Code Division Multiple-Access Techniques in Optical Fiber Networks—Part I: Fundamental Principles

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Abstract—This paper examines fiber-optic code division multiple-access (FO-CDMA) communications techniques. A new class of codes (signature sequences), namely, optical orthogonal codes (OOC’s), that are suitable for FO-CDMA are introduced. An experiment that shows the desired auto- and crosscorrelation properties of these codes and their use in FO-CDMA is reported. Furthermore, the concept of optical disk patterns, an equivalent way of representing OOC’s is introduced. The optical disk patterns are used to derive the probability density functions associated with any two interfering OOC’s. Also presented is a detailed study of different interference patterns from which the strongest and the weakest interference patterns are introduced.

Fig. 1. A fiber-optic communication system using optical encoder and decoder (correlator).

I. INTRODUCTION

The process of optical-to-electrical and electrical-to-optical conversion in a fiber-optic-based optical network for signal processing limits how much fiber bandwidth can be used because of the limited speed of electronic signal processors. It is believed that optical components, once fully developed and integrated, will offer much higher speeds for optical signal processing than electronic counterparts [1], [2], [3]. Therefore, a desirable feature of future optical communications systems would be the ability to perform signal processing functions optically so that the signal conversion from optical to electrical would be done only when desired.

This paper investigates one such scheme, namely, fiber-optic code division multiple access. This scheme takes advantage of excess bandwidth in single mode fibers to map low information rate electrical or optical signals into high rate optical pulse sequences to achieve random, asynchronous communications access, free of network control among many users. A typical FO-CDMA communication system is best represented by an information data source, followed by a laser when the information is in electrical signal form, and an optical encoder that maps each bit of the output information into a very high rate optical sequence, that is then coupled into the single-mode fiber channel (Fig. 1). At the receiver end of the FO-CDMA, the optical pulse sequence would be compared to a stored replica of itself (correlation process) and to a threshold level at the comparator for the data recovery (Fig. 1).

In FO-CDMA there are N such transmitter and receiver pairs (users). Fig. 2 shows one such network in a star configuration. The set of FO-CDMA optical pulse sequences essentially becomes a set of addresses or signature sequences for the network. To send information from user j to user k, the address code for receiver k is impressed upon the data by the encoder at the jth node. One of the primary goals of FO-CDMA is to extract data with the desired optical pulse sequence in the presence of all other users’ optical pulse sequences. We therefore need to design sequences that satisfy two conditions [4]–[7], namely:

C1) each sequence can easily be distinguished from a shifted version of itself and
C2) each sequence can be easily distinguished from (a possibly shifted version of) every other sequence in the set.

In Section II, a new class of sequences for fiber optic signal processing, namely, the ‘optical orthogonal codes’ that satisfy the above two conditions is introduced. Section III discusses a basic experiment demonstrating the principles of OOC’s and their use in FO-CDMA. Section IV introduces the concept of optical disk patterns, an equivalent way of representing OOC’s. These optical disk patterns are used to describe the probability density functions that are associated with any two interfering OOC’s. Section V is devoted to the study of OOC’s in a multiple-access environment. References [8], [9] are two other independent efforts for exploring fiber-optic CDMA.

II. OPTICAL ORTHOGONAL CODES (OOC’S)

Central to any successful code division multiple-access scheme, whether electrical or optical, is the choice of the high rate sequences, namely, the signature sequences, on which the
information data bits of different users is mapped. In CDMA, many asynchronous users occupy the same channel simultaneously. A desired user's receiver must be able to extract its signature sequence in the presence of other user's signature sequences. Therefore, a set of signature sequences that are distinguishable from time shifted versions of themselves and for which any two such signature sequences are easily distinguishable from (a possible time shifted version of) each other is needed. Let the two periodic signals $x(t)$ and $y(t)$ be written as

$$ x(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} x_n P_{T_c}(t-nT_c) \quad (1) $$

and

$$ y(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} y_n P_{T_c}(t-nT_c) \quad (2) $$

where $P_{T_c}(t)$ is a unit rectangular pulse of duration $T_c$. For $x(t) = x(t + T)$ and $y(t) = y(t + T)$ for all $t$, the sequences $(x_n)$ and $(y_n)$ are distinct sequences with period $F = T/T_c$. For some value of $\tau$ such that for $0 \leq \tau \leq T$ and $\tau = IT$, where $I$ is an integer that can take on any integer value between $(0, F - 1)$, the sequence design problem posed in conditions C1 and C2 in Section I reduces to constructing sequences that satisfy the following two conditions.

1) For any sequence $x = (x_n)$ in the set

$$ |Z_{x,l}(l)| = \left| \sum_{n=0}^{N-1} x_n x_{n+l} \right| = \begin{cases} K & \text{for } I = 0 \\ \lambda_x & \text{for } 1 \leq I \leq F - 1 \end{cases} \quad (3) $$

2) For each pair of sequences $x = (x_n)$ and $y = (y_n)$

$$ |Z_{x,y,l}(l)| = \left| \sum_{n=0}^{N-1} x_n y_{n+l} \right| \leq \lambda_c \quad \text{for } 0 \leq I \leq F - 1 \quad (4) $$

Here $K$, $\lambda_x$, and $\lambda_c$ are constants. Strict orthogonality would require that $\lambda_x = \lambda_c = 0$. Here, a sequence is defined to be orthogonal with respect to its shifted version if $\lambda_x$ takes on its minimal value and two sequences are considered to be orthogonal (if $\lambda_c$) takes on its minimal value.

The design of sequences with the above mentioned properties for communications systems, such as spread-spectrum code division multiple access, ranging systems, radar systems, etc., has been a topic of interest to many communications scientists and mathematicians in the last two decades [6].

Typical fiber-optic communications systems and fiber optic signal processors today are modeled as positive systems [10], [11]. A positive system is one that cannot manipulate its signals to add to zero. Therefore, codes or sequences that satisfy the above two conditions based on $+1/-1$ signals would not necessarily maintain those properties once considered for positive systems because the construction of these signals takes advantage of the ability to add the signals to zero. Because fiber-optic signal processing today is equivalent to power measurement (incoherent), and since power is nonnegative, such signals cannot be optically manipulated to add to zero with other induced optical power if coherent interference effects are eliminated. Hence, there is a need for a new class of signature sequences that we call "optical orthogonal codes" for positive systems or specifically for FO-CDMA.

In general, an $(F, K, \lambda_x, \lambda_c)$ optical orthogonal code $C$ is a family of $(0, 1)$ sequences of length $F$ and weight $K$ with auto- and crosscorrelation constraints $\lambda_x$ and $\lambda_c$. In this case, $K$ is the number of 1's in the sequence. From now on, consideration is given to those families of OOC's for which their auto- and crosscorrelation constraints $\lambda_x$ and $\lambda_c$ are equal to one.

The principles of OOC's through the use of an example is introduced. The result of this example will be used to generalize and define new classes of codes. In this example, two OOC's, $A$ and $B$, as in Fig. 3(a) and (b), with length 32 and weight 4, i.e., $F = 32$ and $K = 4$, such that $\lambda_x = \lambda_c = 1$ are constructed. Note that, in this example, $T_c$, the time associated with one bit of information, is divided into 32 equal $T_c$ (chip times), i.e., $F = T/T_c = 32$.

OOC $A$ is represented by placing a chip waveform at the 1st, 10th, 13th, and 28th chip positions and OOC $B$ is represented by placing a chip waveform at the 1st, 5th, 12th, and 31st chip positions. The "fundamental rules" that ensure OOC's $A$ and $B$ to have periodic autocorrelation peak $(K)$ and low periodic correlation (at $\lambda_c$) at any other time shifts, with $T_c$ as the unit of time shift, can best be explained from a simple set theory. The two codes, $A$ and $B$, are represented by their equivalent sets

$$ A = \{ \tau_1^A, \tau_2^A, \tau_3^A, \tau_4^A \} \quad (5) $$

and

$$ B = \{ \tau_1^B, \tau_2^B, \tau_3^B, \tau_4^B \} \quad (6) $$

where $\tau_i^A$ is the relative delay between the beginning of the first pulse to the beginning of the second pulse and $\tau_i^A, \tau_i^B, \tau_i^B$ are the relative delays between the 2nd and 3rd, 3rd and 4th, 1st and 4th of the periodic sequence [Fig. 3(a)]. Similarly, $\tau_i^B, \tau_i^B, \tau_i^B$, and $\tau_i^B$ are the relative delays, as defined above, for code $B$ [Fig. 3(b)].

An extended set of set $A$, $A_{EXT}$, with all linear combinations of jointly connected relative delays of different lengths is constructed by the following.

Step 1: Let the first 4, i.e., $K$, delay elements of the set $A_{EXT}$ be the delay elements of set $A$ in (5), i.e., $\tau_1^A, \tau_2^A, \tau_3^A, \tau_4^A$.

Step 2: Let the next 4, i.e., $K$, delay elements of the set $A_{EXT}$ be the sum of all connected delay elements of size two, i.e., $\tau_1^A + \tau_2^A, \tau_2^A + \tau_3^A, \tau_3^A + \tau_4^A, \tau_4^A + \tau_1^A$ for periodic sequence $A$. Note that the sum of disjoint delay elements $\tau_1^A + \tau_4^A$ and $\tau_2^A + \tau_3^A$ are not elements of the set $A_{EXT}$ because they can never occur independently.

Step $(K - 1)$: Let the $(K - 1)$th (e.g., 3rd for this example) group of $K$, i.e., (4 for this example) delay elements...
of the set $A_{\text{EXT}}$ be the sum of all connected delay elements of
size $(K - 1)$ [e.g., 3 for this example], i.e., $\tau_1^4 + \tau_2^4 + \tau_3^4 + \tau_4^4 + \tau_5^4 + \tau_6^4 + \tau_7^4 + \tau_8^4 + \tau_9^4 + \tau_1^4 + \tau_2^4$ for periodic sequence $A$.

The delay element of size four (K), $\tau_1^4 + \tau_2^4 + \tau_3^4 + \tau_4^4 = T$ is not an element of the set $A_{\text{EXT}}$. Therefore, the set $A_{\text{EXT}}$ can be written for this example as

$$A_{\text{EXT}} = \{\tau_1^4, \tau_2^4, \tau_3^4, \tau_4^4, \tau_4^4 + \tau_2^4, \tau_4^4 + \tau_3^4, \tau_4^4 + \tau_1^4, \tau_4^4 + \tau_1^4 + \tau_2^4, \tau_4^4 + \tau_1^4 + \tau_3^4, \tau_4^4 + \tau_1^4 + \tau_2^4 + \tau_3^4, \tau_4^4 + \tau_1^4 + \tau_2^4 + \tau_3^4 + \tau_4^4, \tau_4^4 + \tau_1^4 + \tau_2^4 + \tau_3^4 + \tau_4^4 + \tau_1^4, \tau_4^4 + \tau_1^4 + \tau_2^4 + \tau_3^4 + \tau_4^4 + \tau_1^4 + \tau_2^4, \tau_4^4 + \tau_1^4 + \tau_2^4 + \tau_3^4 + \tau_4^4 + \tau_1^4 + \tau_2^4 + \tau_3^4}. \quad (7)
$$

It can be observed from (7) that the total number of elements in the set $A_{\text{EXT}}$, $|A_{\text{EXT}}|$, is 12.

In general, for an $(F, K, 1, 1)$ OOC, the construction of its extended set would have $(K - 1)$ steps with $K$ elements in each step. Therefore, the total number of elements in the extended set of an OOC, with $\lambda = \lambda_e = 1$, is $K(K - 1)$.

Similarly, one can construct the extended set of the periodic sequence $B$, $B_{\text{EXT}}$, as

$$B_{\text{EXT}} = \{\tau_1^B, \tau_2^B, \tau_3^B, \tau_4^B, \tau_4^B + \tau_2^B, \tau_4^B + \tau_3^B, \tau_4^B + \tau_1^B, \tau_4^B + \tau_1^B + \tau_2^B, \tau_4^B + \tau_1^B + \tau_3^B, \tau_4^B + \tau_1^B + \tau_2^B + \tau_3^B, \tau_4^B + \tau_1^B + \tau_2^B + \tau_3^B + \tau_4^B, \tau_4^B + \tau_1^B + \tau_2^B + \tau_3^B + \tau_4^B + \tau_1^B, \tau_4^B + \tau_1^B + \tau_2^B + \tau_3^B + \tau_4^B + \tau_1^B + \tau_2^B, \tau_4^B + \tau_1^B + \tau_2^B + \tau_3^B + \tau_4^B + \tau_1^B + \tau_2^B + \tau_3^4}. \quad (8)
$$

Note that $|B_{\text{EXT}}| = 12$.

For the sequence or code $A$ to satisfy the periodic autocorrelation property, with $\lambda_e = 1$, there can be no repeated delay elements in the set $A_{\text{EXT}}$, i.e., no two or more elements of the set $A_{\text{EXT}}$ can be equal. Similarly, for code $B$ to satisfy the periodic autocorrelation property for $\lambda_e = 1$, there can be no repeated delay elements in the set $B_{\text{EXT}}$.

It is desired to construct two sequences, $A$ and $B$, for which, in addition to their periodic autocorrelation property, they satisfy the periodic crosscorrelation property of (4). For example, to guarantee the periodic crosscorrelation property for the two sequences, $A$ and $B$, with equal length $F$ and equal weights $(K)$ and $\lambda_e = 1$, the intersection of their extended sets must be empty, that is

$$A_{\text{EXT}} \cap B_{\text{EXT}} = \phi \quad (9)
$$

where $\phi$ denotes the empty set.

From Fig. 3(a) and (b)

$$A = \{9, 3, 15, 5\} \quad (10)
$$

and

$$B = \{4, 7, 19, 2\}. \quad (11)
$$

Furthermore, from (7) and (8)

$$A_{\text{EXT}} = \{9, 3, 15, 5, 12, 18, 20, 14, 27, 23, 29, 17\} \quad (12)
$$

and

$$B_{\text{EXT}} = \{4, 7, 19, 2, 11, 26, 21, 6, 30, 28, 25, 13\} \quad (13)
$$

From (12) no element of the set $A_{\text{EXT}}$ is repeated twice or more. Similarly, from (13), no element of the set $B_{\text{EXT}}$ is repeated twice or more. This condition guarantees that codes $A$ and $B$ will satisfy the autocorrelation property for $K = 4$ and $\lambda_e = 1$. Furthermore, there is not a common element between the two sets $A_{\text{EXT}}$ and $B_{\text{EXT}}$ of (12) and (13). Therefore, $A_{\text{EXT}} \cap B_{\text{EXT}} = \phi$. This condition guarantees the crosscorrelation property between the two codes $A$ and $B$ for $\lambda_e = 1$.

In general, for a given integer number $F$ (code length) and weight $K$ where $K(K - 1) \leq F - 1$ and $\lambda_a = \lambda_e = 1$, one can construct at most $N$ OOC's, $A'$, such that their corresponding extended sets, $A'_{\text{EXT}}$, for $1 \leq i \leq N$, have no repeated elements and

$$|A'_{\text{EXT}}| = K(K - 1) \quad (14)
$$

where $|A_{\text{EXT}}|$ is the total number of elements in the $i$th extended set and

$$A'_{\text{EXT}} \cap A'_{\text{EXT}} = \phi \quad (15)
$$

for all $1 \leq i, j \leq N$ and $i \neq j$. From (14) and (15) the value of $N$ is upper bounded by

$$N \leq \left\lfloor \frac{F - 1}{K(K - 1)} \right\rfloor \quad (16)
$$

where the symbol $\lfloor x \rfloor$ denotes the integer portion of the real value $x$. Using the above fundamental rules for constructing families of OOC's, there are many methodologies in their design and analysis. In [12] many different techniques (e.g., projective geometry, the greedy algorithm, iterative constructions, algebraic coding theory, block design, and various combinatorial techniques) for constructing families of OOC's have been extensively studied.

III. FO-CDMA EXPERIMENT USING OPTICAL ORTHOGONAL CODES

An experiment was carried out to demonstrate the principles of OOC's and their use in FO-CDMA. In this experiment, the output of each user's data source was mapped into an OOC by means of electronically programmable code generators (encoders). The outputs of the electronic encoders were used to modulate two semiconductor lasers and the output of each laser was coupled into a single mode fiber. The output of each single mode fiber was multiplexed into a common channel (another single mode fiber) by a passive directional optical coupler. The FO-CDMA signal in the common channel can be expressed as

$$r(t) = \sum_{i=1}^{2} s_i(t - \tau_i) \quad (17)
$$

where $s_i(t - \tau_i)$ corresponds to the $i$th user's signal, and $\tau_i$ represents the random time delay associated with the $i$th signal. In this experiment, user number one is denoted as the desired user. In FO-CDMA a binary ‘11’ is represented by transmitting a sequence and a binary ‘00’ is represented by transmitting no sequence. The OOC’s $A' = \{12, 22, 42, 52, 213\}$ and $B' = \{18, 26, 54, 70, 173\}$ were used for the users 1 and 2. In this experiment, we used a fiber optic tapped-delay line [13] (FO-TDL) for code or sequence convolution. For a given sequence, one can design a FO-TDL with an impulse response equivalent to the time reversal of the sequence. The output of a FO-TDL to its sequence is a realizable convolution sum that has a maximum at the sequence frame time (correlation time). The output of the FO-TDL is detected by a high-speed pin photodetector, amplified by series of wideband amplifiers, and displayed on an oscilloscope. Fig. 4(a) shows the response of a FO-TDL when the sequence $A'$ is the input. The maximum value at the output of the FO-TDL is 5 at the correlation time and one or zero at any other time, i.e., it obeys the autocorrelation property, (3), for $K = 5$ and $\lambda_a = 1$. Fig. 4(b) shows the output of a FO-TDL designed for sequence $A'$ when sequence $B'$ is an input. Here the output of FO-TDL takes on two values, namely, zero or one at all times, i.e., the crosscorrelation property, (4), for $\lambda_e = 1$. Fig. 4(c) is the output of FO-TDL when two sequences, $A'$ and $B'$, are present with a relative, random, time delay. It is clear that in the absence of quantum and thermal noise, the desired sequence [in this case, sequence $(A')^3$] can be extracted in the
Fig. 4. The response of optical decoder to (a) OOC A, (b) OOC B, (c) sum of the two OOC’s A and B (A + B) with a relative, random, time shift with respect to each other, and (d) over three bits of information. User 1 transmits the information bits 101 while user 2 transmits the information bits 010.

In FO-CDMA with N users, an error occurs when the desired user transmits a binary “0” that corresponds to sending no sequence for on-off keying, and the interference due to the other N – 1 users’ signals (multiple access interference) cause a false detection. If the total number of interfering signals, i.e., N – 1, is less than the weight K of their corresponding OOC’s (with \( \lambda_e = 1 \)), then one can choose a threshold level \( Th \) greater than N – 1 so that the error associated with the multiple access interference is zero. For example, in the absence of quantum and thermal noise, if \( Th > 1 \) and \( N = 2 \) and \( K = 5 \), an error free FO-CDMA system can be obtained.

In FO-CDMA systems where the number of interfering users is greater than or equal to the weight \( K \), i.e., \( N - 1 \geq K \), then errors due to the interfering signals occur with some probability. For the error analysis of FO-CDMA systems using OOC’s, which is the emphasis of Part II of this paper [14], the probability density functions associated with this multiple-access interference signal must be known. In Section IV, the probability density function for the simple case of two interfering OOC’s with \( \lambda_e = \lambda_e = 1 \) will be derived. This result will be used to evaluate the probability density function for \( N - 1 \) interfering OOC’s in FO-CDMA systems [14].

IV. Probability Density Function for Two Interfering Optical Orthogonal Codes

To visualize the mathematical development of the probability density function associated with any two interfering OOC’s, the concept of optical disk patterns is introduced. For example, the real time periodic OOC A, Fig. 3(a), can be represented as the optical disk pattern shown in Fig. 5(a). There is a one-to-one mapping of the real time periodic OOC’s to optical disk patterns. From Figs. 3(a) and 5(a), the time associated with one bit of information, i.e., \( T \), seconds, corresponds to the perimeter of the disk, and each chip time \( (T_c) \) corresponds to a sector such that \( 2\pi/\Theta = T/T_c \), \( F \) where \( \Theta \) is the phase associated with each sector of the disk. There is a direct correlation between the relative delays between the positions of the pulses in the time domain and the relative phases between the marked sectors. That is \( \tau_i \rightarrow \phi_i \) for \( 1 \leq i \leq K \) where \( \phi_i \) is the angle between the beginning of the \( i \)th mark and the beginning of the \((i + 1)\)th mark. To visualize the periodic autocorrelation of optical disk patterns, i.e., OOC’s, let a new disk, Fig. 5(b), be the complement of the original disk of Fig. 5(a). Now, assume that this disk is placed in a fixed position with a light source underneath to project the disk pattern on a surface. In fact, if an exact replica of this disk is placed on top of the fixed position disk with zero phase difference, the transparent sectors (marks) that correspond to the sequence would fall on top of each other and the actual pattern would still be projected on the projection surface, Fig. 6(a). This corresponds to the peak value of the autocorrelation function with \( K \) transparent sectors overlap. As the upper disk rotates with some integer multiples of \( \Theta \) with respect to the fixed lower disk, the number of transparent sectors overlaps of the upper disk pattern with its lower replica disk is at most one for \( \Theta \) \( \neq 2\pi \) rotation where \( n = 0, 1, 2, \ldots \), Fig. 6(b). This is equivalent to the periodic autocorrelation property, (3), with \( K = 4 \) and \( \lambda_e = 1 \).

The real time OOC B [Fig. 3(b)] can be represented as the optical disk pattern shown in Fig. 7. If disk B is rotated with respect to disk A, by any integer multiple of \( \Theta \), then the maximum number of projected marks on the projection surface would be one (Fig. 8). This is equivalent to the periodic crosscorrelation function, (4), with \( \lambda_e = 1 \). The elements of the sets \( A, B, A_{ext}, \) and \( B_{ext} \) of (5), (6), (7), and (8) can be, equivalently, represented by their corresponding relative phase elements. Because of their equivalence, the probability density function associated with two interfering periodic OOC’s (e.g., A and B) is equivalent to the probability density function associated with their corresponding interfering optical disk patterns. For example, if one rotates disk B
Fig. 6. Demonstrating the autocorrelation value of optical disk pattern A, (a) peak value ($K$), (b) autocorrelation value with some shifts ($\lambda_n = 1$).

with respect to disk A, with some random integer multiples of $\Theta$, then the probability that a mark of disk B would exactly overlap a mark of disk A (each with $K$ marks and $F$ sectors) would be $K^2/F$. This is true because for each mark of disk B, there are $K$ possible marks of disk A where the overlaps could take place. Furthermore, with $K$ marks of disk B, there would be a total of $K^2$ possible overlaps. The probability of each individual overlap is equal to $1/F$, since the random phase rotation is uniformly distributed between $[0, 2\pi]$. With integer multiples of $\Theta$ as the random phase shift and with $K^2$ possible positions, the probability of an overlap would be $K^2/F$. The complement of this event, which corresponds to the probability of no overlap, is $1 - K^2/F$. Therefore, one can define for these events, a random variable $w$ with a probability density function $P_w(w)$ expressed as

$$P_w(w) = \left(1 - \frac{K^2}{F}\right) \delta(w) + \frac{K^2}{F} \delta(w - 1) \quad (18)$$

where $\delta$ denotes Dirac’s delta function. The mean and the variance for the above random variable $w$ are $M_w = K^2/F$ and $\sigma_w^2 = (K^2/F)(1 - K^2/F)$, respectively. Note that the probability density function $P_w(w)$ of (18) corresponds to the probability density function of two interfering OOC’s (e.g., A

Fig. 7. Optical disk pattern B.

Fig. 8. Demonstrating the crosscorrelation value between optical disk patterns A and B ($\lambda_n = 1$).
and B) with shifts that are integer multiples of $T$. This is the case of chip synchronous interference between two OCC’s.

To consider the case of on-off modulation of the sequence representing data transmission, a new random variable $u = dw$ is introduced where $w$ is defined as a random variable with a probability density function defined as in (18) and where $d$ is a random variable that takes on values 1 or 0 with equal probability and corresponds to the state of the light source itself whether it is on or off. Since the expected value of $d$ is 1/2, the probability density function $P_u(u)$ for random variable $u$ becomes

$$P_u(u) = \left(1 - \frac{K^2}{2F}\right) \delta(u) + \frac{K^2}{2F} \delta(u - 1)$$  \hspace{1cm} (19)$$

where its mean and variance are equal to $M_u = K^2/2F$ and $\sigma_u^2 = (K^2/2F)(1 - K^2/2F)$, respectively. Note that this corresponds to the chip synchronous interference between two OCC’s for which one is modulated by an on-off keying.

A less constrained variable $u'$ can be defined when the phase shifts are not integer multiples of $\Omega$ as in the case $u$. Here, the probability density function associated with the random variable $u'$ for the two particular optical disk patterns as in Fig. 5(b) and 7 can be shown to be by the methods that are described in Section V as

$$P_{u'}(u') = \frac{5}{8} \delta(u') + \frac{1}{8} \delta(u' - 1) + \frac{1}{4} \left\lceil \frac{1}{|v|} \right\rceil$$  \hspace{1cm} (20)$$

where $|x|$ is a unit rectangular pulse for $0 < x < 1$ and zero elsewhere. The mean $M_{u'} = 0.25$ and the variance $\sigma_{u'}^2 = 0.146$. The above random variable $u'$ corresponds to the interference of the two OCC’s A and B with time shifts that can take any value on between (0, T), i.e., chip asynchronous interference.

Note that $u'$ in the above case can take on the value 1 with a probability 1/8. This interference effect between two OCC’s occurs when some of their corresponding pulses are adjacent with respect to each other. If $K^2/F \leq 1/2$, then two OCC’s can be constructed that have no adjacent pulses with respect to each other and therefore their corresponding probability density function, $P_{u}(u)$, can be expressed as [see Section V]

$$P_u(u) = \left(1 - \frac{K^2}{F}\right) \delta(u) + \frac{K^2}{|v|}$$  \hspace{1cm} (21)$$

where $v$ is the same as $u'$, except that it corresponds to the mutual interference of two OCC’s for which no pulse of one code is adjacent to a pulse of the other code (no pulse adjacencies). The mean and variance for random variable $v$ are equal to $M_v = K^2/2F$ and $\sigma_v^2 = (K^2/2F)(1/3 - K^2/3F)$, respectively. If $K = 4$ and $F = 32$, then $M_v = M_{u'} = 0.25$ and $\sigma_{u'}^2 < \sigma_v^2 < \sigma_v^2$ where $\sigma_v^2 = 0.104$, $\sigma_{u'}^2 = 0.146$, $\sigma_v^2 = 0.187$.

In general, in a family of OCC’s of size $N$ with length $F$ and weight $K$ where $K^2/F \leq 2/3$ and $\lambda = \lambda = 1$, then [see Section V]

$$\sigma_{u'}^2 < \sigma_{u'}^2 < \sigma_v^2$$  \hspace{1cm} (22)$$

where $\sigma_v^2$ corresponds to the variance of interpolating i-th and j-th, $v_{ij}$ of OCC’s for all $1 \leq i, j \leq N$ and $i \neq j$. In a communications system the variance of an interference or noise signal is the measure of its strength, i.e., the larger the variance the stronger the interference, and it is desired to minimize or reduce this interference strength, i.e., variance. Therefore, from (22), one can deduce that the best designed families of OCC’s are those that do not have any adjacent pulses with respect to each other and the worst designed families are those for which all their corresponding pulses are adjacent. But, in a typical family of periodic OCC’s, some codes would have their pulses adjacent with the pulses of some other codes, some codes would have no adjacent pulses with the pulses of some other codes, and some codes would have only few of their pulses adjacent with the pulses of some other codes. In any case, a complete and exact knowledge for different values of $\sigma_v^2$ in a family of OCC’s with $N$ codes could require the knowledge of $N(N - 1)/2$ probability density functions. It is clear that for large $N$, the above task is lengthy and tedious. But (22) indicates that the variance of chip synchronous interference, $\sigma_{u'}^2$ is approximately equal to the variance of two OCC’s with their pulses are completely adjacent with respect to each other. The variance $\sigma_v^2$ corresponds to the variance of two OCC’s for which there is no adjacent pulse with respect to each other. One can argue that the chip synchronous interference $u'$ is a pessimistic approximation to the actual interference and the ideal chip asynchronous interference $u$ is an optimistic approximation to the actual interference $v_{ij}$ for all $1 \leq i, j \leq N$ and $i \neq j$. [see Section V].

For mathematical convenience, the probability density functions associated with the chip synchronous, (19), and ideal chip asynchronous, (21), will be used as the basis for evaluating the probability density function for $N - 1$ interfering OCC’s in [14].

V. NOTES ON INTERFERING OPTICAL ORTHOGONAL CODES

In this section some quantitative arguments on the probability density functions associated with two chip synchronous and ideal chip asynchronous interfering OCC’s and their use in evaluating the upper and lower bounds on the probability of error per bit are presented.

The approach taken to prove that the chip synchronous and ideal chip asynchronous interferences are the pessimistic and optimistic approximations to the actual interference is by constructing a family of OCC’s and showing that the above two cases are the extreme cases between any two interfering OCC’s in that family (generalization to any other family follows). The easiest family of OCC’s to construct systematically, the family with length $F$ (chips) and $\lambda = \lambda = 1$ and $K = 2$, will be selected. For the above families of OCC’s, the number of elements in their extended set is 2 and they are represented as

$$C_2^\perp = \{ (F - 1, 1), \ (F - 2, 2), \ \ldots, \ (F + 1, F) \}$$  \hspace{1cm} (23)$$

where $C_2^\perp$ denotes the family of OCC’s and $A_i = (F - i, i)$, for $1 \leq i \leq F/2 - 1$, denotes the ith code with $K = 2$ and $\lambda = \lambda = 1$ and, for which $F \geq 2$ is an even integer, and $|C_2^\perp| = F/2 - 1$ where $|\cdot|$ denotes the number (size) of OCC’s in the above families.

For example, for $F = 16$ then

$$C_2^\perp = \{ (15, 1), (14, 2), (13, 3), (12, 4), (11, 5), (10, 6), (9, 7) \}$$  \hspace{1cm} (24)$$

where $|C_2^\perp| = 7$.

For evaluating the probability density functions among some OCC’s, four OCC’s from (24) were first chosen, i.e., $A_1 = (15, 1), A_2 = (14, 2), A_3 = (13, 3)$, and $A_4 = (9, 7)$ [see Fig. 9]. Next, one code was chosen (e.g., $A_3$), as a reference code. Now, circular convolutions between the reference code $A_3$ with the other remaining three codes were done, i.e., $A_1 \oplus A_3, A_2 \oplus A_3, A_4 \oplus A_3$, where $\oplus$ denotes the circular convolution operation. The result of the above circular convolutions are shown in Fig. 10(a)-(c). Note that in these figures, the output of the circular convolutions do not exceed 1. This indicates that when a family of OCC’s are designed for a maximum of one overlap, i.e., $\lambda = 1$, no matter how any two of them are positioned with respect to
projection surface is 2/16 since only 2 chip intervals in Fig. 10(a) are either linearly increasing or linearly decreasing. Therefore, the probability density function for the two interfering OOC’s $A_1$ and $A_2$ can be expressed as

$$P_{i2j}(I_{2j}) = \frac{11}{16} b(I_{2j}) + \frac{3}{16} b(I_{2j} - 1) + \frac{2}{16} I_{2j}$$

(25)

where $I_{2j}$ denotes the random variable associated with the above interfering OOC’s and

$$[x] = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(26)

Similarly, one can express the probability density functions for the other two cases from Fig. 10(b) and (c) as

$$P_{i3j}(I_{3j}) = \frac{9}{16} b(I_{3j}) + \frac{1}{16} b(I_{3j} - 1) + \frac{6}{16} I_{3j}$$

(27)

and

$$P_{i2j}(I_{2j}) = \frac{8}{16} b(I_{2j}) + \frac{8}{16} I_{2j}$$

(28)

where $I_{2j}$ and $I_{2j}$ denote the random variables associated with the interfering OOC’s $A_2$ and $A_1$, respectively.

The assumption for chip synchronous interference is equivalent to Fig. 10(d) where 4, i.e., $K^2$, chip intervals have a value of one and the remaining 12, i.e., $F - K^2$, chip intervals have a value of zero. The probability density function associated with this case is expressed as

$$P_{iCS}(I_{CS}) = \left( 1 - \frac{K^2}{F} \right) b(I_{CS}) + \frac{K^2}{F} b(I_{CS} - 1)$$

(29)

where $I_{CS}$ denotes the random variable associated with chip synchronous interference. If $F = 16$ and $K = 2$ then $E(I_{2j}) = E(I_{3j}) = E(I_{2j}) = E(I_{CS}) = 0.25$, where $E$ denotes the ensemble average. Furthermore, the variances for the above random variables are $\sigma^2_{i2j} = 0.104$, $\sigma^2_{i3j} = 0.143$, $\sigma^2_{i2j} = 0.167$, $\sigma^2_{iCS} = 0.1875$. It is observed that

$$\sigma^2_{i2j} < \sigma^2_{i3j} < \sigma^2_{i2j} < \sigma^2_{iCS}.$$  

(30)

Since a variance of a random variable (interference) is the measure of its strength, then from (30) one can argue that the interference pattern created by interfering the two OOC’s $A_2$ and $A_1$, is the weakest interfering pattern. This “weakest” effect between the two interfering OOC’s $A_2$ and $A_1$ is because of the relative positions of their respective pulses (marks). If the relative delays between the positions of their pulses are sufficiently large, then when a pulse of one code begins to overlap (meet) with a pulse of the other code, and if the overlapping (meeting) continues for a duration of twice the chip time, then, within this duration, no other pulses at any other positions would overlap (meet) with each other. Furthermore, there is no other pulse within these intervals of overlaps. The above properties must be true for any pair of pulses of the codes $A_2$ and $A_1$. Therefore, OOC’s with the above properties create the weakest effect when they interfere with each other.

On the other hand, if two OOC’s have their pulses positioned so that when a pulse of one code begins to overlap (meet) with a pulse of another code, the overlapping (meeting) continues uniformly among all the other pulses for a duration of $K^2 - 1$ chip intervals. These classes of OOC’s create the strongest interfering patterns [see Fig. 10(a)]. In fact, chip synchronous interference, Fig. 10(d), which is considered only for its mathematical convenience, indicates an even
stronger interference with respect to the strongest interference pattern in a family of OOC’s. Any other interference pattern would have a variance between these extremes (weakest and strongest) cases. For example, Fig. 10(b) shows the interference pattern between the two codes and A₂ and A₃.

There are two cases for the above interference pattern (I₂). One case is when a pulse of one code begins to overlap (meet) with a pulse of another code, then during this overlapping (meeting) period, i.e., two chip time, other pulses at other positions could begin to meet, or there is another pulse within these two chip intervals. The other case is when a pulse of one code begins to overlap (meet) with a pulse of the other code, then during this overlapping period no other pulses at any other positions would overlap with each other, and, furthermore, there is no other pulse during these two chip intervals. Note that these kinds of interference patterns (effects) are mixtures of weakest and strongest interference patterns. To judge which effect is a more dominant effect would depend on which two OOC’s are interfering, and usually requires an exact knowledge of the actual interference patterns.

Generally, there are N(N − 1)/2 possible, distinct interference patterns in a family of OOC’s with N codes. For K > 2, the exact calculations for all the interference patterns, i.e., the probability density functions, are somewhat lengthy. But one can bound the effects, i.e., variances, (upper and lower) of these intermediate interference patterns by their extreme cases. It remains to be seen that for a given family of OOC’s with size N, length F, weight K, and λₙ = λₙ = 1, the variances of the intermediate interference patterns are indeed bounded by the weakest and the strongest interference patterns in that family. In fact, one can define a generalized interference pattern with a probability density function expressed as

\[ P_{I}(l) = q(I = 0) + p\delta(l - 1)(1 - p - q)\delta[l] \]

where I is a random variable associated with the above generalized interference pattern, and

\[ q = Pr(I = 0) \]

and

\[ p = Pr(I = 1) \]

and

\[ 1 - p - q = Pr(0 < I < 1) \]

where \( p + q \leq 1 \). Furthermore, the mean \( M_{I} \) and the variance \( \delta_{I}^{2} \) for the above random variable I are equal to

\[ M_{I} = \frac{1}{2}(1 - p - q) \]

and

\[ \delta_{I}^{2} = \frac{1}{2}(q + p) + \frac{1}{2}(1 - q) \]

The above probability density function, (31), is of interest only for those values of p and q for which there exists a family of OOC’s. For example, for \( p = 1 \), a family of OOC’s does not exist. Therefore, the values of p and q for the three cases of chip synchronous, strong interference chip asynchronous, and the weak interference chip asynchronous (ideal chip asynchronous) for which there exists at least one family of OOC’s (a family of OOC’s must contain at least two OOC’s, i.e., \( N \geq 2 \)) will be determined.

A. Chip Synchronous (An Idealized Case)

For all those values of p and q such that \( p + q = 1 \), (31) reduces to the probability density function of chip synchronous interfering OOC’s, Iₜₚ, with a mean \( M_{Iₜₚ} = p = K/F \) and a variance \( \delta_{Iₜₚ}^{2} = pq = (K/F)(1 - K/F) \). The variance, \( pq \), has a maximum value, 0.25, at \( p = q = 1/2 \), and this maximum value occurs when \( F = 2K² \). For example, if \( K = 2 \), then \( F = 8 \) and the three OOC’s that form a family are \{1, 7\}, \{2, 6\}, \{3, 5\}. If \( K = 3 \), then \( F = 18 \) and the two OOC’s that form a family are \{1, 4\}, \{2, 8\}, \{3, 6\}. And if \( K = 4 \), then \( F = 32 \) and the two OOC’s that form a family are the OOC’s A and B discussed in Section II. In fact it is easy to show that for \( K > 2 \) and \( p = q = 1/2 \), the maximum number of OOC’s in a family is limited by 2.

If the values of p, such that \( 0 < p < 1/2 \) or \( 1/2 < p < 1 \) and \( p + q = 1 \), then \( K₁ < F/2 \). If \( K₁ < F/2 \), i.e., \( p < 1/2 \), then \( N \geq 49 \). Therefore, for a FO-CDMA with many users, \( N > 2 \), one must choose \( p \), such that \( p = K₁/F < 1/2 \).

For all those values of p, such that \( 1/2 < p < 1 \), the availability of families of OOC’s diminishes. In fact, if \( p = K₁/F = (F - 1)/F < 1 \) then \( N \leq K/(K - 1) \). If \( K = 2 \), then \( N = 2 \) and if \( K > 2 \) then \( N < 2 \). Since a family of OOC’s must contain at least two OOC’s, it is clear that for \( K > 2 \), one cannot construct any family of OOC’s. The smallest family exists for \( K = 2, N = 2, \) and \( F = 5 \). Here, \( p = 0.8 \) and \( q = 0.2, \delta_{Iₜₚ} = 0.16 \) and the two OOC’s are \{1, 4\}, \{2, 3\} (see Fig. 11). Therefore, it can be concluded that for \( p = K₁/F \geq 0.8 \) or \( K₁/F \geq 0.8F \), one cannot construct any family of OOC’s other than with \( K = 2, N = 2, \) and \( F = 5 \). Note that for any values of p such that \( 0.8F < p < 1 \) and \( p + q = 1 \), one can construct at most one OOC with \( λₙ = 1 \).

B. Strong Chip Asynchronous Interference Pattern

The probability density function for the strongest chip asynchronous interference pattern Iₛ in a family of OOC’s with length F, weight K, and \( λₙ = λₙ = 1 \), is expressed as

\[ P_{Iₛ} = \left(1 - \frac{K₁²}{4F} \right)δ(Iₛ) + \left(\frac{K₁² - 1}{F} \right)δ(Iₛ - 1) + \frac{2}{F}δ[Iₛ]. \]

From (31e) and (32), \( q = 1 - (K₁² + 1)/F, p = (K₁² - 1)/F \) and \( 1 - p - q = 2F \). The above random variable Iₛ with a probability density function defined as in (32) has a mean \( M_{Iₛ} = K₁²/F \) and a variance \( \delta_{Iₛ}² = (K₁²/F)(1 - K₁²/F) - 1/3F \).

Note that the variance for the strong chip asynchronous pattern is always less than the variance of its corresponding chip synchronous interference pattern, i.e., \( \delta_{Iₜₚ}² \), by an amount of \( 1/3F \). For large values of F, the above difference becomes negligible. The largest value for \( 1/3F \) where there exists a family of OOC’s, occurs if \( F = 3, \) and \( 1/3F = 0.6667 \). This family of OOC’s was shown to be the smallest existing family. Therefore, mathematically, one can bound the above differences in variances between any strong chip asynchronous patterns and their corresponding chip synchronous patterns by}

\[ 0 \leq \delta_{Iₜₚ}² - \delta_{Iₛ}² \leq 0.6667. \]

C. Weak Chip Asynchronous Interference Pattern (Ideal Chip Asynchronous Interference)

The probability density function for an ideal chip asynchronous interference pattern Iₛ in a family of OOC’s with length F, weight K, and \( λₙ = λₙ = 1 \) is expressed as

\[ P_{Iₛ} = \delta(Iₛ) + (1 - q)δ[Iₛ]. \]

The random variable Iₛ has a mean \( M_{Iₛ} = 1 - q/F = K₁²/F \) and a variance \( \delta_{Iₛ}² = (1 - q)(1/3 - (1 - q)/4) = 2K₁²/F \).
Equation (34) is true for all those values of \( q \) such that whenever \( p = 0 \) then \( 0 < 2K^2/F \leq 1 \). This implies mathematically that weak interference patterns could exist only in those families of OOC’s such that \( K^2/F \leq 1/2 \). In those families of OOC’s where \( K^2 \ll F/2 \), there are many weak interference patterns. It can be shown that

\[ \sigma_{\text{ICD}}^2 - \sigma_{\text{ICW}}^2 = \frac{K^2}{2F} \]  

(35)

and

\[ \sigma_{\text{IS}}^2 - \sigma_{\text{IW}}^2 = \frac{K^2}{2F} - \frac{1}{2} \]  

(36)

Equations (35) and (36) show the difference between the variances of weak interference patterns with respect to their chip synchronous and strong chip asynchronous interference patterns, respectively.

The maximum values for (35) and (36) are achieved mathematically when \( K^2/F = 1/2 \) for (35) and (36). Then (35) and (36) can be shown to be bounded by

\[ 0 \leq \sigma_{\text{ICD}}^2 - \sigma_{\text{ICW}}^2 < 0.167 \]  

(37)

and

\[ 0 \leq \sigma_{\text{IS}}^2 - \sigma_{\text{IW}}^2 < 0.167. \]  

(38)

From (35) and (36), one can show that

\[ 0 < \sigma_{\text{ICW}}^2 < \sigma_{\text{IS}}^2 < \sigma_{\text{ICD}}^2 < 0.25. \]  

(39)

One can think of the strong chip asynchronous interference pattern as a special case of generalized interference pattern with \( 1 - p - q = 2F \). Therefore, (39) indicates that the variance of a generalized interference pattern is bounded by its corresponding chip synchronous and weak chip asynchronous (ideal chip asynchronous) pattern. To show that the above claim is true for all possible forms of generalized interference patterns, i.e., for all values of \( 1 - p - q \approx 2F \) such that \( p + q < 1 \), (31e) has been plotted in Fig. 12(a) and (b). Fig. 12(a) and (b) show the two dimensional variations of (31e) from different positions. Fig. 12(a) shows the value of (31e), i.e., \( \sigma^2 \), from the back and Fig. 12(b) shows it from the side. In both figures, there are three regions, i.e., Regions 1, 2, and 3. Region 1 is for those values of \( p \) for which there are no families of OOC’s (see Sections V-A and B). Region 2 is for those values of \( p \) where one can make the smallest size, i.e., 2, families of OOC’s with no weak interference pattern effect. Region 3, which is of interest for multiple access, is for those values of \( p \) where there are large size families of OOC’s with weak interference pattern effect. Fig. 12(b) shows Region 3 close up. The dashed line that starts from the beginning of Region 2 all the way down to Region 3 where \( q = 1.0 \), corresponds to the variance of those interference patterns with the values of \( p \) and \( q \) such that \( p + q = 1 \), i.e., chip synchronous. Note that this dashed line has a maximum value at \( p = q = 1/2 \) for which \( \sigma^2 = 0.25 \). The solid line that starts at \( p = q = 0 \) and ends at \( q = 1.0 \) corresponds to the variance of those interference patterns for which their probability density function is defined as in (31e) with \( p = 0 \) and \( q = 1 - 2K^2/F \), i.e., the weak chip asynchronous interference patterns.

The line that starts at \( p = q = 0 \) and ends at \( p = q = 1/2 \) corresponds to the variances of those interference patterns that have equal mean, i.e., 1/2 for this particular line, regardless of what form of interference patterns they are, i.e., chip synchronous, strong chip asynchronous, weak chip asynchronous, and all possible intermediate asynchronous patterns. Fig. 12(a) shows more of these lines for different values of \( q \). Note that in a family of OOC’s with length \( F \), weight \( K \), and size \( N \).
≥ 2, any interference pattern between any two members of that family has a mean equal to $K^r/F$. Therefore, from Fig. 12(a) and (b) one can see that the two cases of chip synchronous and weak chip asynchronous (ideal chip asynchronous) are the pessimistic and optimistic approximations of a given general interference pattern of a family of OOC’s with mean equal to $K^r/F$.

VI. Conclusion

In this paper, fiber-optic code division multiple access was examined, a technique in which low information data rates are mapped into very high rate address codes (signature sequences) for the purpose of achieving random, asynchronous communications free of network control, among many users. Specifically discussed was the need for a special class of signature sequences that can achieve the above multiple-access capability using fiber-optic signal processing techniques. A new class of signature sequences, which are called “optical orthogonal codes” was introduced, for which they satisfy auto- and crosscorrelation properties required for FO-CDMA.

These newly invented codes were used in an experiment to show the principles of FO-CDMA. In this experiment we demonstrated the auto- and crosscorrelation properties of this new class of codes. Furthermore, optical disk patterns were introduced as an equivalent way of representing optical orthogonal codes, and were used to demonstrate the properties of interfering optical orthogonal codes. Described also was an experiment in which the probability density functions for any two interfering OOC’s were developed.

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References


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