

# Evaluation on System Outage Probability Due to Temperature Variation and Statistically Distributed Chromatic Dispersion of Optical Fiber

H. C. Ji, J. H. Lee, *Student Member, IEEE*, and Y. C. Chung, *Senior Member, IEEE*

**Abstract**—This paper discusses the evaluation of the system outage probability caused by the temperature variation and statistically distributed chromatic dispersion of optical fiber in a high-speed ( $> 40$  Gb/s) optical network. This was to identify when the tunable dispersion compensator should be used at every channel.

**Index Terms**—Chromatic dispersion (CD), optical fiber, outage probability, temperature variation, wavelength-division multiplexing (WDM).

## I. INTRODUCTION

RECENTLY, there have been substantial efforts to utilize high-speed ( $> 40$  Gb/s) channels in the wavelength-division-multiplexed (WDM) network [1]–[3]. One of the most critical limiting factors for these networks would be the chromatic dispersion. Thus, it is essential to manage the dispersion of transmission fiber carefully for their proper operation. Previously, the dispersion management was mostly achieved by using the dispersion-compensating fiber (DCF) modules [4]. However, the value of chromatic dispersion is slightly different from transmission fiber to fiber due to the imperfect manufacturing process. In addition, it is well known that the chromatic dispersion of optical fiber is dependent on the ambient temperature [5]–[8]. Thus, it would be extremely difficult to satisfy the stringent dispersion requirement of such high-speed signals by using only the DCF modules.

In this paper, the system outage probability caused by temperature variation and statistically distributed chromatic dispersions of optical fibers is evaluated. For this evaluation, the distribution of chromatic dispersions in 1000 spools of single-mode fiber (SMF) and non-zero dispersion-shifted fiber (NZ-DSF) obtained from five different fiber manufacturers were investigated. These statistically distributed chromatic dispersions of transmission fibers were assumed to be compensated by using the DCF modules with determinate values of dispersions. The system outage probability was then estimated

by using the simple equation derived from the third-order Sellmeier equation. The proposed equation describes the statistical distribution of residual dispersions by using the mean and standard deviation of zero-dispersion wavelengths ( $\lambda_0$ ) and dispersion slopes at these wavelengths ( $S_0$ ) of the transmission fibers. The validity of this equation was confirmed by using Monte Carlo simulation. The results were used to determine when the high-speed WDM network should employ tunable dispersion compensators. For example, when the chromatic dispersion of a typical SMF link was compensated by using only DCF modules (i.e., without tunable dispersion compensators), the maximum transmission distance for 40-Gb/s return-to-zero (RZ) signal could be limited to 27 km  $\sim$  148 km, if the system outage probability (dispersion penalty:  $> 1$  dB) should be less than  $5.5 \times 10^{-5}$ . However, if the statistically distributed fiber dispersions (e.g., the system outage probability is dependent only on the temperature variation) is neglected, this distance would be increased to about 4100 km.

## II. THEORY

To describe the chromatic dispersion of SMF and NZ-DSF as a function of wavelength, we used the third-order Sellmeier equation

$$D(T, \lambda) = \frac{S_0(T)}{4} \left( \lambda - \frac{\lambda_0^4(T)}{\lambda^3} \right) \quad (1)$$

where  $\lambda_0(T)$  and  $S_0(T)$  are the zero-dispersion wavelength and the dispersion slope at zero-dispersion wavelength with respect to temperature  $T$ , respectively. Previously, it has been reported that this equation is valid for both SMF and NZ-DSF across the  $C$  and  $L$  bands over a temperature range from  $-40^\circ\text{C}$  to  $+60^\circ\text{C}$  [7]. To obtain the expression for chromatic dispersion shown in (2), we expanded (1) by using a Taylor polynomial at the mean temperature  $T_0$ , and then simplify the result by assuming that  $d\lambda_0(T_0)/dT$  and  $dS_0(T_0)/dT$  could be considered as constants at any temperature (refer Section III).

$$D(T, \lambda) \approx \frac{S_0}{4} \left( \lambda - \frac{\lambda_0^4}{\lambda^3} \right) + \left\{ \frac{1}{4} \left( \lambda - \frac{\lambda_0^4}{\lambda^3} \right) C_2 - \frac{S_0 \lambda_0^3}{\lambda^3} C_1 \right\} (T - T_0) \quad (2)$$

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The authors are with the Department of Electrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea (e-mail ychung@ee.kaist.ac.kr).

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where  $C_1 (= d\lambda_0(T_0)/dT)$  and  $C_2 (= dS_0(T_0)/dT)$  represent the temperature coefficients of  $\lambda_0$  and  $S_0$ , respectively. In this equation, we denoted  $\lambda_0(T)$  and  $S_0(T)$  as  $\lambda_0$  and  $S_0$ , respectively, for the simplicity. Thus, the mean and variance of  $D(T, \lambda)$  could be described as

$$\begin{aligned} \langle D(T, \lambda) \rangle &= \frac{\langle S_0 \rangle}{4} \left( \lambda - \frac{\langle \lambda_0^4 \rangle}{\lambda^3} \right) \\ &+ \left\{ \frac{1}{4} \left( \lambda - \frac{\langle \lambda_0^4 \rangle}{\lambda^3} \right) C_2 - \frac{\langle S_0 \rangle \langle \lambda_0^3 \rangle}{\lambda^3} C_1 \right\} (\langle T \rangle - T_0) \\ V(D(T, \lambda)) &= \frac{V(S_0)}{16} \left( \lambda^2 - \frac{2\langle \lambda_0^4 \rangle}{\lambda^2} \right) \\ &+ \frac{1}{16} \frac{\{V(S_0) + \langle S_0 \rangle^2\} \langle \lambda_0^8 \rangle - \langle S_0 \rangle^2 \langle \lambda_0^4 \rangle^2}{\lambda^6} \\ &+ \frac{C_2^2 V(T)}{16} \left\{ \lambda^2 - \frac{2\langle \lambda_0^4 \rangle}{\lambda^2} - \frac{2\langle \lambda_0^8 \rangle}{\lambda^6} \right\} \\ &- \frac{C_1 C_2 V(T)}{2} \frac{\langle S_0 \rangle \langle \lambda_0^3 \rangle}{\lambda^2} + \frac{C_1 C_2 V(T)}{2} \frac{\langle S_0 \rangle \langle \lambda_0^7 \rangle}{\lambda^6} \\ &+ \frac{C_1^2 V(T)}{\lambda^6} \{V(S_0) + \langle S_0 \rangle^2\} \langle \lambda_0^6 \rangle. \end{aligned} \quad (3)$$

Equation (3) indicates that if the chromatic dispersion of a fiber-optic transmission link consisted of  $L$  km/spool  $\times$   $N$  spools was periodically compensated by using DCF modules, the distribution of residual dispersions would have the mean value of 0 ps/nm and the standard deviation of  $L \times \{N \times V(D(T, \lambda))\}^{1/2}$  ps/nm.

### III. RESULTS AND DISCUSSIONS

Fig. 1 shows the distributions of  $\lambda_0$  and  $S_0$  for 1000 commercially available fiber spools (six groups of SMF and two groups of NZ-DSF) obtained from five different fiber manufacturers. The length of fiber spool was either 20 or 25 km. This figure shows that the means and variances of  $\lambda_0$  and  $S_0$  were different from fiber group to fiber group. In particular, the old fibers (in group 3) had relatively large variances probably due to the less mature manufacturing process. Nonetheless, the existing fiber-optic networks could have been implemented by using one or mixture of these groups of fibers. For the best-case analysis, we assumed that the network was implemented by using only one group of fibers. However, the dispersion and dispersion slope could still be slightly different from one transmission link to another, since each link would have been implemented by using the randomly selected fiber spools within a fiber group. The DCF module typically has a specific value of dispersion depending on the required compensation length. We assumed that the dispersion of DCF module was matched to the average dispersion of the fiber group used in the network. Thus, it would not be possible to completely compensate the dispersion and dispersion slope of each transmission link by using only DCF modules. In addition, the variance of the residual dispersion (obtained after the compensation by DCF modules) could vary when the environmental temperature changes or the optical path

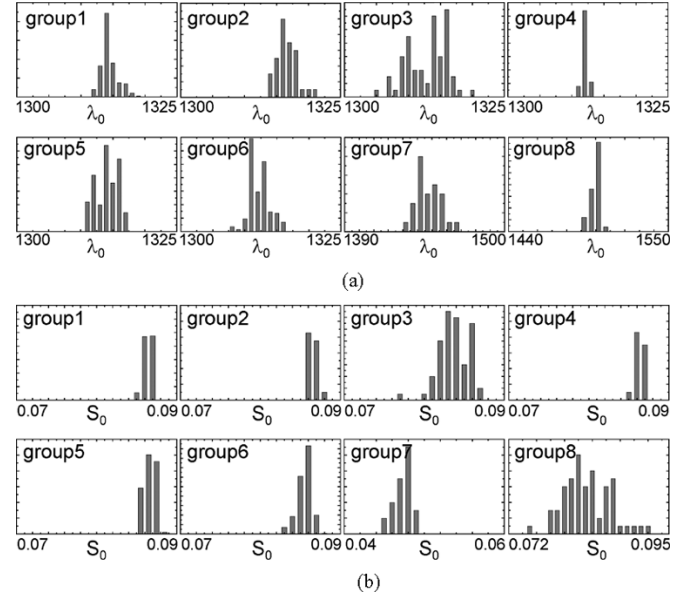


Fig. 1. Distributions of (a) zero-dispersion wavelength  $\lambda_0$  and (b) dispersion slope  $S_0$  of 1000 fiber spools (SMF: group 1 ~ group 6; NZ-DSF: group 7 ~ group 8).

is altered [e.g., by using optical cross connects (OXC)s] within the network. It has been reported that the peak-to-peak seasonal temperature variation is about 20 °C in the fiber buried 40 m under the ground (this variation could be as high as 70 °C in the aerial fiber) [8], [9]. In addition, in a long-distance transmission link, the average temperature could be quite different from region to region. Thus, we assumed that each 20-km (or 25-km)-long fiber section in the transmission link was exposed to the average temperature of 25 °C with a standard deviation of 17 °C. We believe that this assumption would not overestimate the limitations caused by temperature variation. The longitudinal temperature variation along the transmission link was considered by assuming that the temperature variation in each fiber section was statistically independent. To evaluate the effects of temperature changes, we measured the dispersions and dispersion slopes of SMF and NZ-DSF by using an optical network analyzer after placing the fiber spools in a temperature-cyclic chamber.

Fig. 2 shows that the  $C_1$  and  $C_2$  can be approximated as constants at any temperature. The measured values of these coefficients agreed well with the previously reported results [7]. Thus, we calculated the standard deviations of the residual dispersions for all fiber groups by using the measured values of  $C_1$  and  $C_2$  and (3). The results are summarized in Table I. In this calculation, we assumed that the signal's wavelength was 1550 nm and the standard deviation of the temperature variation was 17 °C. We also assumed that the DCF module had determinate values of dispersion and dispersion slope matched to the average values of the specific group of fiber. Any deviation from these assumptions would increase the variance of residual dispersion. Using (3), we calculated the distribution of residual dispersions in a 1000-km-long transmission link made of group 6 fibers and the DCF modules. In this calculation, we assumed that the residual dispersions have a Gaussian distribution. We also assumed that the standard deviation of temperature variation was 17 °C. The

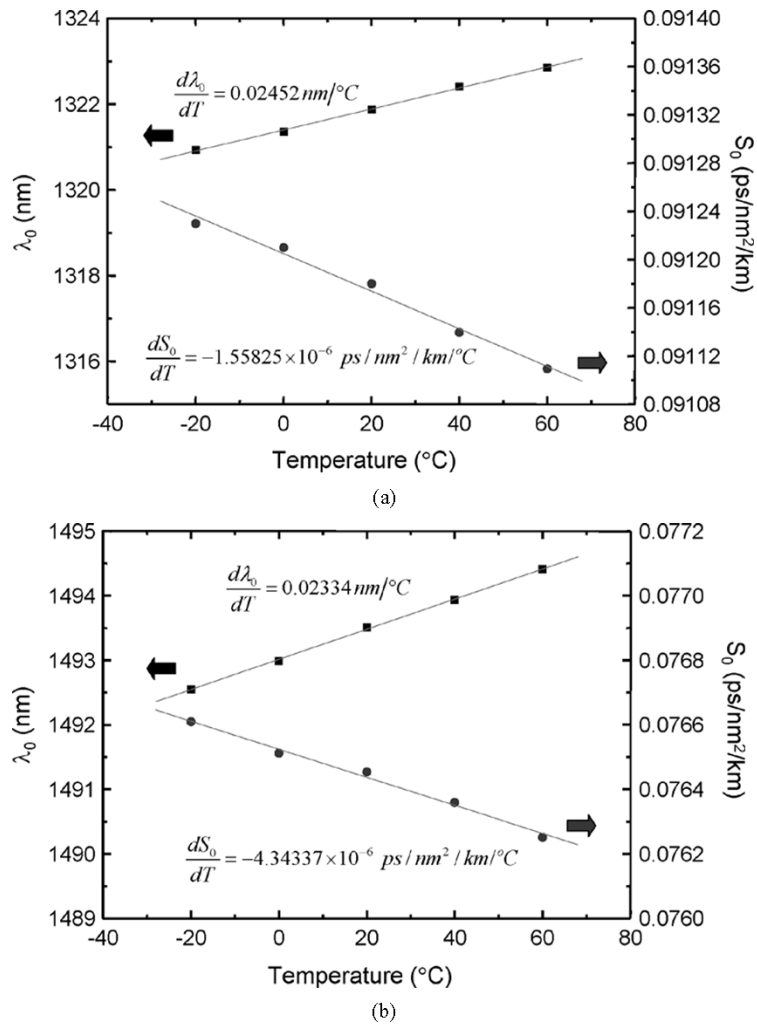


Fig. 2. Measured temperature coefficients of (a) SMF and (b) NZ-DSF.

TABLE I  
STANDARD DEVIATIONS OF RESIDUAL DISPERSIONS CALCULATED FROM 1000 COMMERCIALY AVAILABLE FIBER SPOOLS (SIX GROUPS OF SMF AND TWO GROUPS OF NZ-DSF) OBTAINED FROM FIVE DIFFERENT FIBER MANUFACTURERS

	SMF						NZDSF	
	group1	group2	group3	group4	group5	group6	group7	group8
Length (km)	25	20	20	20	20	25	20	20
# of sample	200	34	66	83	126	400	27	64
$\langle \lambda_0 \rangle$ (nm)	1314.4	1316.6	1312.9	1312.6	1314.1	1312.2	1447.5	1499.7
$\sigma(\lambda_0)$ (nm)	1.3148	1.6165	3.2064	1.2945	1.737	1.6001	8.2160	3.2395
$\langle S_0 \rangle$ (ps/nm <sup>2</sup> /km)	0.0864	0.0866	0.0836	0.0866	0.0866	0.0855	0.0473	0.0832
$\sigma(S_0)$ (ps/nm <sup>2</sup> /km)	0.0006	0.00061	0.0018	0.00065	0.00058	0.00093	0.0011	0.0034
$\sigma(D)$ (ps/nm/km)	0.134	0.144	0.376	0.143	0.143	0.195	0.334	0.296

results are shown in Fig. 3 in comparison with the values obtained by Monte Carlo simulation. For the simulation, we used 1 000 000 samples of 1000-km-long optical link consisting of 40 randomly selected fiber spools from group 6 fiber. We also performed the same simulation by assuming that the dispersions of these fiber spools have either Gaussian or uniform distribu-

tion (using the mean and standard deviation of group 6 fiber of 16.5505 and 0.1857 ps/nm/km, respectively).

Fig. 3 shows that, regardless of the distributions of fiber dispersions, the calculated values obtained by using (3) agreed well with all the simulated results. This result indicates that the residual dispersion of the transmission link of any distance

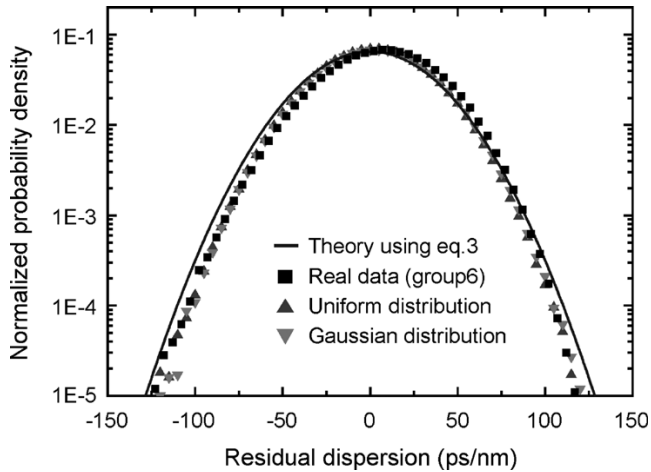


Fig. 3. Distributions of residual dispersions in 1000-km-long transmission link.

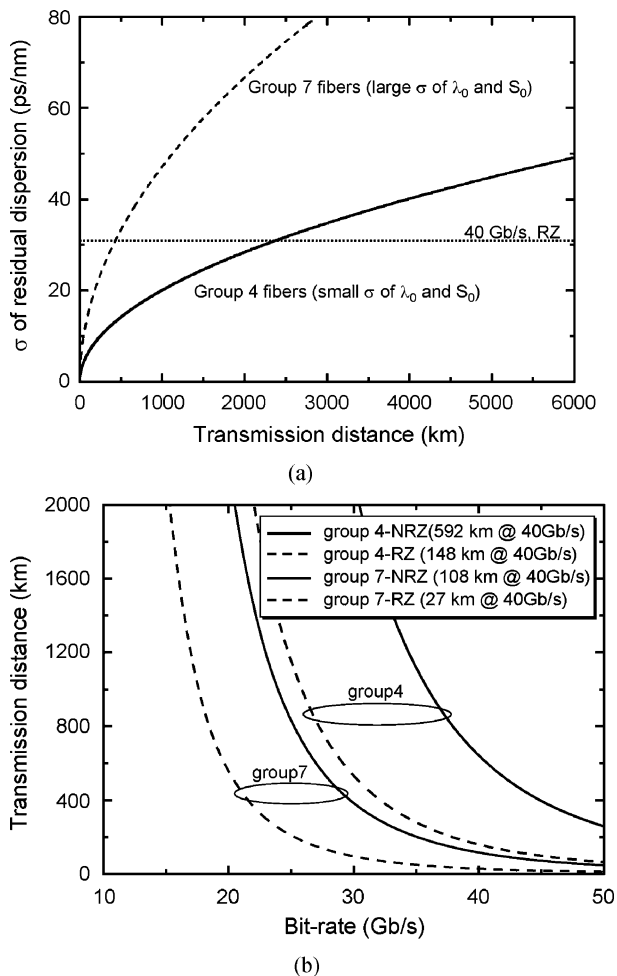


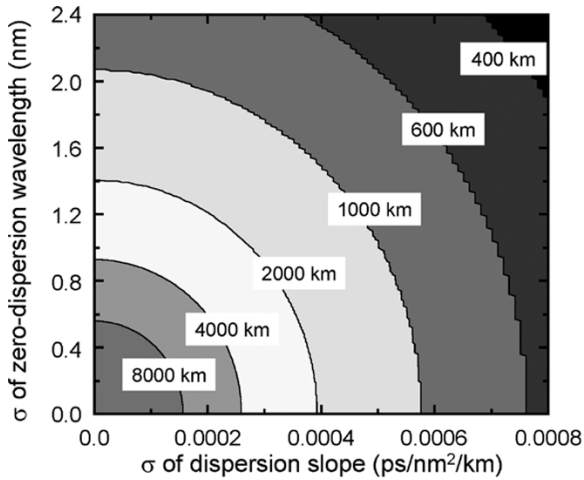
Fig. 4. (a) Standard deviation ( $\sigma$ ) of residual dispersion versus transmission distance. (b) Maximum transmission distance versus bit rate for the outage probability of  $4\sigma$ .

would have a Gaussian distribution. Thus, we could estimate the system outage probability (dispersion penalty:  $> 1$  dB) simply by using (3) and the mean and standard deviation of the measured  $\lambda_0$  and  $S_0$ . Fig. 4(a) shows the standard deviation ( $\sigma$ ) of residual dispersion calculated as a function of the

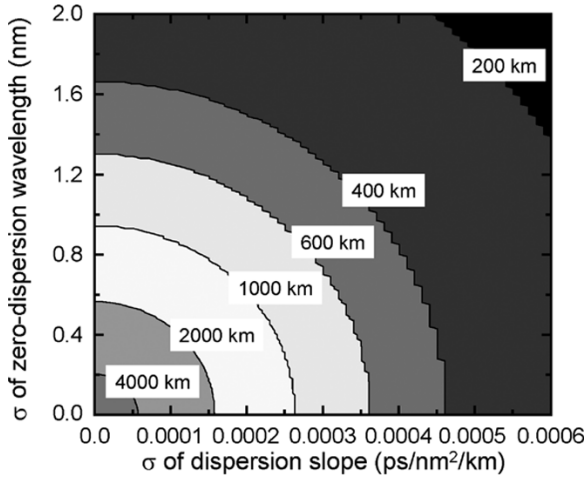
transmission distance. The result shows that the standard deviation of residual dispersion was strongly dependent on the fiber types and increased with transmission distance. For example, if the transmission link were implemented by using the fiber spools in group 4 (which had small  $\sigma$  of  $\lambda_0$  and  $S_0$ ) and group 7 (which had large  $\sigma$  of  $\lambda_0$  and  $S_0$ ), the standard deviation of residual dispersion became 31 ps/nm (e.g., dispersion limit for a 40-Gb/s RZ signal with a 50% duty cycle) at 2400 and 400 km, respectively. This result indicates that the system outage probability becomes 0.334 when the 40-Gb/s RZ signal is transmitted over a 2400 km of a group 4 fiber or 400 km of a group 7 fiber. We assumed that the system outage probability should not exceed  $5.5 \times 10^{-5}$  (which corresponds to  $4\sigma$  on a Gaussian distribution) to ensure the reliability required in modern fiber optic networks. To satisfy this requirement, the standard deviation of residual dispersion should be less than 15.5 and 7.75 ps/nm for 40-Gb/s NRZ and RZ signals, respectively. Fig. 4(b) shows the maximum transmission distance (to maintain the system outage probability to be less than  $5.5 \times 10^{-5}$ ) calculated as a function of bit rate. The result shows that the maximum transmission distance could be seriously limited when the operating speed is higher than 40 Gb/s. For example, when 40-Gb/s NRZ and RZ signals were transmitted through the transmission link implemented by using group 4 fibers, the maximum transmission distances would be limited to be 592 and 148 km, respectively. However, these distances could be reduced to 108 and 27 km if group 7 fibers were used. To overcome this limitation, it would be inevitable to use tunable dispersion compensators at every channel.

Fig. 5 shows the maximum transmission distances calculated as a function of the standard deviations of  $\lambda_0$  and  $S_0$  for 40-Gb/s NRZ and RZ signals. This figure could be used to identify the fiber specifications required for the transmission of 40-Gb/s signals without using tunable dispersion compensators. For example, to transmit 40-Gb/s NRZ signals through a 1000-km-long transmission link (and maintain the system outage probability within  $5.5 \times 10^{-5}$ ), the standard deviations of  $\lambda_0$  and  $S_0$  should be less than 2.06 nm and 0.00057 ps/nm<sup>2</sup>/km, respectively. For 40-Gb/s RZ signals, these values would be further reduced to 0.94 nm and 0.00026 ps/nm<sup>2</sup>/km, respectively.

If the statistically distributed fiber dispersions could be completely compensated (e.g., by carefully measuring the fiber dispersion of every span and using the precisely matched DCF modules), then the system outage probability would be solely dependent on the temperature variation. In this case, the maximum transmission distance would be substantially increased. Fig. 6 shows that if the standard deviation of temperature variation is 17 °C, the maximum transmission distance (to maintain the system outage probability to be less than  $5.5 \times 10^{-5}$ ) for a 40-Gb/s RZ signal could be increased to about 4100 km. However, it would be practically impossible to realize such a precisely compensated fiber link. To solve this problem, one may slightly overcompensate the dispersion of each span with a DCF module and then add a small amount of SMF or other fiber at the end of the transmission link to trim the total dispersion. However, in this case, the system outage probability cannot be improved as in Fig. 6 if the network is dynamically reconfigured (by using optical add/drop multiplexers or OXCs).



(a)



(b)

Fig. 5. Maximum transmission distances (achievable without using tunable dispersion compensators) calculated as a function of the standard deviations of  $\lambda_0$  and  $S_0$  for (a) 40-Gb/s NRZ signals and (b) 40-Gb/s RZ signals. In this calculation, we assumed that the system outage probability should not exceed  $5.5 \times 10^{-5}$ .

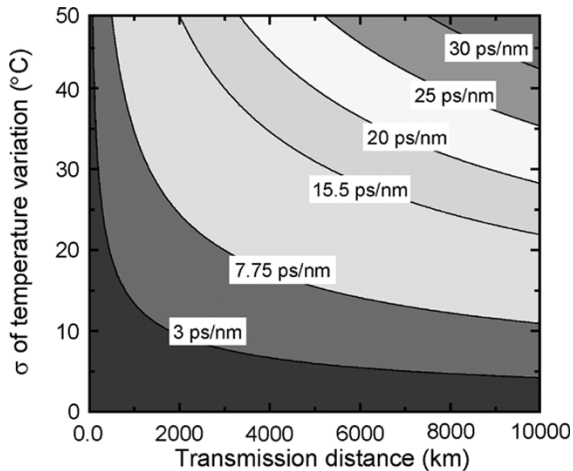
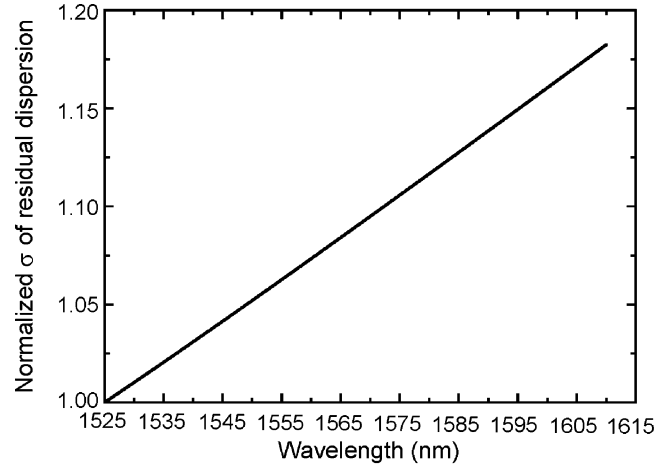
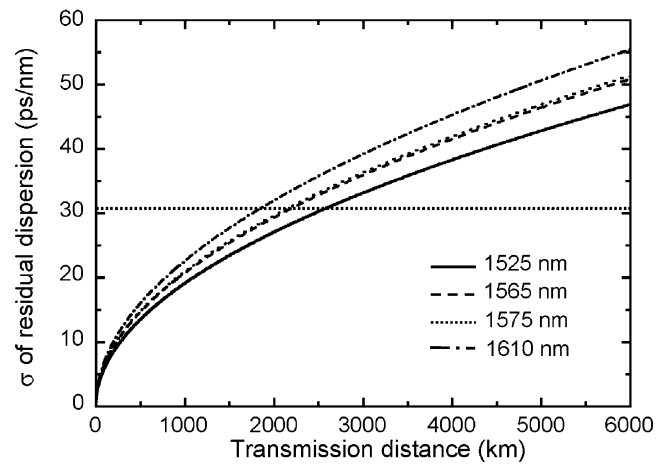


Fig. 6. Standard deviation of residual dispersion caused by temperature variation only (without considering the effect of statistically distributed fiber dispersions). The dispersion limits for 40-Gb/s RZ and NRZ signals are 7.75 and 15.5 ps/nm, respectively, if the system outage probability should not exceed  $5.5 \times 10^{-5}$ .



(a)



(b)

Fig. 7. (a) Normalized standard deviation of residual dispersion versus signal wavelength. (b) Standard deviation of residual dispersion versus transmission distance at various wavelengths.

Up until now, we have evaluated the effect of temperature variation and statistically distributed chromatic dispersion of optical fiber in a single-channel transmission system. However, in WDM networks, the standard deviation of residual dispersion is different from channel to channel. Fig. 7(a) shows the normalized standard deviations of residual dispersion calculated as a function of wavelength by using (3). In this calculation, we assumed that the transmission link was implemented by using group 4 fibers, and their statistically distributed dispersions and dispersion slopes were compensated by using DCF modules. This figure shows that the standard deviation of residual dispersion for the channel operating at 1610 nm is 18% higher than that for the channel operating at 1525 nm. This was mainly because the slight difference in  $S_0$  could amplify the variation of residual dispersion at the long wavelength. This increase in the standard deviation could increase the system outage probability by an order of magnitude (i.e., from  $5.5 \times 10^{-5}$  to  $5.2 \times 10^{-4}$ ). Fig. 7(b) shows the standard deviation of residual dispersion calculated as a function of transmission distance for the channels operating at 1525, 1565, 1575, and 1610 nm. The result shows that the standard deviation of residual dispersion becomes 31 ps/nm at 2500 km and 1800 km for the channels

operating at 1525 and 1610 nm, respectively. Thus, to maintain the system outage probability within  $5.5 \times 10^{-5}$ , the maximum transmission distance would be limited to 164 and 117 km for the 40-Gb/s RZ (duty cycle: 50%) signals operating at 1525 and 1610 nm, respectively. We also found that when the transmission link was implemented by using group 7 fibers (which had unusually large  $\sigma$  of  $\lambda_o$ ), the standard deviation of residual dispersion was decreased (rather than increased) with wavelength. This was due to the nonlinear characteristics of the dispersion curve emerged in the long-wavelength region. As result, when group 7 fibers were used, the maximum transmission distance would be limited to 25 and 29 km for the 40-Gb/s RZ (duty cycle: 50%) signals operating at 1525 and 1610 nm, respectively.

#### IV. CONCLUSION

We evaluated the effect of the residual dispersion caused by temperature variation and statistically distributed chromatic dispersion of optical fiber. For this evaluation, we measured the distribution of chromatic dispersion in 1000 fiber spools of SMF and NZ-DSF obtained from five different manufacturers. In addition, we developed a simple equation to describe the statistical distribution of residual dispersion by using only the mean and standard deviation of  $\lambda_o$  and  $S_o$  of optical fiber. The results showed that the maximum transmission distance for the 40-Gb/s RZ signal operating at 1550 nm could be limited to 27 km  $\sim$  148 km, depending on fiber types, if the system outage probability (dispersion penalty:  $> 1$  dB) caused by this statistical distribution should be less than  $5.5 \times 10^{-5}$ . For the signal operating at 1610 nm, this distance was found to be 29 km  $\sim$  117 km. However, if we neglect the statistically distributed fiber dispersions and assume that the system outage probability is dependent only on the temperature variation, this distance could be increased to about 4100 km. We believe that these results could be used to determine when tunable dispersion compensators should be used in a high-speed WDM network.

#### REFERENCES

- [1] K. Nakamura, H. Ooi, T. Terahara, Y. Akiyama, R. Hainberger, T. Takahara, G. Ishikawa, T. Fukushi, and T. Iwabuchi, "43-Gbit/s  $\times$  40 ch transmission over 1,600 km of conventional single-mode fiber in NRZ modulation scheme," in *Tech. Dig. Optical Fiber Communication Conf. (OFC2003)*, 2003, Paper FN4, pp. 745–746.
  - [2] J. -X. Cai, M. Nissov, C. R. Davidson, Y. Cai, A. N. Pilipetskii, H. Li, M. A. Mills, R. -M. Mu, U. Feiste, L. Xu, A. J. Lucero, D. G. Foursa, and N. S. Bergano, "Transmission of thirty-eight 40 Gb/s channels ( $> 1.5$  Tb/s) over transoceanic distance," in *Tech. Dig. Optical Fiber Communication Conf. (OFC2002)*, 2002, Postdeadline Paper FC4.
  - [3] K. Fukuchi, T. Kasamatsu, M. Morie, R. Ohhira, T. Ito, K. Sekiya, D. Ogasahara, and T. Ono, "10.92-Tb/s (273  $\times$  40-Gb/s) triple-band/ultra-dense WDM optical-repeated transmission experiment," in *Tech. Dig. Optical Fiber Communication Conf. (OFC2001)*, 2001, Postdeadline Paper PD24.
  - [4] S. K. Shin, C. H. Kim, and Y. C. Chung, "Directly modulated 2.5 Gb/s  $\times$  16-channel WDM transmission over 640 km of single-mode fiber using dispersion compensating fiber," *IEEE Photon. Technol. Lett.*, vol. 11, pp. 742–744, June 1999.
  - [5] T. Kato, Y. Koyano, and M. Nishimura, "Temperature dependence of chromatic dispersion in various types of optical fibers," in *Tech. Dig. Optical Fiber Communication Conf. (OFC2000)*, 2000, Paper TuG7–3, pp. 104–106.
  - [6] V. M. Schneider, "Effects of temperature on dispersion of high slope dispersion compensating fibers," *Electron. Lett.*, vol. 37, pp. 1069–1070, Aug. 2001.
  - [7] M. J. Hamp, J. Wright, M. Hubbard, and B. Brimacombe, "Investigation into the temperature dependence of chromatic dispersion in optical fiber," *IEEE Photon. Technol. Lett.*, vol. 14, pp. 1524–1526, Nov. 2002.
  - [8] A. Walter and G. Schaefer, "Chromatic dispersion variations in ultra-long-haul transmission systems arising from seasonal soil temperature variations," in *Tech. Dig. Optical Fiber Communication Conf. (OFC2002)*, 2002, Paper WU4, pp. 332–333.
  - [9] Soil Climate Analysis Network [Online]. Available: <http://www.wcc.nrcs.usda.gov/scan>
- H. C. Ji** was born in Seoul, Korea, in 1974. He received the B.S. and M.S. degrees, both in electrical engineering, from Kwangwoon University, Seoul, Korea, in 1996 and 1998, respectively. He is currently working toward the Ph.D. degree at the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea.
- From 1998 to 2000, he was with Lightwave Systems Research Laboratory, Information and Electronics Research Center, KAIST, where he worked on the development of Korea All-Optical Network (KAON). His current research interests include chromatic dispersion (CD) monitoring and compensation, polarization-mode dispersion (PMD), and polarization-dependent loss (PDL).
- J. H. Lee** (S'99) was born in Yangju, Korea, in 1977. He received the B.S. and M.S. degrees in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea. He is currently working toward the Ph.D. degree at the same institute.
- His current research interests include polarization-mode dispersion (PMD) and optical performance monitoring (OPM).
- Y. C. Chung** (S'81–M'83–SM'03) was with the Los Alamos National Laboratory from 1985 to 1987 under the AWU-DOE graduate fellowship program. From 1987 to 1994, he was with the Lightwave System Research Department at AT&T Bell Laboratories. In 1994, he joined the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, where he is currently Professor of electrical engineering. His current activities include high-capacity wavelength-division-multiplexing (WDM) monitoring techniques, polarization-mode dispersion (PMD), WDM passive optical networks, and fiber-optic networks for wireless communications. He has published more than 300 journals and conference papers in these areas and has more than 50 patents issued or pending.