling that function as channel dropping filters with very low sidelobe levels.

II. Coupler Mach–Zehnder Filters

A Mach–Zehnder interferometer with input and output directional couplers [5] acts as a channel dropping filter, as illustrated in Fig. 1(a). For 3 dB input and output couplers, the power transmission as a function of frequency is sinusoidal

$$T_1 = P_1/P_0 = \cos^2(\Delta\phi/2). \quad (1)$$

Here

$$\Delta\phi = \beta(L_a - L_b) \quad (2)$$

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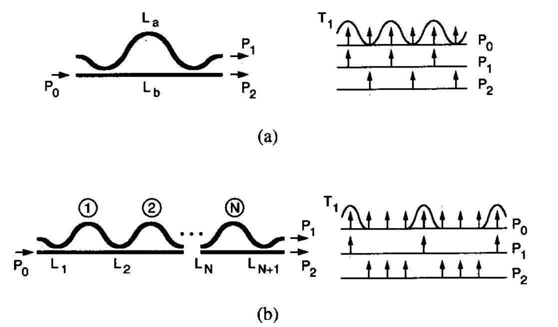

Fig. 1. (a) Single-stage and (b) cascaded Mach–Zehnder (CMZ) filter with channel dropping schematics.

where β is the propagation constant of the waveguides and $L_{a,b}$ are the lengths of the two arms. The filter response is periodic in frequency with the period, or free spectral range, of

$$\Delta f_p = \frac{c}{n_g(L_a - L_b)} \quad (3)$$

where c is the speed of light and n_g is the waveguide group index. Because of power conservation, the two outputs P_1 and P_2 are complementary

$$T_2 = P_2/P_0 = 1 - T_1. \quad (4)$$

Thus, a WDM input stream P_0 with channel separation $\Delta f_p/2$ will be divided between two outputs, with alternate channels going to the outputs P_1 and P_2.

Such an optical filter can be implemented either with discrete optical fibers/couplers or by an integrated optical device [3], [4]. Using the InP integrated optical technology [4], we estimate that a Mach–Zehnder filter can be made with a period $\Delta f_p = 400$ GHz (3.2 nm at $\lambda = 1.55$ μm), a device length of 4.5 mm, and low-loss waveguide bends of 2 mm radius of curvature.

III. Cascaded Coupler Mach–Zehnder Filters

A higher finesse filter is required for selecting every nth channel ($n = 3, 4, \cdots$) out of the WDM signal. This can be accomplished with a cascaded coupler Mach–Zehnder (CMZ) filter [11], as illustrated in Fig. 1(b). Here, the individual Mach–Zehnder stages are identical (the same optical path

length difference $\Delta\phi$ between two branches). However, the lengths (coupling strengths) of the directional couplers between stages need not be identical. The cascaded filter operates on a sim
wavelengths such that the Mach–Zehnder optical path length difference $\Delta\phi$ is a multiple of 2π, the device acts as one long coupler with the total length of

$$L_{tot} = \sum L_i. \quad (5)$$

Here, L_i are the lengths of the individual couplers ($i = 1, 2, \cdots, N+1$ for the N-stage filter), and we have assumed that the coupling coefficient κ is the same in the different sections. Thus, the total length L_{tot} should equal one coupling length ($\kappa L_{tot} = \pi/2$) for a full power transfer to port P_1. Off resonance power transfer to port P_1 drops. Because of power conservation, the two outputs P_1 and P_2 are again complementary. Such a cascaded filter is a direct analog of the grating-assisted co-directional coupler filter [7], [8] or the meander coupler [9], [10]. In those cases, the optical path length difference between the two interferometer arms is achieved by the waveguide propagation constant difference, whereas in our case it is given by the geometrical path length difference.

One can describe the CMZ filter operation using transmission matrices for the Mach–Zehnder and the coupler sections. If the normalized fields in the top and bottom waveguides are given by a_1 and a_2, respectively, the ith coupler is characterized by its transmission matrix [12] $T_c(L_i)$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\text{out}} = T_c(L_i)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\text{in}} = \begin{bmatrix} \cos(\kappa L_i) & -j\sin(\kappa L_i) \\ -j\sin(\kappa L_i) & \cos(\kappa L_i) \end{bmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\text{in}}. \quad (6)$$

The transmission matrix for the Mach–Zehnder two arm section is

$$T_{MZ} = \begin{bmatrix} \exp(j\Delta\phi/2) & 0 \\ 0 & \exp(-j\Delta\phi/2) \end{bmatrix} \quad (7)$$

where we have dropped the common phase delay term. The transmission of the N-stage filter is then

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\text{out}} = T_c(L_{N+1}) \cdots T_{MZ}T_c(L_2)T_{MZ}T_c(L_1)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\text{in}}. \quad (8)$$

The power transmission is

$$T_1 = \frac{|a_1|^2_{\text{out}}}{|a_2|^2_{\text{in}}}. \quad (9)$$

IV. Weighted Coupler Distribution

For the N-stage CMZ filter, the total coupler length L_{tot} can be distributed in different ways over the $(N+1)$ individual couplers. This coupler length distribution controls the shape of the filter transmission characteristic [13], such as the filter passband width and the sidelobe level. A variety of *continuous* distributions has been considered in the context of grating-assisted filters [8], [10], [13]. In contrast, for the CMZ filter,

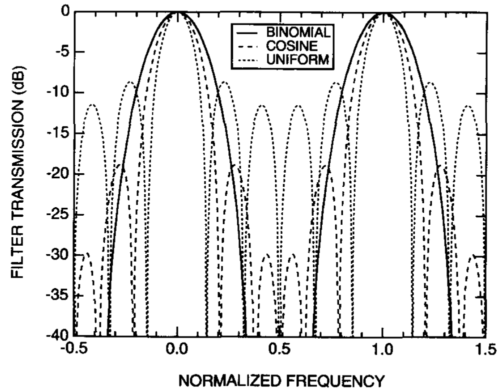

Fig. 2. $N = 5$ stage CMZ filter transmission for the binomial (solid line), cosine ($a = 0.8$) (dashed line), and uniform (dotted line) coupler weight distributions.

the distribution is *discrete* and the transformation that connects the weight distribution to the filter shape is different from that for the grating filters. Since an optimal discrete distribution is unknown, we have investigated several simple weight distributions to characterize the CMZ filters:

$$\text{Uniform: } w_i = 1 \qquad (i = 1, 2, \cdots, N+1) \quad (10a)$$

$$\text{Cosine: } w_i = \cos(\pi a((i-(N+2)/2)/N)) \quad (10b)$$

$$\text{Binomial: } w_i = B(N, i-1) = \frac{N!}{(i-1)!(N-i+1)!}. \quad (10c)$$

The individual coupler lengths are then

$$L_i = w_i L_{tot} \Big/ \sum w_i. \quad (11)$$

More generally, the coupler strengths of the individual sections, given by the products $\kappa_i L_i$, can follow the weight distributions by adjusting either the lengths L_i or the coupling coefficients κ_i of the individual sections.

Fig. 2 shows the $N = 5$ stage filter transmission as a function of normalized frequency for uniform, cosine, and binomial weight distributions. The transmission is periodic and the frequency normalization factor is the period Δf_p. Clearly, the uniform distribution gives unacceptably high sidelobe levels of -8 dB, the cosine distribution lowers them to -18 dB, while for the binomial distribution the sidelobes are very low at -47 dB. The lower sidelobes are achieved at the expense of the wider transmission peak [13]. Sidelobe levels below -30 dB would be required for WDM applications.

As we increase the number of stages, the width of the main peak decreases and, correspondingly, the filter finesse increases. For the binomial distribution, the sidelobe level remains below -45 dB and the transmission function T_1 appears to be close to

$$T_1 \approx \cos^{2N}(\Delta\phi/2) \quad (12)$$

for the N-stage filter. This is just the transmission function of the single-stage filter raised to the Nth power. Approximating the cosine in (12) by a parabola, we obtain a relation, accurate

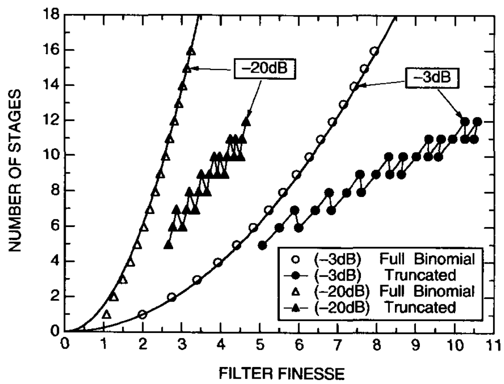

Fig. 3. The required number of CMZ filter stages as a function of the filter finesse (−3 dB and −20 dB levels). Open and filled symbols correspond, respectively, to the full and truncated binomial distributions of the coupler lengths. Solid lines are fits of (13a) to the data.

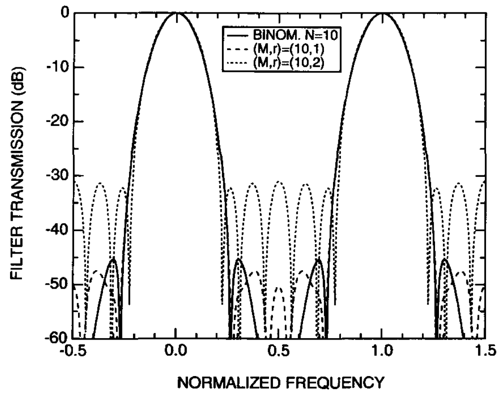

Fig. 4. Effect of stage truncation on the binomial CMZ filter transmission. Filter transmission for the binomial $N = 10$ stage (solid line) and the truncated binomial $(M, r) = (10, 1)$, $N = 8$ stage (dashed line) and $(M, r) = (10, 2)$, $N = 6$ stage (dotted line) coupler weight distributions.

for large N, between the filter finesse F and the number of stages N for the binomial distribution

$$N = B_b F^2 \tag{13a}$$

where

$$B_b = \frac{4}{\pi^2} \ln (1/T_f). \tag{13b}$$

Here, we define the finesse F as the ratio of the transmission period to the transmission peak width, where the peak width is measured at the T_f transmission level below the transmission maximum. The conventional finesse is a particular case for $T_f = 0.5 = -3$ dB. In Fig. 3, we plot with open symbols the number of stages N required to achieve a given filter finesse F for the binomial coupling weight distribution; the finesse is measured for both the $T_f = -3$ dB and the $T_f = -20$ dB level. The points in the plot were obtained from the calculated transmission functions, while the lines through the open symbols are fits to (13a) with $B_b = 0.25$ and 1.52 for $T_f = -3$ dB and −20 dB, respectively. Equation (13b) gives, correspondingly, the values $B_b = 0.28$ and 1.87, in good agreement with the fit values above.

V. Truncated Binomial Weight Distribution

For the full binomial distribution, the number of filter stages grows very rapidly (quadratically) as a function of the desired finesse [see Fig. 3 and equation (13a)]. For a practical, physically compact implementation of the filter, a smaller number of stages is desired. One way to reduce the number of stages is found by observing the binomial distribution, e.g., for $N = 10$: $\{1\,10\,45\,120\,210\,252\,210\,120\,45\,10\,1\}$. The first (and last) one or two weights are relatively small; these couplers contribute negligible power exchange between two waveguides and thus can be eliminated. Therefore, we introduce the *truncated binomial* distribution for the $N = (M - 2r)$ stage filter

$$w_i = B_t(M, r, i) \equiv B(M, i - 1 + r) \quad (i = 1, 2, \cdots, M - 2r + 1) \tag{14}$$

which is just the order M binomial distribution with the first and last r elements dropped.

Fig. 4 shows the transmission of an $N = 10$ stage filter with full binomial weights, together with transmissions of $(M, r) = (10, 1)$ eight-stage and $(M, r) = (10, 2)$ six-stage filters with truncated binomial distributions. Truncating stages leaves the filter width essentially unchanged, while the sidelobe level rises, reaching a remarkably uniform −30 dB level for the $(10, 2)$ distribution. Further truncation raises the sidelobe level above −30 dB. How many stages can be "truncated" depends on the sidelobe level allowed for a particular application. We have tried a number of other weight distributions known from the antenna array theory, such as Chebyshev distribution; however, none gave results as good as the truncated binomial distribution.

In Fig. 3, we plot with filled symbols the number of stages required to achieve a given filter finesse for the truncated binomial coupling weight distribution. For each data point, the binomial distribution was maximally truncated, keeping the sidelobe level below −30 dB. The first point corresponds to the $(M, r) = (7, 1)$ five-stage distribution, and the last one to the $(M, r) = (31, 10)$ eleven-stage distribution; note that the same number of stages $(M - 2r)$ can be obtained in several different ways. We have observed that the −30 dB sidelobe level is achieved when the largest-to-smallest coupler weight ratio is of the order 6–7, regardless of the number of stages. To achieve lower sidelobes, this ratio has to be higher. For the truncated binomial distribution and a fixed sidelobe level, the required number of stages appears to grow only linearly with finesse, as compared to the quadratic dependence for the full binomial distribution. This reduction in the number of stages, by as much as a factor of 3 for a filter with finesse of 12, is very important for reducing the physical size of the CMZ filter in the integrated optical implementation.

VI. The Required Number of Stages

A WDM optical communication system requires channel adding/dropping filters that can select one out of every N_{ch}

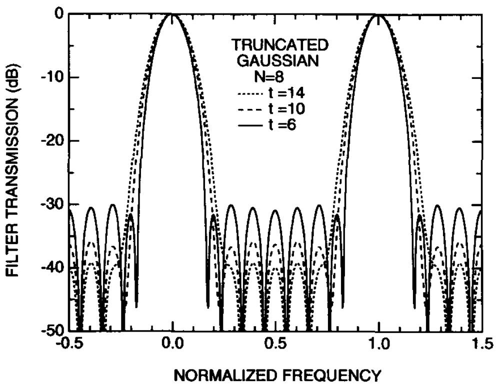

Fig. 5. Transmission of the fixed-length $N = 8$ stage CMZ filter with truncated Gaussian weight distribution; the center-to-wing ratios are $t = 6$ (solid line), $t = 10$ (dashed line), and $t = 14$ (dotted line).

channels. For the truncated binomial distribution, which appears to be optimal, we now determine the analytical relation between the number N of CMZ filter stages and the desired filter finesse F or the channel number N_{ch}.

If the filter is to reject undesired channels with a cross-talk level of T_f, say, -30 dB, then the number of selectable channels N_{ch} is given by twice the filter finesse F as defined at the T_f level

$$N_{ch} = 2 \cdot F. \tag{15}$$

Of course, the filter sidelobes also have to be kept below the T_f level.

It is well known that for large N, the *binomial* distribution (10c) can be approximated by the *Gaussian* distribution. Thus, we can approximate the *truncated binomial* by the *truncated Gaussian* distribution, both for convenient filter design and for analytically determining the number of stages dependence on the filter finesse. The binomial distribution (10c) with a mean $\mu = N/2$ and variance $v = N/4$ can be approximated by a Gaussian distribution

$$G(\mu, v) = \exp\left(-\frac{(i-1-\mu)^2}{2v}\right) \tag{16}$$

with $i = 1, 2, \cdots, (N+1)$. The truncated binomial distribution (14) can be approximated by the truncated Gaussian distribution

$$G_t(\mu, v_t) = \exp\left(-\frac{(i-1-\mu)^2}{2v_t}\right) \tag{17}$$

where $i = 1, 2, \cdots, (N+1)$, $\mu = N/2$, and $v_t = N^2/(8\ln(t))$. Here, the center-to-wing ratio t of the distribution should be of the order 6–7 in order to achieve the -30 dB filter sidelobe level, as we have observed empirically. Fig. 5 illustrates the transmission of the $N = 8$ stage filter with truncated Gaussian weight distribution with $t = 14$, 10, and 6. For this fixed-length filter, as the center-to-wing ratio t of the distribution decreases, the filter transmission narrows down at the expense of the rising sidelobe levels.

We now determine, for the truncated binomial and Gaussian distributions, the analytical relation between the number of stages N and the filter finesse F. Approximating the full binomial distribution (10c) by the Gaussian (16), we truncate the wings of the two distributions so that their center-to-wing ratio becomes t [(14) and (17)]. Starting with a full binomial with M stages, after truncation, we are left with approximately $N = \sqrt{2M\ln(t)}$ stages. Assuming that the filter shape remains unchanged (12) and only the sidelobe levels rise, the finesse is still given by $M = B_b F^2$ [see (13a)]. Thus, for the truncated binomial and Gaussian distributions, the number of stages N as a function of the finesse F_f is given by

$$N = B_g F \tag{18a}$$

where

$$B_g = \sqrt{2B_b \ln(t)} = \frac{2}{\pi}\sqrt{2\ln(1/T_f)\ln(t)}. \tag{18b}$$

For the truncated distributions, the required number of CMZ filter stages grows only linearly with finesse and the selectable number of channels [see (15)]. Compare this to the much worse quadratic growth (13a) for the full binomial distribution.

These dependences are illustrated in Fig. 6, where the bottom and top scales give the finesse F and the number of selectable channels N_{ch}, while the left scale is the required number of stages N. The finesse reference level is chosen to be $T_f = -30$ dB. Here, the open triangle symbols correspond to the full binomial distribution, and the solid line through them is a fit to the quadratic dependence of (13a). The fit gives $B_b = 2.1$, as compared to the analytical estimate of $B_b = 2.8$ from (13b). The solid triangle symbols in Fig. 6 correspond to the truncated binomial distribution, using maximal truncation with sidelobes below -30 dB. The open circles correspond to the truncated Gaussian distribution with the truncation level of $t = 6$; here, the sidelobe level was approximately -30 dB. In Fig. 6, the truncated Gaussian filters follow closely the behavior of the truncated binomial filters. As expected from (18a), the required number of stages grows linearly with finesse. The solid line through the open circles in Fig. 6 is a linear fit to the points. The fit gives $B_g = 2.5$, as compared to $B_g = 3.2$ from the analytical estimate in (18b). The numerical estimates from (18b) and (13b) work better for the moderate finesse reference levels, say, $T_f \approx -(3-10)$ dB, and are worse for the low levels of $T_f \approx -30$ dB, where the filter shape is more sensitive to the level of truncation, the exact weight distribution, and deviates further from the approximation in (12).

VII. Conclusion

In conclusion, we have described cascaded coupler Mach–Zehnder (CMZ) channel adding/dropping filters for use in WDM systems. Such filters allow selection of every nth channel ($n = 2, 3, 4, \cdots$) from the WDM signal with very low (< -30 dB) cross-talk levels. Channel dropping function

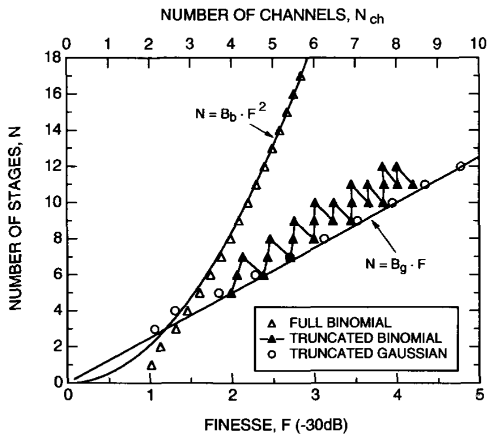

Fig. 6. Number of CMZ filter stages required for the given number of selectable channels or the finesse. Shown are dependences for the full binomial weight distribution, truncated binomial and Gaussian ($t = 6$) distributions, and the quadratic and linear fits to the two dependences. The finesse is measured at the $T_f = -30$ dB level.

of the CMZ filters is periodic, which can be advantageous for certain applications. Using weighted coupler lengths with truncated binomial distribution, which appears to be optimal, relatively few stages are required for a filter with good finesse and low sidelobe characteristics. The truncated Gaussian distribution can be used conveniently to approximate the truncated binomial distribution in designing such multistage filters. Importantly, we have shown that the number of CMZ filter stages grows only linearly with the required finesse and thus the number of selectable channels. This is important for keeping down the physical size of integrated optical CMZ filters. Physical device size will limit the ultimate number of stages and the achievable finesse of these filters. Integrated optical technology on silicon [3] or InP [4] can be used to implement the CMZ channel dropping filters and, perhaps, integrate them with other optoelectronic devices.

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