Influence of Fibre Dispersion on the Performance of Optical Fibre Code Division Multiple Access Systems

Paulo M. MENDES[1] and Henrique J. A. DA SILVA[2]
[1] Polytechnic Institute of Castelo Branco, Superior School of Technology
Av. do Empresário, P-6000 Castelo Branco
Phone: +351 39 796318; Fax: +351 39 796293; Email: pmendes@co.it.pt
[2]Institute of Telecommunications and Department of Electrical Engineering,
University of Coimbra - Pole II, Pinhal de Marrocos, P-3030 Coimbra, Portugal
Phone: +351 39 796248; Fax: +351 39 796293; E-mail: hjas@ci.uc.pt

Abstract

Numerical simulation is frequently used to analyse direct sequence code division multiple access (DS/CDMA) systems and understand the influence on their performance of some important parameters. With this purpose, a *Simulink/Matlab*® based simulator has been developed for performance assessment of a DS/CDMA optical network. Considerable research effort has already been dedicated to this subject by others, but very few published results take into account some parameters which may be critical in high speed optical networks, namely the fibre dispersion and the non-linear of the laser sources. The study described here uses the moment generating function with the saddle point approximation for performance evaluation of DS/CDMA systems without neglecting those impairments.

I. INTRODUCTION

CDMA has been proposed for use in optical networks in 1985 [1], and a considerable research effort has been focused on this multiple access technique during the last decade. Several authors have used various analytical and numerical methods to evaluate the performance of this kind of systems, such as combinatorial analysis [2], Gaussian approximation [3], moment generating functions (MGF) [4], or numerical simulation [5, 6]. However, in spite of the generalised use of numerical simulation as a powerful tool for system performance assessment, a serious effort to take into consideration critical system parameters has not been done, in the studies dedicated to CDMA optical networks reported so far.

In a DS/CDMA system, each transmitted symbol is multiplied by a code sequence [4]. The performance measures of an IM/DD CDMA fibre link are directly related to the code sequence used by each transmitter. The selection of candidate sets of pulse sequences is basically a problem of code design, and has already been rigorously formulated, as in [7].

Due to its main characteristics, relatively low information rate and asynchronous transmission, optical CDMA is especially suitable for LANs [3]. Therefore, the main objective of the study reported here was to develop a numerical simulation tool with the possibility of handling some system parameters which are usually neglected in the analysis of CDMA systems, such as the effect of fibre dispersion and attenuation, the effect of laser non-linear response, and the effect of the receiver filter, for the estimation of system performance.

Actually, the tool which we have developed uses a semi-analytical approach. Firstly, a simulation method is used to describe the transmission of the optical signal trough the network. After that, the MGF is used to describe the receiver statistics and to estimate the system performance. The MGF with saddle point approximation was chosen because it gives a very close approximation to the true probability of error [8].

II. SYSTEM MODEL

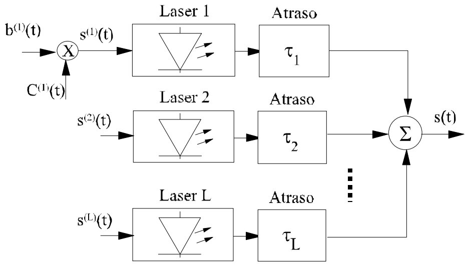

Fig. 1 - CDMA optical transmitter model.

In an optical DS/CDMA communication system, there are L simultaneous sources (users) transmitting data asynchronously (Fig. 1). So, each of them produces an NRZ intensity modulated signal. A $\{0, 1\}$-valued periodic spreading waveform $C^{(n)}(t)$, with period $T = NT_c$ and chip duration T_c, is assigned to the nth user, representing the message symbol 1. For the symbol 0 no code is generated [4]. If the laser sources are assumed to be non coherent, then the intensity of the signal received by each user is the sum of the intensities contributed by the L sources. Considering the usual star topology, the base band signal model generally considered in each branch of the network is:

$$s(t)=\sum_{n=1}^{L}\sum_{l=-\infty}^{\infty}\sum_{j=0}^{N-1} m_{l,j}^{(n)}\, p_{T_c}\left(t-lT-jT_c-\tau_n\right) \qquad (1)$$

where $m_{l,j}^{(n)}=c_j^{(n)}b_l^{(n)}$, $b_l^{(n)}$ is the lth equally probable data symbol of the nth user, $c_j^{(n)}$ is the jth chip of the lth data symbol, τ_n is the *n*th user delay, and p_{T_c} is the pulse shape function.

The light pulses emitted by the *L* transmitter lasers travel along the fibre and are collected by each optical receiver. At the end of the fibre, the light pulses are detected by an avalanche photodiode which generates primary electrons, which in turn produce secondary carriers by impact ionisation. If a light pulse corresponding to $m_{l,j}^{(n)} = 1$ was transmitted, the signal received by user 1 would generate primary electrons with rate [4]:

$$\lambda = \frac{\Lambda_1 h(t)}{T_c} \tag{2}$$

with

$$\Lambda_1 = \frac{R P_s T_c}{q} \tag{3}$$

where *R* is the photodiode responsivity (*A/W*), P_s is the signal average power received in the chip interval T_c, *q* is the electron charge, and *h(t)* is the normalised fibre impulse response. Therefore, Λ_1 is the average number of primary electrons generated during an interval of duration T_c if *h(t)* = 1.

When $c_j^{(n)} = 0$, a smaller number of primary electrons, $\eta\Lambda_1$ ($0 \le \eta < 1$), where η is the extinction ratio, is generated because of the need to bias the laser above threshold, for high speed transmission. Another contribution to the primary current are the electrons due to the dark current λ_d. The total rate of the primary electrons at the input of the APD is then:

$$\lambda(t) = \frac{\Lambda_1}{T_c} \sum_{n=1}^{L} \sum_{l=-\infty}^{\infty} \sum_{j=0}^{N-1} \lambda_{l,j}^{(n)} h(t - lT - jT_c - \tau_n) + \lambda_d \tag{4}$$

where:

$$\lambda_{l,j}^{(n)} = (1-\eta) m_{l,j}^{(n)} + \eta \tag{5}$$

Equations (4) and (5) can be used to obtain:

$$\begin{aligned} \lambda(t) = {} & (1-\eta)\frac{\Lambda_1}{T_c} \sum_{n=1}^{L} \sum_{l=-\infty}^{\infty} \sum_{j=0}^{N-1} m_{l,j}^{(n)} h(t - lT - jT_c - \tau_n) \\ & + \eta \frac{\Lambda_1}{T_c} \sum_{n=1}^{L} \sum_{l=-\infty}^{\infty} \sum_{j=0}^{N-1} m_{l,j}^{(n)} h(t - lT - jT_c - \tau_n) + \lambda_d \end{aligned} \tag{6}$$

Considering the statistical properties of the APD, Ho [4] obtained the following expression for the moment generating functions which describe the received signal *V*(*t*) (for symbols “1” and “0”):

$$\mu_{b_0^{(1)}}(s) = \exp\left\{\frac{1}{2}\sigma^2 s^2 + F_d D_0(s) + w b_0^{(1)} D_1(s)\right\} \prod_{n=2}^{L} \Gamma_n(s) \tag{7}$$

where *w* is the code height, $b_0^{(1)}$ is the symbol transmitted at t=0 by user 1 (chosen as the reference user), and σ^2 is the thermal noise variance.

To obtain the probability of error of the system we have to compute the following equation:

$$P_e = \frac{1}{2} f_0(V_{T_h}) + \frac{1}{2} f_1(V_{T_h}) \tag{8}$$

with:

$$\begin{aligned} f_0(V_{T_h}) &= \Pr\left[V(t_s) \ge V_{T_h} \;\middle|\; b_0^{(1)} = 0\right] \\ f_1(V_{T_h}) &= \Pr\left[V(t_s) < V_{T_h} \;\middle|\; b_0^{(1)} = 1\right] \end{aligned} \tag{9}$$

where f_i are the probability density functions associated with the detection process *V*(*t*) when the transmitter sends a *one* (*i* = 1)or a *zero* (*i* = 0), which may be obtained by:

$$f_k(V_{T_h}) = \int_{c_k} e^{\psi_k(s)} ds / 2\pi i, \qquad k = 0{,}1 \tag{10}$$

where

$$\psi_k(s) = \ln \mu_k(s) - sV_{T_h} - \ln\left[(-1)^k s\right] \tag{11}$$

Since the solution of equation (10) is non-trivial, an approximation is usually used. Ho [4] considered the Gaussian and the saddle point approximations, and he has shown that the Gaussian approximation cannot provide a good approximation to the probability of error in CDMA systems, but that the saddle point approximation produces a result very close to the true value.

The saddle point approximation for the probability density functions is obtained with the following equation:

$$f_k(V_{T_h}) = \frac{1}{\sqrt{2\pi \psi_k''(s_k)}} \exp\left[\psi_k(s_k)\right] \tag{12}$$

The saddle point is the solution of the equation:

$$\psi_k'(s) = 0 \tag{13}$$

with:

$$\psi_k'(s) = \left[\ln \mu_k(s)\right]' - V_{T_h} - s^{-1} \tag{14}$$

where the prime indicates derivation with respect to *s*.

The Newton method is used to solve equation (14). The calculation is iterated until the variation of the calculated value falls bellow 10^{-5}. An initial trial value is selected and at each stage of iteration we obtain a new trial value given by:

$$M \leftarrow M - \frac{\psi_k'(s)}{\psi_k''(s)} M'(s) \tag{15}$$

where the second derivative is given by:

$$\psi_k''(s) = \left[\ln \mu_k(s)\right]'' + s^{-2} \tag{16}$$

As it is well known, in IM/DD systems the threshold V_{T_h} may have influence on the probability of error. So, it is necessary to determine an optimum decision level, which may be obtained by solving:

$$f_0(V_{T_h}) = f_1(V_{T_h}) \tag{17}$$

or, equivalently, the equation:

$$R\left(V_{T_h}; \Lambda_s\right) = \ln f_0(V_{T_h}) - \ln f_1(V_{T_h}) \tag{18}$$

Ho [4] suggested the use of the secant method to solve this equation. At each stage, a trial value of V_{T_h} is obtained by the iteration:

$$V_{T_h} \leftarrow V_{T_h} - \frac{R\left(V_{T_h};\Lambda_s\right)}{R\left(V_{T_h}+\Delta V_{T_h};\Lambda_s\right) - R\left(V_{T_h};\Lambda_s\right)}\Delta V_{T_h} \quad (19)$$

where $\Lambda_s=(1-\eta\lambda_1)\Delta V_{T_h}$ is about $0.01 V_{T_h}$. The initial trial value used is:

$$V_{T_h} \approx \frac{m_0+m_1}{2} = G\Lambda_s w\left(m_d + m_I + \frac{1}{2}\right) \quad (20)$$

where m_0, m_1, m_d, and m_I are defined in [4].

The previous calculations allow us to obtain the system performance in the presence of an ideal rectangular pulse shape at the input of a particular receiver. However, we want to observe the influence of the optical fibre propagation characteristics on the system performance, and therefore the previous formulation is not adequate.

To include the fibre influence on the estimation of the probability of error, we have developed a numerical simulator (Fig. 2) which models the generation, propagation, and reception of the optical signal transmitted by a particular transmitter to a particular receiver.

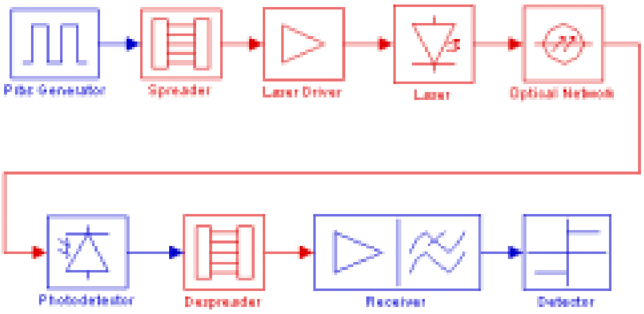

Fig 2 - Block Diagram of a fibre optic CDMA System.

The first block in the diagram is a pseudo random binary sequence (PRBS) generator. After this block we have considered the spreader, which is used to convert the incoming sequence of symbols into a coded chip sequence. This chip sequence is obtained by multiplying each information symbol by the user code [5], which is used to determine the laser drive current:

$$I(t) = I_b + \sum_{k=-\infty}^{\infty} A_k I_p(t-kT_c) \quad (21)$$

where I_b is the bias current, A_k denotes the coded sequence, $1/T_c$ is the chip rate and $I_p(t)$ is the applied current pulse. With this model we take also into account the exponential falling and rising edges of the modulating current.

The optical power response of the semiconductor laser to the current wave form $I(t)$ is determined by solving the large signal rate equations [9]. We have considered the possibility of using two kind of single frequency lasers, DFB or MQW [10]. The laser rate equations are solved with the 4th/5th order Runge-Kutta algorithm. The laser parameters considered where obtained from [9].

The optical network was modelled as a linear fibre channel. Therefore, we considered the usual low-pass equivalent model for the fibre frequency response.

The despreader is used only to introduce an attenuation factor in the calculations, since the method used to compute the probability of error, above described, is based on the chip sequence.

Finally we have considered a third-order Butterworth receiver filter, whit a bandwidth of 0.7xD, where D is the chip rate [11].

Since the performance of a IM/DD DS/CDMA system depends on the extinction ratio [4], we select the bias and modulating current in order to obtain a high extinction ratio. In fact, the signal distortion observed after the Butterworth receiver filter includes the influence of the extinction ratio. Therefore, to obtain the extinction ratio required by the method implemented to estimate the probability of error we used:

$$\eta = \frac{\frac{1}{T}\int_0^T \overline{C^{(n)}(t)}\, i^{(n)}(t)\,dt}{\frac{1}{T}\int_0^T C^{(n)}(t)\, i^{(n)}(t)\,dt} \quad (22)$$

where $C^{(n)}(t)$ is the code sequence for the particular user, $i^{(n)}(t)$ is the shape of the received signal, and $\overline{C^{(n)}(t)}$ is the complementary sequence of $C^{(n)}(t)$, that is, the sequence $\overline{C^{(n)}(t)}$ is formed by zeros where $C^{(n)}(t)$ have ones, and vice versa.

III. NUMERICAL RESULTS

To assess the system performance taking into consideration the influence of the source response and optical fibre propagation characteristics, we use the sequence of light pulses generated by driving the laser with a current wave form obtained by the multiplication of the data symbol by the selected code. This is propagated along the optical fibre, and processed by the optical receiver. The simulator has two purposes: one is to simulate the generation, transmission, and reception of the signal emitted by a particular source (user), the other is to compute the system performance in the presence of interfering signals and system impairments such as those above referred.

The simulator computes the received signal taking into account the effect of exponential rising and falling edges of the NRZ current driver, the real shape of the pulses emitted by the laser source, the fibre dispersion, and the transfer function of the optical receiver (modelled by a third-order Butterworth filter). The probability of error is estimated by solving equation (8).

A DFB laser with a chip rate of 10 Gchip/s and three codes, two prime codes with lengths 529 and 121 and an orthogonal optical code (OOC) with length 127, have been considered to obtain the results presented here. With these parameters, the system has an overall channel data rate of 18.9 Mbit/s, 82.6 Mbit/s, and 78.7 Mbit/s for each code, respectively. In order to have a high extinction ratio, the laser was considered as being polarised close to threshold. To determine the extinction ratio used in the estimation of error probability, we have used equation (22). Since the system is subjected to power loss mechanisms, namely coding and fibre attenuation, we have considered the possibility of several values for the received power. The receiver parameters used for simulation can be found in [4].

The results shown in Fig. 3 were obtained considering the system operating at chip rate of 10 Gchip/s, using a prime code of length 121, for three values of fibre length, L_f, and with 2 and 5 interfering users.

We considered also the case of using a prime code with length 529. As we can observe in Fig. 4, for the same received power we have a lower probability of error. The performance improvement is due to the increase of code length. Besides, the penalty due to the fibre propagation characteristics is less significant for this longer code. The reason for this is that, in the longer code, the relation between the code length and the chip number (code height) is larger. Therefore, the inter-chip interference (ICI) is less significant.

Another aspect that we may observe in these results is that the influence of fibre length depends on the number of interfering users.

We have also made a comparison of the performance obtained using an optical orthogonal code (OOC) [7], and a prime code with approximately the same length. This assumption guarantee that the chips are subjected to the same ICI. In the analysis we assumed an OOC with length 127 and a prime code with length 121. When we consider two interfering users in the system, we obtain the results shown in Fig. 5.

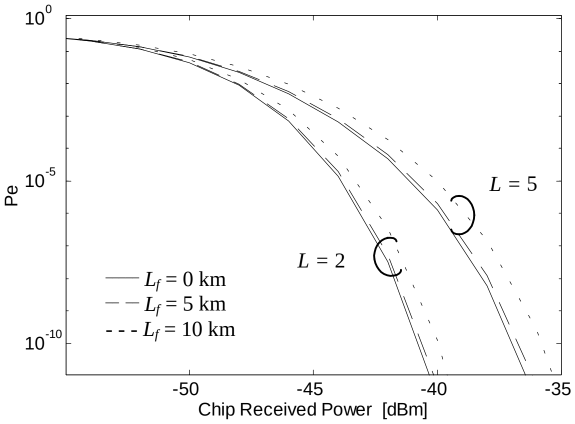

Fig. 3 - Prime code with length 121.

We can conclude that the prime code leads to a better performance and is more sensitive to the length of the optical fibre. The performance improvement in this case is due to the fact that we have considered the same power for each chip and the prime code has 11 chips, whereas the OOC has only 3 chip. Besides, due to the larger relation between length and chip number in the OOC, this code is less sensitive to the fibre length than the prime code.

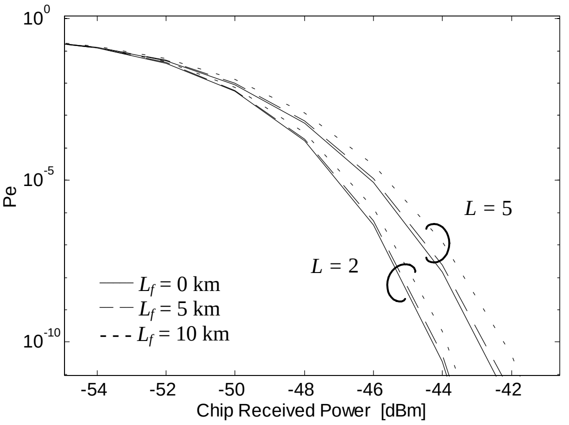

Fig. 4 - Prime code with length 529.

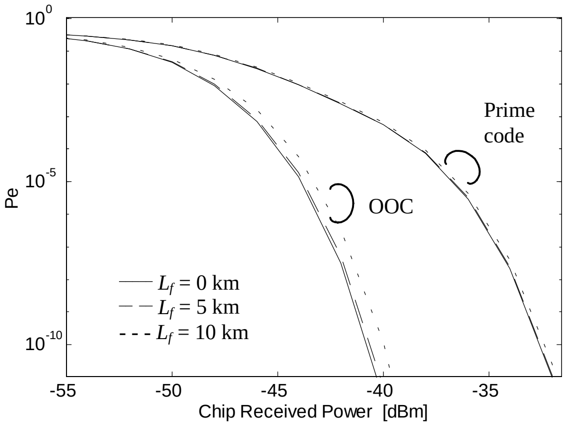

Fig. 5 - Comparison between an OOC with length 127 and a prime code with length 121.

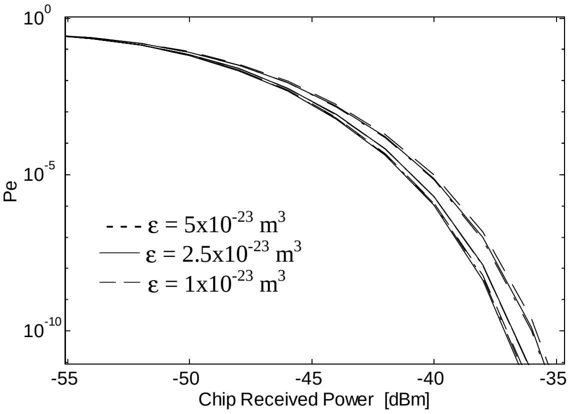

Fig. 6 – Probability of error for three values of the gain compression factor.

In the last case study presented here, we considered three different values for the laser gain compression factor, $\varepsilon = 5\mathrm{x}10^{-23}\ \mathrm{m}^3$, $\varepsilon = 2.5\mathrm{x}10^{-23}\ \mathrm{m}^{3,}$, and $\varepsilon = 5\mathrm{x}10^{-23}\ \mathrm{m}^3$. The results obtained are shown in Fig. 6, for a system using a prime code with length 121. As we can see, this parameter can influence the system performance, but not significantly.

IV. CONCLUSION

We have applied a new methodology for performance evaluation of optical DS/CDMA systems: the combination of signal transmission simulation with the MGF method for estimation of the error probability (BER).

In earlier studies, as far as we know, only one of these strategies has been used: both the computation and characterisation of the received signal [5, 6], without estimation of BER, or the performance estimation assuming a given signal at the input of the receiver [1-4]. The latter used either the MGF or the Gaussian approximation methods.

A simulator has been developed which enables the study of many critical parameters of an optical CDMA system.

From our results, we can conclude that the available technology allows the implementation of an optical network, with acceptable values for the probability of error, where the number of simultaneous users may be limited by the performance of some critical subsystem parameters.

Usually, in the analysis of the codes used in DS/CDMA, the main objective is to find the code set with the best correlation properties. From our results, we may conclude that the codes used in this kind of systems must be selected taking into consideration not only their correlation properties, but also their dispersion tolerance. Besides, we have found that the dispersion tolerance depends on the relation between the code length and number of chips in the particular coding set used by the system. The influence of this effect increases with the chip rate.

V. ACNOWLEDGEMENT

This work was partially supported by JNICT, in the scope of Programme PRAXIS II.

VI. REFERENCES

[1] Mestdagh, D. J. G., *Fundamentals of Multiaccess Optical Fiber Networks*, Artech House Publishers, Cap. 10, pp. 341-353, 1995

[2] Salehi, J.A, Brackett, C. A.,“Code Division Multiple-Access Techniques in Optical Fiber Networks-Part II: Systems Performance Analysis”, *IEEE Trans. Commun*., Vol 37, No. 8, pp. 834-842, August 1989.

[3] Maric, S. V., *et al.* “Multimedia Transmission in Fiber-Optic LAN’s Using Optical CDMA”, *IEEE J. Lightwave Technol.*, Vol. 14, No. 10, pp. 2149-2153, October 1996.

[4] Ho, Chia-Lu, Wu, Cheng-yu, “Performance Analysis of CDMA Optical Communications Systems with Avalanche Photodiodes”, *IEEE J. Lightwave Technol.*, Vol. 12, No. 6, pp. 1062-1072, June 1994.

[5] Wu, Jyh-Horng, *et. al.*, “The Effect of Laser Chirping on Synchronous Fiber-Optic Code Division Multiple Access Networks”, *IEEE Photon. Tech. Lett.*, Vol. 5, No. 2, pp. 239-242, February 1993.

[6] Yao, X. Steve, *et. al.*, “Limitations on Peak Pulse Power, Pulse Width, and Coding Mask Misalignment in a Fiber-Optic Code-Division Multiple-Access System”, *IEEE J. Lightwave Technol.*, Vol. 11, No. 5/6, pp. 836-846, May/June 1993.

[7] Chung, F. R. K., *et al.*, “Optical Orthogonal Codes: Design, Analysis and Applications”, IEEE Trans. Inform. Theory, Vol. 35, No.3, pp. 595-604, May 1989.

[8] Nelson, Laurie B., Poor, Vincent H., “Performance of Multiuser Detection for Optical CDMA – Part I: Error Probabilities”, *IEEE Trans. Commun.*, Vol. 43, No. 11, pp. 2803-2811, November, 1995.

[9] Goulart, Paulo Alexandre dos Santos, *Optimização da Transmissão a 10 Gbit/s em Fibra Óptica Monomodo*, Tese de Mestrado, Instituto Superior Técnico, December 1996.

[10] Freire, Mário M., da Silva, Henrique J. A., “Performance Assessment of Dispersion Supported Transmission at 20 Gbit/s Using a Semiconductor Optical Booster Amplifier”, in Long-Haul, ATM and Multi-Media Networks, Networks and Optical Communications 1998 - Volume II (NOC’98), D. W. Faulkner and A.L. Harmer (Eds.), IOS Press, Amsterdam, pp. 228-235 1998.

[11] Kitayama, Ken-ichi, “Code Division Multiplexing Lightwave Networks Based upon Optical Code Conversion”, IEEE J. Select. Areas Commun., Vol. 16, No. 7, pp. 1309-1319, September 1998.