System Outage Probability Due to First- and Second-Order PMD

Henning Bülow

***Abstract*—A theoretical approach is proposed that allows one to quantify the impact of fiber polarization mode dispersion (PMD) on optical binary transmission taking into account not only first-order polarization mode dispersion, but also signal distortion induced by second-order PMD. Using this approach the impact of the spectral signal width on PMD-induced system outage probability could be studied for the first time. An analysis of 10-Gb/s transmission exhibits that, as long as the mean PMD remains below the commonly accepted limit (about 10 ps) for negligible outage, a linear chirp of up to 30 GHz does not lead to an additional increase of the system outage. This result confirms that low bandwidth modulation schemes (external modulator, low chirp laser) do not suffer from additional outage degradation due to second-order PMD.**

***Index Terms*— Fiber impairments, polarization mode dispersion, system outage.**

I. Introduction

POLARIZATION mode dispersion (PMD) has been identified as one of the mechanisms posing a limit on the transmission capacity of multigigabit-per-second optical transmission. PMD can be described as birefringence of the transmission fiber fluctuating with wavelength and time. As long as the signal spectrum is sufficiently narrow the first-order effect of PMD is dual path propagation causing inter symbol interference. When using directly modulated chirping lasers or other broad spectrum sources, the PMD may change within the signal spectrum which will give rise to additional signal distortions due to so called higher order PMD [1]–[3]. Up to now the degradation due to first-order PMD was quantified by criteria like the probability of system outage or eye penalty [4], [5]. To date, bit distortion induced by second-order PMD has been investigated, but the effect on a system has only been estimated by extrapolating results based on a limited number of statistically independent PMD samples obtained from numerical simulation or fiber measurements [6], [7]. However, the impact on transmission has not been quantified. In this letter, we will evaluate the system outage probability theoretically by calculating the bit-error rate (BER) in the presence of first- and second-order PMD, and by approaching the probability density of the associated PMD distortion. Finally, the outage degradation of 10-Gb/s transmission due to a transmitter chirp will be discussed.

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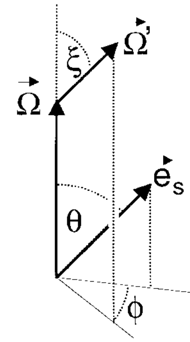

Fig. 1. PMD dispersion vectors $\vec{\Omega}$ and $\vec{\Omega}'$ and signal SOP Stokes vector $\vec{e}_s$.

II. BER Under PMD

The PMD is described by the dispersion vector $\vec{\Omega} = \Delta\tau\vec{e}_+$ with $\Delta\tau$ denoting the differential group delay (DGD) between slow and fast principal axes and with $\vec{e}_+$ denoting the fast axis output state of polarization (SOP, unity Stokes vector on the Poincaré sphere). The complex vector $\hat{E}$ of the output field distorted by first-order PMD can be decomposed in two undistorted and differently delayed signals $\hat{E}(\omega) = c_+\hat{E}_+ + c_-\hat{E}_-$ [1]. The amplitudes $c_\pm = \hat{e}_s\hat{e}_\pm^*$ are determined by the signal SOP given by the Jones vectors $\hat{e}_s$, and $\hat{e}_\pm$, respectively. Taking into account a wavelength dependency by evaluating $\vec{\Omega} = \vec{\Omega}(\omega_0) + \vec{\Omega}'\Delta\omega$ at signal center frequency ω_0 with the prime denoting the derivative with respect to ω and $\Delta\omega = \omega - \omega_0$, the distorted field in each axis can be approached by [1]

$$\hat{E}_\pm(\omega) = \hat{e}_\pm\left[s_{\text{in}}(\Delta\omega)\exp\left(i\phi_\pm \pm i\frac{\Delta\tau}{2}\Delta\omega \pm i\frac{\Delta\tau'}{4}\Delta\omega^2\right)\right] \pm \hat{e}_\mp\left[s_{\text{in}}(\Delta\omega)\exp(i\phi_\mp)i\frac{\Delta\tau}{2}(\hat{e}'_\pm\hat{e}^*_\mp)\Delta\omega^2\right]. \quad (1)$$

Inserting (1) into $\hat{E}(\omega)$ yields an expression where first-order PMD distortion is quantified by the parameters $c_\pm$ and $\Delta\tau$ and the second-order PMD distortion is quantified by $\Delta\tau'$ and $\hat{e}'_\pm\hat{e}^*_\mp$. $\phi_\pm$ denotes phase terms and $s_{\text{in}}(\Delta\omega)$ is the Fourier transform of the input field, which we assume to be a Gaussian pulse $s_{\text{in}}(t) = \Delta t_{\text{in}}^{-1/2}\exp[-2\ln 2t^2(\Delta t_{\text{in}}^{-2} - i\dot{\omega}/4\ \ln 2)]$ with a width Δt_{in} [full-width at half-maximum (FWHM) of the optical power] and a linear chirp $\dot{\omega} = d\omega/dt$.

In order to minimize the set of parameters quantifying the PMD we will evaluate first and second-order distortion in terms of $\vec{\Omega}, \vec{\Omega}'$ and the Stokes vector $\vec{e}_s$ denoting the signal output SOP. These three vectors are shown in Fig. 1. θ, ϕ, and ξ are the angles describing the mutual orientation between

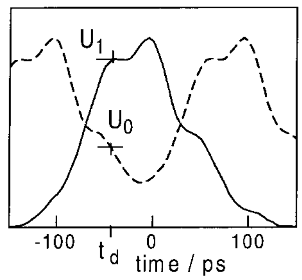

Fig. 2. 10 Gb/s "010" and "101" patterns (solid and dashed line, respectively, ($t_{\rm in}$ = 80 ps, $\Delta\nu$ = 20 GHz) distorted by a PMD of $(|\vec{\Omega}|, |\vec{\Omega}'|, \theta, \phi, \xi)$ = (80 ps, 1500 ps^2, 0.4π, 0, 0.5π).

them. Taking into account the relations between the complex two-element Jones vector representation of the SOP and the real three-element Stokes vector [8] it is straightforward to derive the following identities: $c_+ = \cos(\theta/2)e^{i\phi/2}, c_- = \sin(\theta/2)e^{-i\phi/2}$, and $\pm c_x = (\Delta\tau/2)(\hat{e}'_\pm \hat{e}^*_\mp) = \pm\frac{1}{4}|\vec{\Omega}'|\sin\xi$. Finally, by applying the inverse Fourier transform on $\hat{E}(\omega)$ we obtain the distorted output field in the time domain:

$$\begin{aligned}\hat{E}(t) = &\ \hat{e}_+[c_+ s(t+\Delta\tau/2) + ic_- c_x \ddot{s}_{\rm in}(t)] \\ &+ \hat{e}_-[c_- s(t-\Delta\tau/2) + ic_+ c_x \ddot{s}_{\rm in}(t)] \qquad (2)\end{aligned}$$

where $\ddot{s}_{\rm in}$ denotes the second derivative of the Gaussian input pulse with respect to time and $s(t \pm \Delta\tau/2)$ are the differently retarded Gaussian output pulses $s(t) = \Delta t^{-1/2}\exp(-2\ln 2 t^2 \Delta t^{-2})$ of fast and slow axis that are, in general, also differently broadened and chirped by the PMD induced group velocity dispersion $\pm\Delta\tau'/2 = \pm(|\vec{\Omega}'|\cos\xi)/2$ due to its opposite sign in both axes [1]. The temporal pulsewidth $\Delta t = (\mathrm{Re}\{\Delta t^{-2}\})^{-1/2}$ and the chirp are determined by real and imaginary part of $\Delta t^{-2} = (1/(\Delta t_{\rm in}^{-2} - i\dot{\omega}/4\ln 2) \pm i2\ln 2\ |\vec{\Omega}'|\cos\xi)^{-1}$, respectively. Note, that in (2) the first- and second-order PMD distortion is quantified by a set of the five distortion parameters $|\vec{\Omega}|, |\vec{\Omega}'|, \theta, \phi$, and ξ.

In Fig. 2, the optical power $|\hat{E}(t)|^2$ of the PMD distorted 10-Gb/s "010" and "101" patterns ($\Delta t_{\rm in}$ = 80 ps), shown as solid and dashed lines, respectively, have been calculated using (2). Since the BER is mainly determined by these two three-bit patterns, which corresponding to the minimum eye opening and which appear with a probability of 1/8, the BER for a specific PMD distortion can be approached with aid of the eye opening $U_1 - U_0$ extracted from the two three-bit patterns shown in Fig. 2. The BER is given by

$$\mathrm{BER} = (1/8)\,\mathrm{erfc}(Q(t_d)/\sqrt{2}) \qquad (3)$$

where the Q factor at decision time t_d. $Q(t_d) = (U_1 - U_0)/(\sqrt{N_1} + \sqrt{N_0})$ can be calculated using (2), with $U_1 = A|\hat{E}(t_d)|^2$ and $U_0 = A(|\hat{E}(t_d - T)|^2 + |\hat{E}(t_d + T)|^2)$ and the dominating signal-spontaneous and spontaneous-spontaneous beat-noise powers $N_{0,1} = (2n_{sp}U_{0,1} + A^2 n_{sp}^2 \Delta\nu)B$ of an erbium-doped fiber (EDF) preamplified receiver [4]. In this equation, A denotes a constant factor, n_{sp} (= 3 dB) the inversion factor of the erbium-doped fiber apmlifier (EDFA), $\Delta\nu$ (= 125 GHz, 1 nm) the bandwidth of the optical filter inserted between EDFA and receiver photo diode, $B(= 0.75/T)$ the noise bandwidth of the receiver, and T (= 100 ps) the bit period of the signal. The decision time t_d was determined with an analytic approach [4] taking into account receiver clock recovery phase walkoff due to first-order distortion only determined by the parameters $|\vec{\Omega}|$ and θ.

III. Outage Probability

For first-order PMD distortions the cumulative probability CP that the BER degradation exceeds the value of 10^{-12} (outage) was introduced as a measure for the PMD sensitivity [4], [5]. An extension of this definition to second-order distortions yields

$$\mathrm{CP} = \iiiint\!\!\int_{\mathrm{BER} > 10^{-12}} p_j\ d|\vec{\Omega}|\ d|\vec{\Omega}'|\ d\theta\ d\phi\ d\xi. \qquad (4)$$

Here, p_j denotes the joint probability density function (pdf) for the occurrence of a first- and second-order PMD distortion. The pdf has to be integrated over the area, span by the five distortion parameters, where BER exceeds 10^{-12}. Equation (3) allows one to determine this outage area.

The pdf p_j for a PMD distortion can be calculated using an approach for a joint pdf $p_{\rm PMD}$ for the occurrence of $\vec{\Omega}$ and $\vec{\Omega}'$ [9]:

$$\begin{aligned}p_{\rm PMD}(\vec{\Omega}, \vec{\Omega}', \xi) = &\left(\frac{9}{2\pi}\right)^3 \left(\frac{8}{3\pi}\right)^{9/2} \cdot \langle\Delta\tau\rangle^{-9} \cdot (1 + 3\Omega^2)^{-1} \\ &\cdot \exp\left(-\frac{3}{2}\left[\Omega^2 + 3\frac{\Omega'^2 + 3(\Omega\Omega'\cos\xi)^2}{\Omega^2 + 1/3}\right]\right) \qquad (5)\end{aligned}$$

with $\Omega = \sqrt{8/3\pi}(|\vec{\Omega}|/\langle\Delta\tau\rangle)$ and $\Omega' = (8|\vec{\Omega}'|/3\pi\langle\Delta\tau\rangle^2)$ where $\langle\Delta\tau\rangle$ denotes the mean fiber PMD. In addition, we have to consider the pdf $p_s(\vec{e}_s) = 1/(4\pi)$ of a distinct signal SOP $\vec{e}_s$ with the spherical coordinates θ and ϕ in the coordinate system determined by $\vec{\Omega}$ and $\vec{\Omega}'$, as illustrated in Fig. 1. Thus, the joint pdf of a first- and second-order PMD distortion is approached by the product

$$p_j(|\vec{\Omega}|, |\vec{\Omega}'|, \theta, \phi, \xi) = p_{\rm PMD} p_s. \qquad (6)$$

With (3) and (6), we possess all tools to determine the PMD sensitivity of a system using (4).

IV. Results and Discussion

The impact of transmitter bandwidth on PMD sensitivity of 10-Gb/s transmission under the assumption of zero or compensated chromatic dispersion can be assessed by calculating the outage probability CP for differently chirped signals. Fig. 3 shows CP (solid lines) against transmitter chirp $\Delta\nu$ ($\dot{\omega} = 2\pi\Delta\nu/\Delta t_{\rm in}$) for three mean PMD's of 10, 20, and 30 ps. A 2-dB power margin and nonreturn-to-zero (NRZ) bit format, approached by a pulsewidth of 80 ps, has been assumed (BER$< 10^{-19}$ without PMD). The dashed lines are the corresponding outages considering only first-order PMD distortion. The calculations reveal that within the commonly accepted PMD limit of about 10% of the bit period ($\langle\Delta\tau\rangle$ = 10 ps in Fig. 3) up to a transmitter chirp

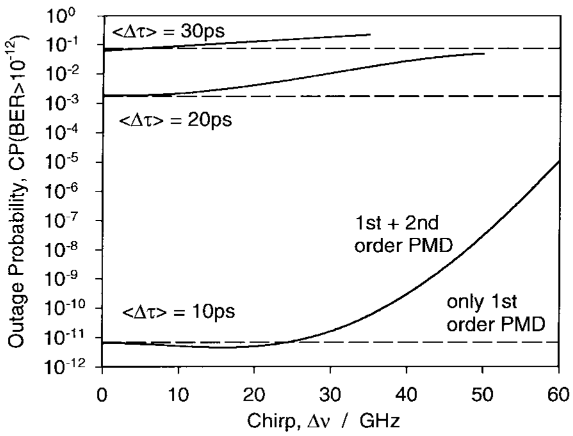

Fig. 3. Solid lines show outage probability at 10 Gb/s versus transmitter chirp for mean PMD's of 10, 20, and 30 ps. Dashed lines show probabilities taking into account only first-order distortion.

of about 30 GHz the increase of system outage of $\mathrm{CP} = 10^{-11}$ due to second-order PMD is negligible. Hence, we do not await any significant increase of outage probability for systems working with low-bandwidth sources like external modulators (EAM, MZI) or directly modulated low chirp laser diodes. As it is illustrated by the 20- and 30-ps PMD curves of Fig. 3, transmission over high PMD fibers is accompanied not only by a high-outage probability ($\mathrm{CP} > 10^{-3}$), but also by additional outage degradation for signal bandwidth broadening in excess of about 15 GHz. This has been confirmed in [10] by a transmission experiment over a high PMD fiber (>40 ps) showing that low bandwidth 10-Gb/s phase-shaped binary transmission format exhibits lower BER degradation than NRZ format.

V. Conclusion

An approach was presented that allows one to calculate the system outage ($\mathrm{BER} > 10^{-12}$) probability of a binary data signal spectrally broadened by a chirp in the presence of first- and second-order PMD. In the theoretical model signal bits are represented by Gaussian pulses with a linear chirp. Thus the impact of transmission bandwidth on PMD induced system outage could be analyzed quantitatively even for system relevant low-outage probabilities. Calculation of 10-Gb/s transmission under PMD exhibits negligible outage degradation due to second-order PMD up to a transmitter chirp of 30 GHz provided the mean PMD remains below the low-outage PMD limit of about 10 ps. Furthermore, an analysis of transmission over a mean PMD of 20 and 30 ps is accompanied by additional outage degradation beyond a chirp of only 15 GHz. This shows that transmission over a fiber with a higher PMD is accompanied not only by an increase of the outage probability, but also by an increase of the sensitivity to an additional spectral broadening. This result demonstrates that low bandwidth modulation schemes like external modulation may not suffer from outage degradation induced by second-order PMD.

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