BER Evaluation for Phase and Polarization Diversity Optical Homodyne Receivers Using Noncoherent ASK and DPSK Demodulation

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***Abstract*—An analysis of the performance of phase diversity receivers using ASK and DPSK is done. Both $\{2 \times 2\}$ and $\{3 \times 3\}$ multiport receivers are investigated. Asymptotic methods are employed, to give estimates of the BER(SNR) dependence for each type of the receiver. The analysis favors the squarers as the demodulators for ASK whose performance approaches that of the ideal heterodyne detector in the limit of large SNR. The SNR necessary for the linear envelope detectors to obtain BER of 10^{-9} is 1.6 dB ($\{2 \times 2\}$) and 1.1 dB ($\{3 \times 3\}$) greater than that required by the ideal heterodyne detector when ASK is used. The corresponding values for the squarers are 0.25 dB ($\{2 \times 2\}$) and 0.4 dB ($\{3 \times 3\}$). A modification of the ASK ($\{3 \times 3\}$) receiver which cancels the local oscillator intensity noise is proposed. The values of SNR required for DPSK phase diversity receivers to obtain BER $= 10^{-9}$ are found to be 0.45 dB (the $\{2 \times 2\}$ receiver) and 0.7 dB (the $\{3 \times 3\}$ receiver) worse than that required by the ideal heterodyne PSK receiver.**

Receivers which comprise polarization and phase diversity techniques are also investigated for both ASK and DPSK. Their performance is independent of the polarization state of the received signal and the value of SNR required to obtain the BER of 10^{-9} is only a few tenths of a decibel greater than that needed by the phase diversity receivers.

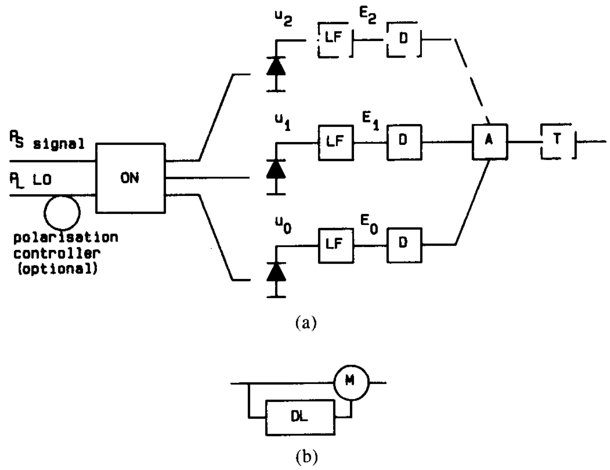

Fig. 1. (a) Multiport optical receiver: PC—polarization controller, ON—$\{2 \times 2\}$ or $\{3 \times 3\}$ optical network, LF—low pass (or matched) filter, D—demodulator, A—adder, T—threshold comparator. (b) DPSK demodulator: DL—delay by one bit line, M—multiplier.

I. Introduction

THE SENSITIVITY of homodyne optical coherent detection is offset by the difficulty of optical phase locking of two independent coherent sources. One solution to this problem uses a multiport optical network which provides a means for recovering the amplitude and phase of the optical signal without phase locking [1]–[3]. Fig. 1(a) shows a block diagram of such a receiver. It consists of a $\{2 \times 2\}$ or $\{3 \times 3\}$ optical network, photodetectors, low pass filters, demodulators (squarers or linear envelope detectors for ASK, and a circuit delaying one bit and a multiplier for DPSK—see Fig. 1(b)), an adder, and a threshold comparator. The $\{2 \times 2\}$ optical network is assumed to be a 90° optical hybrid [4]–[6] and that is not a standard $\{2 \times 2\}$ optical directional coupler as is in the case of the $\{3 \times 3\}$ device. An ideal $\{4 \times 4\}$ standard coupler [7] is needed to replace the optical hybrid. It is necessary to stress that the case of the $\{2 \times 2\}$ receiver includes also some kinds of $\{4 \times 4\}$ receivers [8], in which the photodiodes are cascaded to produce two quadrature components.

The ASK receivers are often claimed to have the same performance as the ideal heterodyne detector [3] or 1.8 dB worse than this for the $\{3 \times 3\}$ coupler and linear envelope detectors [2]. However, all these claims are based purely on the SNR considerations, disregarding completely the probability density function (pdf) of the process at the input of the threshold comparator. This pdf is definitely non-Gaussian and great care must be taken when using any Gaussian approximation or expressing the receiver performance by means of SNR.

In a conventional coherent optical receiver the polarization state of the local oscillator laser must be exactly the same as that of the received signal. This condition requires sophisticated polarization control schemes. A simpler solution is to use the polarization diversity technique [9]–[12] in which the input light is divided between two orthogonal polarization states. These are detected independently and then processed electronically.

In this paper we shall investigate polarization diversity

Manuscript received February 8, 1988; revised June 9, 1988.
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IEEE Log Number 8824814.

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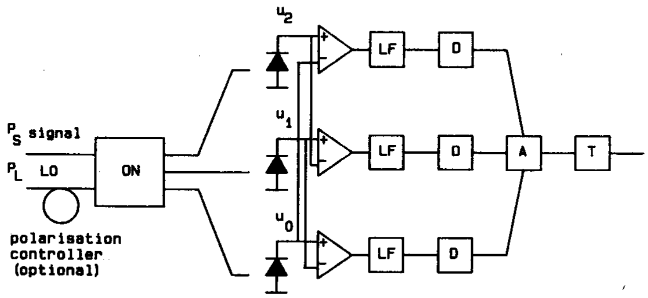

Fig. 2. Modified receiver: PC—polarization controller, ON—{ 3 × 3 } optical network, LF—low pass filter, D—squarer, A—adder, T—threshold comparator.

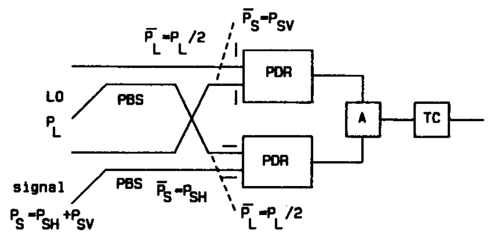

Fig. 3. Polarization diversity receiver: PBS—polarization beamsplitter, PDR—phase diversity receiver, A—adder, TC—threshold comparator.

receivers for ASK and DPSK which combine the advantages of both the polarization and phase diversity techniques. In the sequel we investigate two types of receivers:

1) phase diversity receivers in which the polarization states of the received and local oscillator signals are matched by means of some polarization control scheme, and
2) phase and polarization diversity receivers without such a scheme.

We will express the BER of these ASK and DPSK receivers by means of asymptotic approximations of the exact pdf's. We will estimate the relative errors of these approximations which do not exceed a few percent in the range of interest. We will also find the optimum threshold levels as a function of SNR for each ASK receiver. We shall propose a modification of the { 3 × 3 } ASK receiver which makes the suppression of the local oscillator intensity noise possible.

This receiver is presented in Fig. 2. It uses subtractors which are inserted between the squarers and the matched filters. Due to the equal distribution of the power in the { 3 × 3 } coupler, the cancellation of the LO intensity noise at the output of each subtractor is obtained.

We investigate ASK and DPSK under the assumption that the shot noise related to the local oscillator laser dominates other noise sources (for example the thermal noise). We assume that intersymbol interference is absent. The local oscillator intensity noise is not included in the analysis. Its influence on the receivers performance is considered elsewhere [13]. However, there are receiver structures, e.g., a { 4 × 4 } optical network with cascaded photodiodes [8], which suppress this kind of noise. For the sake of clarity some mathematical derivations are placed in appendices.

II. Receivers Preliminary

The analyzed polarization diversity receiver is shown in Fig. 3. It consists of two polarization beamsplitters [14], two ASK or DPSK phase diversity receivers of either { 2 × 2 } or { 3 × 3 } type (without the comparators and polarization controllers), an adder, and a threshold comparator. The ASK phase diversity receivers use squarers as the demodulators because we shall see that they give better performance. Both the signal and LO light are fed to polarization splitting couplers which produce orthogonal polarization components of the incoming light. These components are inputs to the two phase diversity receivers of any type. The polarization plane of linearly polarized LO light is chosen so that its power is equally divided between two orthogonal polarization states. The outputs of the two multiport receivers are then added. The first who proposed the polarization diversity receiver were Glance [11], Okoshi [12], and Kuwahara [15].

The signals at the outputs of the photodetectors of either phase diversity receiver are [2], [3]:

$$u_k = R\sqrt{\bar{P}_S\bar{P}_L}\, b(t) \cos\left[\alpha(t) + k\pi/2\right], \qquad k = 0, 1, \tag{1}$$

$$u_k = (2/3) R\sqrt{\bar{P}_S\bar{P}_L}\, b(t) \cos\left[\alpha(t) + k(2/3)\pi\right], \qquad k = 0, 1, 2. \tag{2}$$

Equation (1) holds for the { 2 × 2 } and (2) for the { 3 × 3 } receiver. Here R is the responsivity of the photodetectors

$$R = \eta q / h\nu \tag{3}$$

where η is the quantum efficiency, q is the electron charge, h is the Planck constant, and ν is the light frequency.

$\alpha(t)$ stands for the phase noises of both the local oscillator and the transmitter and for the frequency offset between them. For the phase diversity receivers with the polarization control scheme, $\bar{P}_S$ denotes the power of the received signal P_S and $\bar{P}_L$ denotes the power of the local oscillator P_L. In the case of the polarization and phase diversity receiver $\bar{P}_S$ denote either the power of horizontally P_{SH} or the power of vertically P_{SV} polarized component of the signal, depending on which of two parts of the phase diversity receiver is being considered. $\bar{P}_L$ denotes half of the LO signal power. We have $P_S = P_{SV} + P_{SH}$.

In (1) and (2), $b(t)$ is defined as a digital baseband signal, i.e.,

$$b(t) = \sum_k b_k \operatorname{rect}(t - kT_0). \tag{4}$$

$\{b_k\}$ is the symbol sequence, taking on the values +1 or −1 for DPSK, and 0 or 1 for ASK. The function rect (t)

is equal to 1 if $t \in (0, T_0)$ and 0 elsewhere. Here T_0 is the bit duration.

The shot noises at the outputs of the photodetectors are independent, it is reasonable to assume them to be Gaussian. They have spectral power densities

$$N_2 = qR\overline{P}_L/2, \quad (-\infty < f < +\infty), \tag{5}$$

$$N_3 = qR\overline{P}_L/3, \quad (-\infty < f < +\infty) \tag{6}$$

when $\overline{P}_L >> \overline{P}_S$ as it should. Here N_2 corresponds to the $\{2 \times 2\}$ receiver, and N_3 to the $\{3 \times 3\}$ receiver, and q is again the electron charge.

In the case of ASK, the low pass filters, which follow the photodetectors, are assumed to pass all the signal power and suppress the intersymbol interference. When the laser linewidth is much smaller than the bit rate, the filter matched to the signal rect (t) may be applied. Its impulse response is

$$h(t) = \begin{cases} 1/T_0, & \text{for } 0 < t < T_0 \\ 0, & \text{elsewhere.} \end{cases} \tag{7}$$

It is well known that DPSK is effective only when the linewidths of both the transmitter and local oscillator lasers are substantially smaller than the bit rate $1/T_0$ [16]. We can then ignore the IF instability and take it to be equal to 0 Hz. In this case the low pass filters which follow the photodetectors may be replaced by filters matched to the signal rect (t).

Under the above assumptions the maximum value of the signal at the filter's output is for both modulations given by $(\alpha = \alpha(t))$:

$$E_k = R\sqrt{\overline{P}_L\overline{P}_S} \cdot b \cos[\alpha + k\pi/2], \quad k = 0, 1 \tag{8}$$

for the $\{2 \times 2\}$ receiver and

$$E_k = (2/3)R\sqrt{\overline{P}_L\overline{P}_S} \cdot b \cos[\alpha + k(2/3)\pi], \quad k = 0, 1, 2 \tag{9}$$

for the $\{3 \times 3\}$ receiver. Here $b = 0$ or 1 for ASK, and $b = -1$ or $+1$ for DPSK. The noise power at the output of each filter is given by

$$i_k^2 = N_2 B = qR\overline{P}_L B/2, \quad k = 0, 1 \tag{10}$$

for the $\{2 \times 2\}$ receiver and

$$i_k^2 = N_3 B = qR\overline{P}_L B/3, \quad k = 0, 1, 2 \tag{11}$$

for the $\{3 \times 3\}$ receiver. Here B is the noise bandwidth of the low pass filter which is equal to the integral of the spectral response of the filter. For the filter (7), it is given by

$$B = 1/T_0. \tag{12}$$

The following analysis will be simpler if we normalize the signal and the noise to unity noise variance in each channel. This does not influence the BER characteristics since it is equivalent to division by a constant. After normalization the noise has unity variance in each channel. The values of the signals are then given by

$$s_k = \sqrt{2\,\mathrm{SNR}'}b \cos[\alpha + k\pi/2], \quad k = 0, 1. \tag{13}$$

for the $\{2 \times 2\}$ receiver and

$$s_k = (2/\sqrt{3})\sqrt{\mathrm{SNR}'}b \cos[\alpha + (2/3)k\pi], \quad k = 0, 1, 2. \tag{14}$$

Here $\mathrm{SNR}' = R\overline{P}_S/qB$. This parameter has the meaning of signal to noise ratio. Let us denote $\mathrm{SNR} = RP_S/qB$. For any phase diversity receiver with the polarization control scheme we have $\mathrm{SNR} = \mathrm{SNR}'$ as in this case $\overline{P}_S = P_S$ and this value should be inserted into (13) and (14) instead of SNR′. In the case of the polarization diversity receiver, SNR′ expresses the signal to noise ratio for each polarization component of the signal. In the case of the matched filter (7) $B = 1/T_0$, the value of SNR may be treated as the number of electrons for one bit when $b = 1$. When $b = 0$ (as for ASK) is also included, the average number of electrons per bit is two times less than SNR.

We have for any type for the phase diversity receiver from (13) and (14)

$$\sum_k s_k^2 = 2\,\mathrm{SNR} \tag{15}$$

and for any type of the phase and polarization diversity receiver

$$\sum_k s_{kV}^2 + \sum_k s_{kH}^2 = 2\,\mathrm{SNR}. \tag{16}$$

Here the subscripts V, H denote the signals with vertical and horizontal polarization, respectively. Equations (15) and (16) are valid for both ASK and DPSK.

Since the following nonlinear part of the ASK receiver is memoryless and the noises at the outputs of the filters are Gaussian, (13) and (14) are sufficient to find the probability density function of the noise at the input of the threshold comparator. Equations (13) and (14) are also sufficient to find this pdf for DPSK. Note that in this case the noises at the two inputs of any multiplier are independent as one of them results from integrating a white shot noise during the interval $(0, T_0)$ whereas the second results from integrating the same noise during the interval $(-T_0, 0)$.

If $b = 1$ and $b = 0$ $(b = -1)$ have equal probabilities the BER is determined by

$$\mathrm{BER} = 0.5\big(P(a > T \mid b = 0\ (b = -1)) + P(a < T \mid b = 1)\big). \tag{17}$$

Here a is the realization of the process at the input of the threshold comparator at the moment of sampling, T is the threshold, subject of optimization, $p_0 = P(a > T \mid b = 0\ (b = -1))$ is the probability that $a > T$ when $b = 0$ $(b = -1)$ is sent, and $p_1 = P(a < T \mid b = 1)$ is the probability that $a < T$ when $b = 1$ is sent. In the following paragraphs we will find these probabilities for all types of the receivers.

III. Error Probability of ASK Receivers

A. {2 × 2} Phase Diversity Receiver with Squarers

In the case when $b = 0$ ("0" message sent) the pdf of the process at the input of the threshold comparator is that of the sum of two squared Gaussian variables. When these variables are independent (they are in our case) and have unit variances (which also holds after normalization) the resulting pdf is chi-square with two degrees of freedom [17]:

$$p(z) = \exp(-z/2)/2. \tag{18}$$

We assume that $b = 0$ when the signal at the input of the threshold comparator is less than the threshold at the moment of sampling, and $b = 1$ when it is greater than the threshold. Thus the probability of error when $b = 0$ was sent is equal to the probability that the realization of the process exceeded the threshold. If the threshold was set at $2k^2$ SNR (k is a parameter) it is given by

$$p_0 = \int_{2k^2\mathrm{SNR}}^{\infty} \exp(-z/2)/2\, dz = \exp(-k^2\,\mathrm{SNR}). \tag{19}$$

When $b = 1$ the probability of error p_1 for the same threshold T is equal to the probability that the realization of the process is less than T. Due to the structure of the receiver this realization is $x^2 + y^2$, where $x = s_0 + i_0$ and $y = s_1 + i_1$ are independent and jointly Gaussian with unit variances [17].

The asymptotic expression for this probability is computed in Appendix A and it is given by

$$p_1 = \sqrt{k} \exp[-(1-k)^2\,\mathrm{SNR}]\Big\{1 - \big(1/(2(1-k)^2) + 1/(4k(1-k))\big)/\mathrm{SNR}\Big\}/\Big\{2\sqrt{\pi\,\mathrm{SNR}}\,(1-k)\Big\}. \tag{20}$$

The relative error of this approximation for $k = 0.5$ is less than 4 percent for SNR > 70. As we shall see later on this k is close to the optimum value of k which minimizes the BER. It may be shown that the optimum value of k approaches 0.5 when SNR is large.

Note that the pdf of $x^2 + y^2$ for $b = 1$ is the noncentral chi with 2 degrees of freedom [18], [19] and the noncentral parameter 2 SNR. The general form of the noncentral chi distribution is given as follows [18], [19]. Let us consider an N dimensional Gaussian vector, each component of which is an independent Gaussian stochastic variable g_i with mean m_i and unit variance ($i = 1, 2, \cdots, N$). The probability density function of the variable

$$\chi = \sqrt{\sum_{i=1}^{N} g_i^2} \tag{21}$$

is given by [18], [19] (noncentral chi distribution):

$$p(\chi) = \chi^{N/2} I_{N/2-1}(A\chi) \exp[-(\chi^2 + A^2)/2]/A^{N/2-1} \tag{22}$$

where I represents the modified Bessel function of the first kind and A is the noncentral parameter defined as

$$A^2 = \sum_{i=1}^{N} m_i^2. \tag{23}$$

B. {2 × 2} Phase Diversity Receiver with Linear Envelope Detectors

Such a detector gives at the output the absolute value of its input. When $b = 0$ the probability of error is equal to the probability that the process at the input of the threshold comparator exceeds the threshold at the moment of sampling. The asymptotic approximation to this probability is (for derivation see Appendix B)

$$p_0 = 2\sqrt{2} \exp(-k^2\,\mathrm{SNR}/2) \cdot (1 - 1/k^2\,\mathrm{SNR})/(k\sqrt{\pi}\,\mathrm{SNR}) \tag{24}$$

where $T = k\sqrt{2\,\mathrm{SNR}}$. As we shall see later the optimum value of k which minimizes BER is close to 0.6. For this value we can estimate the relative error of (24) to be less than 2 percent for SNR > 50 and less than 1 percent for SNR > 70.

When $b = 1$ the error p_1 depends on the phase angle α. We shall compute this error in the worst case when the signal reaches its minimum, i.e., when either s_0 or s_1 is equal to zero. The worst case error determines the whole performance of the receiver. This will be discussed later. We have (Appendix B) for $T = k\sqrt{2\,\mathrm{SNR}}$:

$$p_1 = \exp[-(1-k)^2\,\mathrm{SNR}] \cdot [1 - 2/(\mathrm{SNR}\,(1-k)^2)]/[2\pi\,\mathrm{SNR}\,(1-k)^2]. \tag{25}$$

The relative error of this approximation is less than 4 percent for $k = 0.6$ (close to the optimum) and SNR > 80. The error estimation is based on the accuracy of the erfc function expansion.

C. {3 × 3} Phase Diversity Receiver with Squarers

In the case when $b = 0$ ("0" message sent) the pdf of the process at the input of the threshold comparator is that of the sum of three squared Gaussian variables. These variables are independent and have unit variances so the resulting pdf is chi-square with three degrees of freedom [17] and the value of p_0 can be approximated as (for details see Appendix C):

$$p_0 = (2/\sqrt{\pi})\, k\sqrt{\mathrm{SNR}} \exp(-k^2\,\mathrm{SNR}) \cdot [1 + 1/(2k^2\,\mathrm{SNR})]. \tag{26}$$

The error of this approximation is less than 1 percent for $k = 0.5$ (close to the optimum) and SNR > 20.

The pdf of the square root of the process at the moment of sampling for $b = 1$ is the noncentral chi with three degrees of freedom and the noncentral parameter equal to 2 SNR as this value is the square root of the sum of three squared Gaussian variables with nonzero means and unit variances. Thus we have (Appendix C):

$$p_1 = k \exp\left[-(1-k)^2 \mathrm{SNR}\right] \cdot \left[1 - 1/(2k(1-k)^2 \mathrm{SNR})\right] / \left[2(1-k)\sqrt{\pi \mathrm{SNR}}\right]. \tag{27}$$

The relative error of this approximation is less than 1 percent for $k = 0.5$ (close to the optimum) and SNR > 50.

D. {3 × 3} Phase Diversity Receiver with Linear Envelope Detectors

The value of p_0 is given in this case by (Appendix D):

$$p_0 = 4\sqrt{3} \exp\left[-k^2 \mathrm{SNR}/3\right] \cdot \left[1 - 3/(2k^2 \mathrm{SNR})\right] / (\sqrt{\pi \mathrm{SNR}}\, k) \tag{28}$$

with the relative error less than 2 percent for $k = 0.8$ (close to the optimum) and SNR greater than 50. Here $T = k\sqrt{2\,\mathrm{SNR}}$.

The value of p_1 depends on the phase angle α. We shall determine it in the worst case when the signal reaches its minimum. This happens for example for $\alpha = 90°$. The computation is done in Appendix D. Here we present only the final result

$$p_1 = \exp\left[-\mathrm{SNR}\,(1 - k/\sqrt{2})^2\right] \cdot \left[1 - 2.5/(\mathrm{SNR}\,(1-k/\sqrt{2})^2)\right] / \left[\sqrt{2\pi \mathrm{SNR}}\,(1-k/\sqrt{2})^2\right]. \tag{29}$$

The relative error of this approximation is less than 4 percent for $k = 0.8$ (close to the optimum) and SNR > 80.

E. Modified {3 × 3} Phase Diversity Receiver

This receiver is presented in Fig. 2. It uses subtractors and squarers as the demodulators. Due to the equal distribution of the power in the $\{3 \times 3\}$ coupler, the LO intensity noise is suppressed at the output of each subtractor. It is shown in Appendix E that the performance of the modified receiver is the same as that of the $\{2 \times 2\}$ phase diversity receiver with squarers as the demodulators. The only difference is that the threshold is set at a different value.

F. {2 × 2} and Modified {3 × 3} Phase and Polarization Diversity Receivers

It was shown that the BER performance of the modified receiver is exactly the same as that of the $\{2 \times 2\}$ receiver. Thus all the results obtained for the $\{2 \times 2\}$ receiver are valid also for the modified receiver.

When $b = 0$ the process at the input of the threshold comparator is the sum of four squared Gaussian processes with zero means and unit variances. The pdf of such a process is chi square with four degrees of freedom [17]. Thus the probability p_0 is given by

$$p_0 = (1/4)\int_T^\infty z \exp(-z/2)\,dz = \exp(-k^2 \mathrm{SNR})(1 + k^2 \mathrm{SNR}). \tag{30}$$

We set the threshold at $T = 2k^2$ SNR.

When the message $b = 1$ is sent the pdf of the square root of the variable at the input of the threshold comparator is the noncentral chi with four degrees of freedom (22) as this variable is the square root of the sum of four squared independent Gaussian variables with nonzero means and unity variances.

The probability of the error p_1 is computed in Appendix F and it is given by

$$p_1 = \frac{k^{3/2} \exp\left[-\mathrm{SNR}\,(1-k)^2\right] \left\{1 - \left[3/(16k) + 0.5/(1-k)^2 + 0.75/(k(1-k))\right]/\mathrm{SNR}\right\}}{2\sqrt{\pi}\,\mathrm{SNR}\,(1-k)}. \tag{31}$$

The error of the above approximation does not exceed 7.5 percent for $k = 0.5$ and SNR > 70. We shall see that this value of k is close to the optimum.

G. {3 × 3} Phase and Polarization Diversity Receiver

When $b = 0$ the process at the input of the threshold comparator is the sum of six squared Gaussian processes with zero means and unit variances. The pdf of such a process is chi square with six degrees of freedom [17]. Thus the probability p_0 is given by

$$p_0 = (1/16)\int_T^\infty z^2 \exp(-z/2)\,dz = \exp(-k^2 \mathrm{SNR})(1 + k^2 \mathrm{SNR} + k^4 \mathrm{SNR}^2/2). \tag{32}$$

Here we set the threshold at $T = 2k^2$ SNR.

When the message $b = 1$ is sent the pdf of the square root of the variable at the input of the threshold is the noncentral chi with six degrees of freedom and the noncentral parameter of 2 SNR. The probability of the error p_1 is computed in Appendix G and it is given by

$$p_1 = \frac{k^{5/2} \exp\left[-\mathrm{SNR}\,(1-k)^2\right] \left\{1 - \left[15/(16k) + 0.5/(1-k)^2 + 1.25/(k(1-k))\right]/\mathrm{SNR}\right\}}{2\sqrt{\pi}\,\mathrm{SNR}\,(1-k)}. \tag{33}$$

The error of the above approximation does not exceed 4 percent for $k = 0.5$ and SNR > 70. We shall see that this value of k is close to the optimum.

IV. BER Determination for ASK Receivers

The value of BER is determined for each receiver by (17) with the appropriate values of p_0 and p_1 inserted into it. The value of BER depends on the threshold which is set by the parameter k. Thus, it is necessary to minimize BER with regard to this parameter for each value of SNR. This was done using the simplex method and the results for the phase diversity receivers are presented in Fig. 4. We have also marked BER for the ideal heterodyne receiver. In this case the value of BER is given by

$$\begin{aligned} \text{BER} &= 0.5 \operatorname{erfc}\left(0.5\sqrt{\text{SNR}}\right) \\ &= \exp\left[-\text{SNR}/4\right](1 - 2/\text{SNR})/\sqrt{\pi\,\text{SNR}} \end{aligned} \tag{34}$$

with the relative error less than 0.5 percent for SNR $>$ 50. This formula implies that SNR $= 72$ is required to obtain BER $= 10^{-9}$; i.e., if $b = 0$ is also included it corresponds to 36 photoelectrons/bit in average [20], [21]. We can readily see that the performance of the receivers with squarers is close to that of the heterodyne receiver. The performance of the receivers with the linear envelope detectors is substantially worse. The losses for BER $= 10^{-9}$ are 1.6 and 1.1 dB for the $\{2 \times 2\}$ and $\{3 \times 3\}$ receivers, respectively. The corresponding numbers for the receivers with the squarers are 0.25 and 0.4 dB for the $\{2 \times 2\}$ (modified $\{3 \times 3\}$) and $\{3 \times 3\}$ receivers, respectively. The optimum values of the parameter k are depicted in Fig. 5 for each type of the receiver. They depend rather slightly on SNR.

The results for polarization and phase diversity receivers are depicted in Fig. 6. The losses as compared with the ideal heterodyne receiver for BER $= 10^{-9}$ are 0.5 dB and 0.7 dB for the $\{2 \times 2\}$ (modified $\{3 \times 3\}$) and $\{3 \times 3\}$ receivers, respectively, i.e., they are only a few tenths of a decibel worse than for polarization control scheme. The optimum values of k depend only slightly on SNR. For the values of SNR in the range of 50–110 the optimum $k = 0.53$–0.56 for the $\{2 \times 2\}$ receiver and $k = 0.55$–0.59 for the $\{3 \times 3\}$ receiver.

The optimum value of the parameter k depends mainly on the exponents in the expressions for p_0 and p_1, namely on their symmetry with regard to $k = 0.5$. If these exponents are symmetric, as in the case of all the receivers with the squarers, the optimum value of k is close to 0.5. It may be shown that it approaches 0.5 when SNR tends to infinity. On the other hand, if the number of branches of the receiver increases, then the pdf's are closer to the normal pdf (from the central limit theorem). For both the normal pdf's, the optimum value of k is given by $k^2 = 0.5$, and this explains a slight rise of the optimum value of k when the number of the branches of the receiver increases. Both the exponents and the threshold definition are different for the linear detectors. Therefore the optimum values of k are different since these values may be

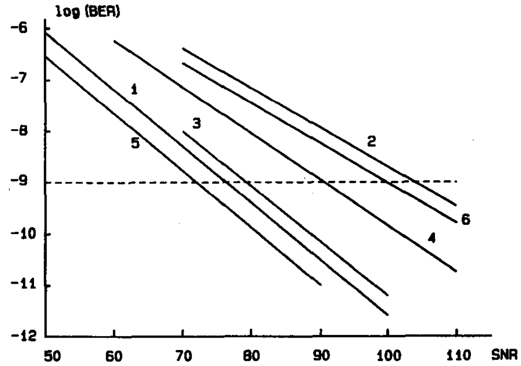

Fig. 4. BER against SNR for various types of the phase diversity ASK receivers. 1: $\{2 \times 2\}$ with squarers, or the modified $\{3 \times 3\}$ receiver. 2: $\{2 \times 2\}$ with linear envelope detectors. 3: $\{3 \times 3\}$ with squarers. 4: $\{3 \times 3\}$ with linear envelope detectors. 5: Ideal heterodyne receiver. 6: $\{2 \times 2\}$ with linear envelope detectors for the uniform distribution of the phase angle α.

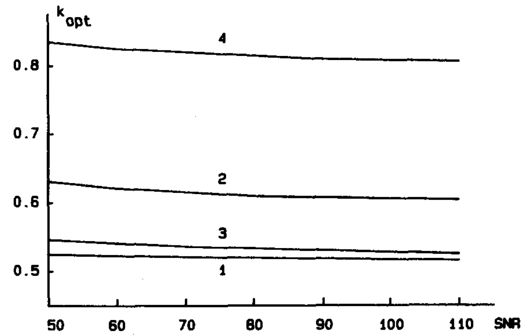

Fig. 5. The optimum values of the parameter k against SNR for ASK. The description of the curves corresponds to that of Fig. 4.

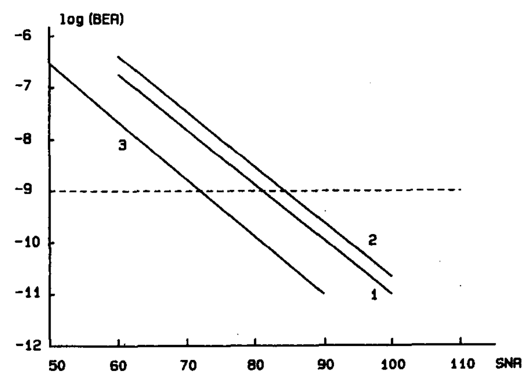

Fig. 6. BER versus SNR for the polarization diversity ASK receivers. 1: $\{2 \times 2\}$ receiver (or modified $\{3 \times 3\}$ receiver). 2: $\{3 \times 3\}$ receiver. 3: Ideal heterodyne receiver.

roughly found as the values of k for which the exponents are equal. For smaller SNR, the other factors in the expressions for p_0, p_1 start to play a role, and this explains a rise of the optimum value of k for smaller SNR.

One may have doubts to the results for the linear detectors, however, as the performance of the receivers with these detectors was computed in the worst case. We shall discuss this treating the $\{2 \times 2\}$ receiver as an example.

Let us assume that the phase angle α is uniformely distributed over the $(0, 2\pi)$ range. Then we can define the mean value of p_1 as

$$p_{1m} = 1/(2\pi) \int_0^{2\pi} p_1(\alpha)\, d\alpha. \tag{35}$$

We can compute BER inserting p_{1m} instead of p_1 into (17) (the value of p_0 stays the same). The value of p_{1m} is computed in Appendix H. The function BER(SNR) in this case is very close to the curve for the worst case confirming our previous statement that the worst case determines the overall performance.

V. BER Evaluation for DPSK

Let us denote the two inputs of the kth multiplier of the phase diversity receiver by x_k, y_k, and by x_{kV}, y_{kV} (x_{kH}, y_{kH}) the inputs of the kth multiplier of one part of the phase and polarization diversity receiver. Here the subscript $V(H)$ denotes the part with vertical (horizontal) polarization.

Since IF is taken to be equal to 0 Hz, the means of x_k and y_k have approximately the same absolute values

$$|\langle x_k \rangle| = |\langle y_k \rangle| = |s_k|, \qquad k = 0, 1, (2). \tag{36}$$

This also holds for x_{kV}, y_{kV}, s_{kV} (x_{kH}, y_{kH}, s_{kH}).

In order to determine which bit was sent the following sum is tested:

$$A = \sum_k x_k y_k \tag{37}$$

for the phase diversity receiver, and

$$A = \sum_k x_{kV} y_{kV} + \sum_k x_{kH} y_{kH} \tag{38}$$

for the phase and polarization diversity receiver. Because of the symmetry the threshold should be set at zero. The true value of this bit is given by the sign of the following sum:

$$D = \sum_k s_k s_k' \tag{39}$$

for the phase diversity receiver and

$$D = \sum_k s_{kV} s_{kV}' + \sum_k s_{kH} s_{kH}' \tag{40}$$

for the phase and polarization diversity receiver. Here s_k' denotes the value of the signal delayed T_0. If the two bits have equal probability the value of the BER is given by

$$\begin{aligned}\text{BER} &= 0.5(P(A > 0 \mid D < 0) + P(A < 0 \mid D > 0)) \\ &= P(A < 0 \mid D > 0).\end{aligned} \tag{41}$$

Here $P(A > 0 \mid D < 0)$ is the conditional probability that $A > 0$ provided that $D < 0$, and $P(A < 0 \mid D > 0)$ is the conditional probability that $A < 0$ provided that $D > 0$. These error probabilities are equal because of the symmetry. Thus without loss of generality we can assume that $D > 0$. It means that both s and s' have the same signs. The sign of A is the same as the sign of [22]:

$$C = \sum_k (x_k + y_k)^2/2 - \sum_k (x_k - y_k)^2/2. \tag{42}$$

This holds for the phase diversity receiver. For the phase and polarization diversity receiver, it is necessary to sum over the subscripts V, H as was done in (38) and (40). Let us consider the random variables $z_k = (x_k + y_k)/\sqrt{2}$ and $v_k = (x_k - y_k)/\sqrt{2}$. They are Gaussian and have unit variances. Their means are given by

$$\langle z_k \rangle = \sqrt{2}\, s_k, \qquad \langle v_k \rangle = 0. \tag{43}$$

By examining their covariance functions it is easy to prove that they are not correlated. Since they are Gaussian they are also independent. Let us denote

$$Z^2 = \sum_k z_k^2, \qquad W^2 = \sum_k v_k^2, \qquad Z, W > 0. \tag{44}$$

This holds for the polarization diversity receiver. For the phase and polarization diversity receiver the sum should include both polarizations. In this way our problem reduces to finding the probability that $W > Z$ under the assumption that $D > 0$ since $C < 0$ only when $W > Z$. Let us denote the probability density function (pdf) of W by $f_W(x)$ and the pdf of Z by $f_Z(y)$. Since W and Z are independent we have

$$\begin{aligned}\text{BER} &= P(C < 0 \mid D > 0) = P(W > Z) \\ &= \int_0^\infty dy\, f_Z(y) \int_y^\infty dx\, f_W(x).\end{aligned} \tag{45}$$

Thus it is necessary to find both the probability density functions for each receiver and then use (20). Let us do this beginning with the $f_W(x)$.

It is well known that the square root W of the sum of the squares of N independent Gaussian random variables with unit variances and zero means has chi probability density functions with N degrees of freedom in the form [23]:

$$f_W(x) = 2/[2^{N/2}\Gamma(N/2)]x^{N-1} \exp(-x^2/2), \quad x > 0. \tag{46}$$

Here Γ is the gamma function. Using (46) we can compute the value of the internal integral $G = \int_y^\infty dx\, f_W(x)$ in (45) for each type of the receiver. We have

$$G_2(y) = \exp(-y^2/2), \qquad y > 0 \tag{47}$$

for the $\{2 \times 2\}$ phase diversity receiver ($N = 2$), and

$$G_3(y) = \sqrt{2/\pi}\, y \exp(-y^2/2) + \operatorname{erfc}(y/\sqrt{2}), \quad y > 0 \tag{48}$$

for the $\{3 \times 3\}$ phase diversity receiver ($N = 3$). Here $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$. To obtain (48) we integrated by parts. We have further

$$G_4(y) = (1 + y^2/2)\exp(-y^2/2), \qquad y > 0, \tag{49}$$

for the $\{2 \times 2\}$ phase and polarization diversity receiver ($N = 4$), and

$$G_6(y) = (y^4 + 4y^2 + 8) \exp(-y^2/2)/8, \qquad y > 0, \tag{50}$$

for the $\{3 \times 3\}$ phase and polarization diversity receiver ($N = 6$). The other pdf is the noncentral chi with N degrees of freedom ($N = 2, 3, 4, 6$) and the noncentral parameter 4 SNR. It is given by (22). Writing these pdf's explicitly we have

$$f_{Z2}(y) = y \exp[-(y^2 + 4\,\mathrm{SNR})/2]\, I_0(2\sqrt{\mathrm{SNR}}\, y) \tag{51}$$

for the $\{2 \times 2\}$ phase diversity receiver; and

$$f_{Z3}(y) = y\left[\exp\left(-(y - 2\sqrt{\mathrm{SNR}})^2/2\right) - \exp\left(-(y + 2\sqrt{\mathrm{SNR}})^2/2\right)\right] / \left[2\sqrt{2\pi\,\mathrm{SNR}}\right] \tag{52}$$

for the $\{3 \times 3\}$ phase diversity receiver; and

$$f_{Z4}(y) = y^2 \exp[-(y^2 + 4\,\mathrm{SNR})/2] \cdot I_1(2\sqrt{\mathrm{SNR}}\, y)/(2\sqrt{\mathrm{SNR}}) \tag{53}$$

for the $\{2 \times 2\}$ phase and polarization diversity receiver; and

$$f_{Z6}(y) = y^3 \exp[-(y^2 + 4\,\mathrm{SNR})/4] \cdot I_2(2\sqrt{\mathrm{SNR}}\, y)/(4\,\mathrm{SNR}) \tag{54}$$

for the $\{3 \times 3\}$ phase and polarization diversity receiver. Here I denotes the modified Bessel function of the first kind. It is now possible to compute the values of the BER using (45) together with (47)–(54). Closed form expressions are to be obtained in three cases in Appendix I. These are as follows:

$$\mathrm{BER} = 0.5 \exp(-\mathrm{SNR}) \tag{55}$$

for the $\{2 \times 2\}$ phase diversity receiver. This is in fact well known result [24] for the ideal heterodyne DPSK, and it means that the performance of this phase diversity receiver is the same.

$$\mathrm{BER} = 0.5 \exp(-\mathrm{SNR})(1 + \mathrm{SNR}/4) \tag{56}$$

for the $\{2 \times 2\}$ phase and polarization diversity receiver (the same result has been published lately by Okoshi *et al.* [19]) and

$$\mathrm{BER} = 0.5 \exp(-\mathrm{SNR})(1 + 3\,\mathrm{SNR}/8 + \mathrm{SNR}^2/16) \tag{57}$$

for the $\{3 \times 3\}$ phase and polarization diversity receiver.

In the case of the $\{3 \times 3\}$ phase diversity receiver we can obtain only the asymptotic approximation of the BER. To do this, note that the essential for the integration region in (45) is located near $y = \sqrt{\mathrm{SNR}}$. The second term in (52) is then negligible. Using the asymptotic expansion of the erfc function, and the steepest descent method [25] near the point $y = \sqrt{\mathrm{SNR}}$ we get finally

$$\mathrm{BER} = 0.5 \exp(-\mathrm{SNR})(\mathrm{SNR} + 1.5)/\sqrt{\pi\,\mathrm{SNR}}, \qquad \mathrm{SNR} \gg 1. \tag{58}$$

The relative error of this approximation is less than 5 percent for SNR > 10 and less than 2.5 percent for SNR > 20. The values of BER for all kinds of the receivers are shown in Fig. 7 against the values of SNR.

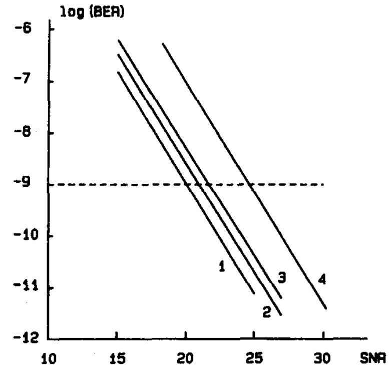

Fig. 7. BER versus SNR for DPSK. 1: $\{2 \times 2\}$ phase diversity receiver. 2: $\{3 \times 3\}$ phase diversity receiver. 3: $\{2 \times 2\}$ phase and polarization diversity receiver. 4: $\{3 \times 3\}$ phase and polarization diversity receiver.

VI. Discussion

On the basis of the results which are presented in Figs. 4, 6, and 7 we may find the numbers of electrons per bit necessary to obtain the value of BER equal to 10^{-9}. These are given in Table I. Note that the values for ASK are average values, i.e., they include also the case when $b = 0$. It means that in fact the maximum power (i.e., for $b = 1$) required to obtain the same performance is four times greater for ASK than for DPSK (the average power is twice greater). Losses as compared to the ideal heterodyne receiver are given in parentheses. It is necessary to stress that a few percent errors of determining the probabilities $p_0(p_1)$ do not influence these values. The results for ASK are contrary to some statements [2], [26] which favor the $\{2 \times 2\}$ to the $\{3 \times 3\}$ receiver with linear envelope detectors.

To make comparison between receivers with and without polarization diversity more meaningful, let us note that any polarization control scheme is not ideal. There is always some tracking error φ between the signal and the LO polarization states. This error reduces the value of SNR by a factor of $\cos^2 \varphi$. The excess of power necessary for the $\{2 \times 2\}$ phase diversity receiver to maintain the same value of BER as the $\{2 \times 2\}$ phase and polarization diversity receiver in the presence of polarization tracking error φ is shown in Fig. 8. The performance of the $\{3 \times 3\}$ receivers is similar. It is readily seen that the polarization control scheme slightly outperforms the phase diversity scheme only for relatively small tracking errors (less than 14° for ASK and less than 17° for DPSK). For

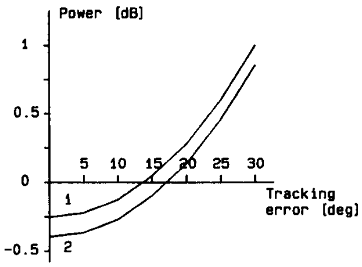

Fig. 8. Excess of power necessary for the $\{2 \times 2\}$ phase diversity receiver to maintain the same value of BER as the $\{2 \times 2\}$ phase and polarization diversity receiver in the presence of polarization tracking error φ. 1: ASK. 2: DPSK.

TABLE I
NUMBER OF ELECTRONS PER BIT REQUIRED TO OBTAIN BER-10^{-9} FOR VARIOUS TYPES OF RECEIVERS

Ideal heterodyne ASK receiver	36
Phase diversity ASK receivers	
{2x2} receiver with squarers	38 (0.25 dB)
Modified {3x3} receiver	38 (0.25 dB)
{2x2} receiver with linear detectors	52 (1.6 dB)
{3x3} receiver with squarers	40 (0.4 dB)
{3x3} receiver with linear detectors	45 (1.1 dB)
Phase and polarisation diversity ASK receivers	
Polarisation diversity receiver ({2x2})	41 (0.6 dB)
Polarisation diversity modified receiver ({3x3})	41 (0.6 dB)
Polarisation diversity receiver ({3x3})	42.5(0.7 dB)
Ideal heterodyne PSK receiver	18
DPSK receivers	
{2x2} phase diversity receiver	20 (0.45 dB)
{3x3} phase diversity receiver	21 (0.7 dB)
{2x2} phase and polarisation diversity receiver	22 (0.9 dB)
{3x3} phase and polarisation diversity receiver	24 (1.2 dB)

larger tracking errors the performance of the polarization diversity receiver is better.

To study the impact of the laser linewidth on the performance of the ASK receivers let us assume a Lorentzian laser lineshape [27]. If IF is zero this results in the following spectrum of the lasers phase noise at the receiver

$$L(f) = \frac{c}{\pi^2 B_L \left(1 + 4(f/B_L)^2\right)}. \qquad (59)$$

Here c is a factor proportional to the lasers powers and B_L is the sum of the total (two-sided) linewidths of two lasers. If the intersymbol interference may be neglected the power of the signal at the output of the low pass filter is

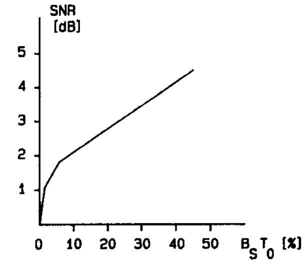

Fig. 9. Loss of SNR versus the normalized laser linewidth $B_S T_0$.

given by the following integral:

$$\int_{-\infty}^{+\infty} \left(L(f) * \left|H(f)\right|^2\right) \left|F(f)\right|^2 df. \qquad (60)$$

Here $H(f)$ is the Fourier transform of $h(t)$ (equation (7)), $*$ stands for the convolution, and $F(f)$ is the frequency response of the low pass filter. When B_L increases, the spectrum of the signal at the input of the low pass filter broadens. Consequently, the bandwidth of the low pass filter should be greater to accommodate the same portion of the signal power. However, this increases also the noise bandwidth and reduces SNR. Let us approximate the frequency response of the low pass filter by a rectangle having unity gain between 0 and cutoff frequency, and zero elsewhere. (Note that we do not assume that the actual filter is the ideal rectangular filter, which is neither casual nor it provides suppression of the intersymbol interference in general case. Such approximation was chosen to simplify the calculations in (60).) The loss of the SNR due to the increase of the noise bandwidth is shown in Fig. 9 against the normalized linewidth $B_L T_0$ for the filter which passes 90 percent of the power of the incoming signal.

When large filter bandwidths are used for ASK to overcome the signal spectrum spreading due to the nonnegligible laser linewidths, the post detection filtering is usually used as it improves the SNR [3], [28]. The operation of the post detection filter may be roughly visualized as the adding of M ($M > 1$, not necessarily integer) independent random variables having (generalized) chi square pdf's. (If M is not integer appropriate weighting coefficients must be applied.) The value of M may be approximated [28] as the ratio of the noise bandwidths at the input and output of the post detection filter ($M > 1$). To illustrate briefly the influence of the post detection filtering let us consider the $\{2 \times 2\}$ phase diversity receiver with the post detection filter such that $M = 2$. It follows from the above that the pdf's at the input of the threshold comparator should be well approximated by those computed previously for the $\{2 \times 2\}$ phase and polarization diversity receiver. In the other words, the post detection filtering substantially improves the SNR and BER for wide laser linewidths, however, there is a small additional penalty (0.4 dB in the described case) due to the change of the pdf's. It means also that the optimum value of the parameter k increases as it is greater for the $\{2 \times 2\}$ phase and polarization diversity receiver.

We can compare our results for the $\{3 \times 3\}$ ASK phase diversity receiver with those obtained by Kazovsky *et al.* [3]. We can do it directly only for the case without post detection filtering. The results are very similar in both cases. The results of Kazovsky tend to slightly overestimate the BER. For instance, the number of electrons per bit required to reach BER $= 10^{-9}$, computed from [3, eq. (33)], is 43 (in our analysis it is 40). This slight discrepancy is caused by the assumption about the normal pdf of the processes that is taken in [3]. Comparing Fig. 9 with results obtained in [3] for the receiver with post detection filtering we can see the advantage of such a receiver for broad laser linewidths as it can handle lasers linewidths of the order of the bit rate with small losses (~ 2 dB).

One may pose a question: what type of detector (i.e., linear, square, cubic, etc.) gives the optimum performance of the ASK phase diversity receiver. To answer this question let us consider the $\{2 \times 2\}$ receiver. The probability of the error p_0 is determined in fact by the shortest distance d_0 between the point $[0, 0]$ and the curve $|x|^q + |y|^q = T$ where q gives the functional dependence of the single detector. We have $p_0 \sim \exp(-d_0^2/2)$. In the same way, the probability of the error p_1 is determined by the shortest distance d_1 between the curve $|x|^q + |y|^q = T$ and a circle $x^2 + y^2 = 2$ SNR ($T < \sqrt{2 \text{ SNR}}$). We have again $p_1 \sim \exp(-d_1^2/2)$. The value of BER will be minimized if $d_0 + d_1 = \sqrt{2 \text{ SNR}}$. This can happen only when $q = 2$, i.e., for the squarers as the detectors. In the same way we can extend this result to 3 dimensions (the $\{3 \times 3\}$ receiver) and in general to n dimensions. It means that the squarers are the optimum detectors.

So far, the perfect receivers with all the branches absolutely similar have been assumed. It is well known that this ideal situation is very difficult to reach. In this section, we briefly discuss the degradation of the performance due to the not perfectly balanced arms of the receiver. The ASK modulation and the $\{2 \times 2\}$ and $\{3 \times 3\}$ phase diversity receivers will be used to illustrate this effect.

Let us assume that the gain in one branch of the receiver is slightly different than the unity gain in the other(s). Let us denote this gain by μ, and assume that $\mu > 1$ (the case of $\mu < 1$ is dual). The probability of the error p_0 is again determined by the shortest distance d_0 between the point $[0, 0]$ and the ellipse $\mu^2 x^2 + y^2 = T$. We have $d_0 = \sqrt{T}/\mu$ for $\mu > 1$. The probability of the error p_1 is determined by the distance $d_1 = \sqrt{2 \text{ SNR}} - T$ (for $\mu > 1$). To minimize BER, the value of T should be chosen so that $d_0 \approx d_1$ from where $d_0 = \sqrt{2 \text{ SNR}}/(\mu + 1)$ and

$$\text{BER} \sim \exp\left[-\text{SNR}/(\mu + 1)^2\right], \qquad \mu > 1. \tag{61}$$

It means that it is necessary to increase SNR by a factor of $(\mu + 1)^2$ to get the same performance as in the ideal case.

The situation for the $\{3 \times 3\}$ receiver is more complicated since there are three independent gains. Let us assume, without loss of generality, that they are $\mu > 1$, 1, $\eta < 1$. The probability of the error p_0 is determined by the shortest distance d_0 between the point $[0, 0, 0]$ and the ellipsoid $\mu^2 x^2 + y^2 + \eta^2 z^2 = T$ given again by $d_0 = \sqrt{T}/\mu$. However, the distance d_1 decreases as compared to the previous case since $\eta < 1$ and the ellipsoid is closer to the sphere $x^2 + y^2 + z^2 = 2$ SNR. That is, the performance of the $\{3 \times 3\}$ receiver is worse. This fact is intuitively clear as the equalizing of three branches is more difficult than doing the same for two.

Sometimes a parameter γ [3] is used to describe the performance of the receiver

$$\gamma = \frac{M(b = 1) - M(b = 0)}{S(b = 1) + S(b = 0)} \tag{62}$$

where M and S are the mean and the standard deviation of the signal at the input of the threshold comparator. In the case of the ideal heterodyne detector $\gamma = \sqrt{0.5 \text{ SNR}}$ [3]. Let us compute the value of γ for the $\{3 \times 3\}$ receiver with linear envelope detectors in the worst case. We have for SNR $\gg 1$:

$$M(b = 1) - M(b = 0) = 2\sqrt{\text{SNR}}$$
$$S(b = 1) = \sqrt{3}$$
$$S(b = 0) = \sqrt{3}\,\sqrt{(1 - 2/\pi)}$$

(the standard deviation of the rectified Gaussian process). These values give $\gamma = \sqrt{0.52 \text{ SNR}}$ what suggests the performance better than the heterodyne receiver! Thus, high caution must be preserved when using this parameter.

VII. Conclusions

The main advantage of all the receivers which are considered here is the fact that they do not require phase locking. The analysis shows that the receivers with $\{2 \times 2\}$ and $\{3 \times 3\}$ optical networks with or without polarization diversity perform very similarly (within 0.8 dB as compared to the ideal heterodyne receiver) for both ASK and DPSK. (Of course DPSK outperforms ASK by roughly 6 dB if maximum powers are considered.) The ASK receivers with linear detectors are the only exception as they are less efficient. Thus the practical reasons are the most important to make any choice between the receivers.

The polarization diversity receivers avoid the use of the sophisticated polarization control scheme (often microprocessor controlled). However, this solution doubles the receiver high frequency electronics, which may be very difficult to balance unless the whole receiver is integrated. Furthermore, if the number of the branches of the receiver increases, the shot noise limit operation of the receiver may be difficult to reach, since it requires substantial increase of the LO laser power.

The $\{3 \times 3\}$ optical network may be relatively easy manufactured as $\{3 \times 3\}$ optical fiber fused coupler [1].

This is not the case with the $\{2 \times 2\}$ optical network which may be realized as a sophisticated 90° optical hybrid [4], [5]. An ideal $\{4 \times 4\}$ fused fiber coupler [7] is required to replace the optical hybrid. In addition the $\{3 \times 3\}$ modified receiver offers the possibility of the suppression of the LO intensity noise. (Note that this holds true also for other types of modulation not considered here.) This appears to be the best choice unless the problems with balancing of the electronics are encountered.

It is necessary to stress that the receivers performance depends neither on the relative phases nor on the polarization state of the received signal (for polarization diversity). In fact our analysis covers also other types of receivers which were not mentioned explicitely. The $\{2 \times 2\}$ phase and polarization diversity receiver performance is identical to that of the polarization diversity receiver proposed by Glance [11]. The performance of the $\{2 \times 2\}$ polarization and phase diversity receiver is fully equivalent to that of any phase diversity receiver using $\{4 \times 4\}$ optical network (i.e., such as that in [29]).

Appendix A

We have for the second error probability

$$p_1 = 1/(2\pi) \iint_{x^2+y^2<2k^2\mathrm{SNR}} \exp\left[-((x-s_0)^2 + (y-s_1)^2)/2\right] dx\,dy. \tag{A1}$$

Using (13), changing the coordinates to polar ones, and using the integral representation of the modified Bessel function I_0 [30], we get

$$p_1 = 2\exp(-\mathrm{SNR}) \int_0^{k\sqrt{\mathrm{SNR}}} r \cdot \exp(-r^2)\, I_0(2\sqrt{\mathrm{SNR}}\, r)\, dr. \tag{A2}$$

Let us substitute for the modified Bessel function in (A2) its asymptotic expansion [30] $I_0(x) = \exp(x)/\sqrt{2\pi x}$. The relative error of this approximation is less than $1/8x$ [30]. We have then

$$p_1 = 1/\left[\sqrt{\pi}\,\sqrt[4]{\mathrm{SNR}}\right] \int_0^{k\sqrt{\mathrm{SNR}}} \sqrt{u} \cdot \exp\left[-\left(u - \sqrt{\mathrm{SNR}}\right)^2\right] du. \tag{A3}$$

The essential for the integration region is located near $u = k\sqrt{\mathrm{SNR}}$.

Thus, we can replace the lower limit of the integral by minus infinity and expand $\sqrt{u}$ in its Taylor series retaining only the two first terms. This is in fact equivalent to the use of the steepest descent method [25]. After substitution, $x = u - \sqrt{\mathrm{SNR}}$ we get

$$p_1 = 1/\left[\sqrt{\pi}\,\sqrt[4]{\mathrm{SNR}}\right] \int_{-\infty}^{(k-1)\sqrt{\mathrm{SNR}}} \cdot \left\{\sqrt[4]{\mathrm{SNR}}\,\sqrt{k} - \left[x - (k-1)\sqrt{\mathrm{SNR}}\right] / \left[2\sqrt{k}\,\sqrt[4]{\mathrm{SNR}}\right]\right\} \exp(-x^2)\, dx. \tag{A4}$$

The above integral may be easily expressed using the asymptotic expression of the erfc function. Here $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$ where erf is given by

$$\mathrm{erf}(x) = 2/\sqrt{\pi} \int_0^x \exp(-t^2)\, dt. \tag{A5}$$

This asymptotic expansion is [30]

$$\mathrm{erfc}(x) = \exp(-x^2)/(\sqrt{\pi}x)\left[1 - 1/(2x^2)\right] \tag{A6}$$

with the relative error less than $3/(4x^4)$. Using (A6) in (A4) we get finally

$$p_1 = \sqrt{k}\exp\left[-(1-k)^2\,\mathrm{SNR}\right]\left\{1 - \left(1/(2(1-k)^2) + 1/(4k(1-k))\right)/\mathrm{SNR}\right\} / \left\{2\sqrt{\pi\,\mathrm{SNR}}\,(1-k)\right\}. \tag{A7}$$

The integral (A1) was computed numerically for various values of k and SNR. As we shall see later the optimum value of k is close to 0.5. The relative errors of the approximation (A6) are less than 4 percent for this k and $\mathrm{SNR} > 70$.

Appendix B

The realization of the process at the input of the threshold comparator is given by $|x| + |y|$ where $x = i_0$ and $y = i_1$ are independent, jointly Gaussian with zero means and unity variances. Thus we have

$$p_0 = 1/(2\pi) \iint_{|x|+|y|>T} \exp\left[-(x^2+y^2)/2\right] dx\,dy \tag{B1}$$

where T is the threshold. By turning the coordinates by 45° this reduces to

$$\begin{aligned} p_0 &= 1 - \left\{1/\sqrt{2\pi}\int_{-T/\sqrt{2}}^{T/\sqrt{2}} \exp(-t^2/2)\, dt\right\}^2 \\ &= 1 - \left[\mathrm{erf}(T/2)\right]^2 = 2\,\mathrm{erfc}(T/2) \\ &= 2\sqrt{2}\exp(-k^2\,\mathrm{SNR}/2) \\ &\quad \cdot (1 - 1/k^2\,\mathrm{SNR})/(k\sqrt{\pi\,\mathrm{SNR}}). \end{aligned} \tag{B2}$$

We substituted here $T = k\sqrt{2\,\mathrm{SNR}}$ and used the asymptotic expansion of the function erfc [30] (see Appendix A).

As we shall see later the optimum value of k which minimizes BER is close to 0.6. For this value we can estimate the relative error of (19) to be less than 2 percent for $\mathrm{SNR} > 50$ and less than 1 percent for $\mathrm{SNR} > 70$. This is done based on the estimation of the accuracy of the erfc function expansion given in Appendix A.

We have for the value of p_1 in the worst case

$$p_1 = 1/(2\pi) \iint_{|x|+|y|>T} \cdot \exp\left[-\left(x^2 + (y - \sqrt{2\,\mathrm{SNR}})^2\right)/2\right] dx\,dy. \tag{B3}$$

By turning the coordinates by 45° we can reduce this to

$$p_1 = \left\{ 1/\sqrt{2\pi} \int_{-T/\sqrt{2}}^{T/\sqrt{2}} \exp\left[-(x - \sqrt{\mathrm{SNR}})^2/2\right] dx \right\}^2. \tag{B4}$$

The value of this integral is determined by the region near $T/\sqrt{2}$ so we can replace the lower limit of integration by minus infinity. Then p_1 may be expressed by the erfc function which in turn can be replaced by its asymptotic expansion. We have for $T = k\sqrt{2\,\mathrm{SNR}}$:

$$\begin{aligned} p_1 &= (1/4)\left[\mathrm{erfc}\left((1-k)\sqrt{\mathrm{SNR}/2}\right)\right]^2 \\ &= \exp\left[-(1-k)^2\,\mathrm{SNR}\right] \\ &\quad \cdot \left[1 - 2/\left(\mathrm{SNR}\,(1-k)^2\right)\right]/\left[2\pi\,\mathrm{SNR}\,(1-k)^2\right]. \end{aligned} \tag{B5}$$

The relative error of this approximation is less than 4 percent for $k = 0.6$ (close to the optimum) and SNR > 80. The error estimation is based on the accuracy of the erfc function expansion.

Appendix C

The pdf (the chi square with three degrees of freedom) is

$$p(z) = \left(1/\sqrt{2\pi}\right)\sqrt{z}\exp(-z/2). \tag{C1}$$

Thus the value of the error p_0 may be expressed after substitution $u^2 = z/2$ ($T = 2k^2$ SNR) as

$$\begin{aligned} p_0 &= \int_T^\infty p(z)\,dz = 4/\sqrt{\pi}\int_{k\sqrt{\mathrm{SNR}}}^\infty u^2\exp(-u^2)\,du \\ &= \left(2/\sqrt{\pi}\right)k\sqrt{\mathrm{SNR}}\exp\left(-k^2\,\mathrm{SNR}\right) \\ &\quad + \mathrm{erfc}\left(k\sqrt{\mathrm{SNR}}\right) \\ &= \left(2/\sqrt{\pi}\right)k\sqrt{\mathrm{SNR}}\exp\left(-k^2\,\mathrm{SNR}\right) \\ &\quad \cdot \left[1 + 1/(2k^2\,\mathrm{SNR})\right]. \end{aligned} \tag{C2}$$

We have integrated by parts and made use of the asymptotic expansion of the erfc function. The error of this approximation is less than 1 percent for $k = 0.5$ (close to the optimum) and SNR > 20.

We have from (22) for p_1($T = 2k^2$ SNR)

$$\begin{aligned} p_1 = 1/\left(\sqrt{\pi}\,\mathrm{SNR}\right)\int_0^{k\sqrt{\mathrm{SNR}}} & u\left[\exp\left(-(u - \sqrt{\mathrm{SNR}})^2\right)\right. \\ & \left. - \exp\left(-(u + \sqrt{\mathrm{SNR}})^2\right)\right] du. \end{aligned} \tag{C3}$$

We used here the fact [31] that $I_{1/2}(z)\sqrt{0.5\pi/z} = \sinh(z)/z$.

The second term in the square brackets is negligible for k SNR >> 1. Under the same condition we can replace the lower limit in this integral by minus infinity. Making use of the asymptotic expansion of the erfc function we get

$$\begin{aligned} p_1 &= k\exp\left[-(1-k)^2\,\mathrm{SNR}\right] \\ &\quad \cdot \left[1 - 1/\left(2k(1-k)^2\,\mathrm{SNR}\right)\right]/ \\ &\quad \left[2(1-k)\sqrt{\pi\,\mathrm{SNR}}\right]. \end{aligned} \tag{C4}$$

The relative error of this approximation is less than 1% for $k = 0.5$ (close to the optimum) and SNR > 50.

Appendix D

The value of p_0 is given in this case by

$$\begin{aligned} p_0 &= 1/\left(\sqrt{2\pi}\right)^3 \iiint_{|x|+|y|+|z|>T} \\ &\quad \cdot \exp\left[-(x^2+y^2+z^2)/2\right] dx\,dy\,dz \\ &= 8/\left(\sqrt{2\pi}\right)^3 \iiint_{\substack{x+y+z>T \\ x>0,y>0,z>0}} \\ &\quad \cdot \exp\left[-(x^2+y^2+z^2)/2\right] dx\,dy\,dz. \end{aligned} \tag{D1}$$

The region of integration in (D1) is shown in Fig. 10. Simple geometry shows that the distance between the plain $x + y + z = T$ and the point [0, 0, 0] is $T/\sqrt{3}$. Here $T = k\sqrt{2\,\mathrm{SNR}}$. Because of the symmetry we can turn the coordinates over any angle and it does not change the value of the integral (D1). We chose the turning in which the plain $x + y + z = T$ becomes perpendicular to the y axis (see Fig. 10). The value of the integral (D1) then satisfies the following inequality:

$$v > p_0/8 > v/(2\pi)\int_0^{T/\sqrt{6}} dr \int_0^{2\pi} d\varphi\; r\exp\left[-r^2/2\right], \tag{D2}$$

where the value of v is equal to

$$v = 1/\left(\sqrt{2\pi}\right)\int_{T/\sqrt{3}}^\infty \exp(-y^2/2)\,dy. \tag{D3}$$

The values of the left and right hand sides of the inequality (D2) are within 1 percent for k^2 SNR > 28 (for $k = 0.8$ (close to the optimum) it corresponds to SNR > 44). Thus we can express p_0 by means of v:

$$\begin{aligned} p_0 = 8v &= 4\sqrt{3}\exp\left[-k^2\,\mathrm{SNR}/3\right] \\ &\quad \cdot \left[1 - 3/2k^2\,\mathrm{SNR}\right]/\left(\sqrt{\pi\,\mathrm{SNR}}\,k\right). \end{aligned} \tag{D4}$$

We made use of the asymptotic expansion of the erfc function. The error of this expansion is less than 1 percent for $k = 0.8$ (close to the optimum) and SNR > 50. Therefore the total error does not exceed 2 percent for these values of the parameters.

The value of p_1 in the worst case is given by

$$\begin{aligned} p_1 = 1/\left(\sqrt{2\pi}\right)^3 \iiint_{|x|+|y|+|z|<T} & \exp\left[-\left((x - \sqrt{\mathrm{SNR}})^2\right.\right. \\ & \left.\left. + (y - \sqrt{\mathrm{SNR}})^2 + z^2\right)/2\right] dx\,dy\,dz. \end{aligned} \tag{D5}$$

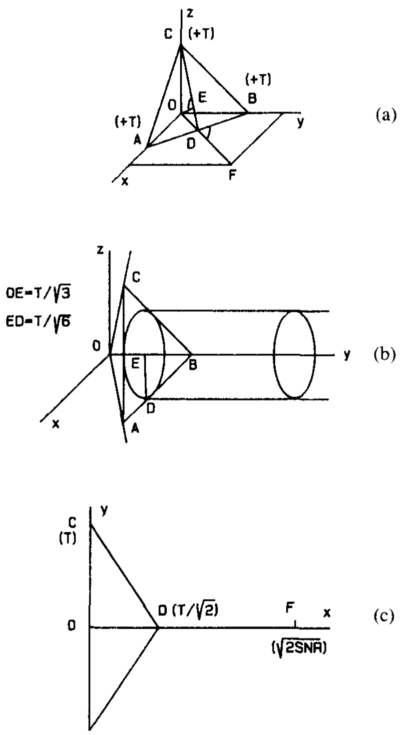

Fig. 10. Regions of integration (a) in (D1), (b) after coordinates turn, (c) in (D6).

By the same argumentation as for (D3) we can, with negligible error for SNR >> 1, change the three dimensional integral (D5) to a two-dimensional one. The region of integration A for this new integral is shown in Fig. 10(c). We can also extend, with negligible error, the limits of the integration to infinity. Thus we have

$$\begin{aligned} p_1 &= 1/(2\pi) \iint_A \exp\left[-\left((x - \sqrt{2\,\mathrm{SNR}})^2 + y^2\right)/2\right] dx\, dy \\ &= 2\int_0^\infty dy \exp\left[-y^2/2\right]/(\sqrt{2\pi}) \int_{-\infty}^{T/\sqrt{2} - y\sqrt{2}} dx \\ &\quad \cdot \exp\left[-\left(x - \sqrt{2\,\mathrm{SNR}}\right)^2\right]/(\sqrt{2}\pi) \\ &= \exp\left[-\mathrm{SNR}\left(1 - k/\sqrt{2}\right)^2\right] \\ &\quad \cdot \left[1 - 2.5/\left(\mathrm{SNR}\left(1 - k/\sqrt{2}\right)^2\right)\right]/ \\ &\quad \left[\sqrt{2}\pi\,\mathrm{SNR}\left(1 - k/\sqrt{2}\right)^2\right]. \end{aligned} \tag{D6}$$

We used the same method as in Appendix A. We expanded the function erfc in its asymptotic series and then used the steepest descent method. To estimate the error of the approximation we computed the integral (D6) numerically. It is less than 4 percent for $k = 0.8$ (close to the optimum) and SNR > 80.

Appendix E

The value of p_0 is

$$p_0 = 1/\left(\sqrt{2\pi}\right)^3 \iiint_{(x-y)^2 + (y-z)^2 + (z-x)^2 > T} \cdot \exp\left[-(x^2 + y^2 + z^2)/2\right] dx\, dy\, dz. \tag{E1}$$

Here we set the threshold at $T = 6k^2$ SNR. Let us introduce new variables of integration $\bar{x}$, $\bar{y}$, $\bar{z}$:

$$\begin{aligned} x &= l_1\bar{x} + l_2\bar{y} + l_3\bar{z} \\ y &= m_1\bar{x} + m_2\bar{y} + m_3\bar{z} \\ z &= n_1\bar{x} + n_2\bar{y} + n_3\bar{z} \end{aligned} \tag{E2}$$

where

$$\begin{aligned} -n_1 &= -n_2 = l_3 = m_3 = n_3 = 1/\sqrt{3} \\ l_1 &= m_2 = 0.5(1 + 1/\sqrt{3}) \\ m_1 &= l_2 = -0.5(1 - 1/\sqrt{3}). \end{aligned} \tag{E3}$$

After this transformation the integral (E1) reduces to

$$\begin{aligned} p_0 &= 1/(2\pi) \iint_{\bar{x}^2 + \bar{y}^2 > 2k^2\mathrm{SNR}} \exp\left[-(\bar{x}^2 + \bar{y}^2)/2\right] d\bar{x}\, d\bar{y} \\ &= \int_{k\sqrt{2\,\mathrm{SNR}}}^\infty r \exp(-r^2/2)\, dr = \exp(-k^2\,\mathrm{SNR}) \end{aligned} \tag{E4}$$

which is the same as (19). Here we used the polar coordinates. When $b = 1$ the value of p_1 is given by

$$\begin{aligned} p_1 = 1/\left(\sqrt{2\pi}\right)^3 &\iiint_{(x-y)^2 + (y-z)^2 + (z-x)^2 < T} \\ &\cdot \exp\left[-\left((x - s_0)^2 + (y - s_1)^2 + (z - s_2)^2\right)/2\right] \\ &\cdot dx\, dy\, dz. \end{aligned} \tag{E5}$$

Changing the coordinates to those given by (E2) and (E3) we get

$$\begin{aligned} p_1 = 1/(2\pi) &\iint_{\bar{x}^2 + \bar{y}^2 < 2k^2\mathrm{SNR}} \\ &\cdot \exp\left[-\left((\bar{x} - \bar{x}_0)^2 + (\bar{y} - \bar{y}_0)^2\right)/2\right] d\bar{x}\, d\bar{y}. \end{aligned} \tag{E6}$$

Here the points $\bar{x}_0$, $\bar{y}_0$ satisfy

$$\bar{x}_0^2 + \bar{y}_0^2 = \bar{s}_0^2 + \bar{s}_1^2 + \bar{s}_2^2 = 2\,\mathrm{SNR}. \tag{E7}$$

Equation (E6) with the above condition is fully equivalent to (A1). This means that the performance of the modified receiver is the same as that of the $\{2 \times 2\}$ receiver with squarers as demodulators. The only difference is that the threshold is set at a different value.

Appendix F

We have from (22):

$$p_1 = \int_0^{\sqrt{T}} r^2 I_1(\sqrt{2\,\text{SNR}}\,r) \cdot \exp\left[-(r^2/2 + \text{SNR})\right]/\sqrt{2\,\text{SNR}}\,dr. \quad \text{(F1)}$$

Since the value of this integral is determined by the region near $T = k\sqrt{2\,\text{SNR}}$ we can replace the Bessel function by its asymptotic expansion [30]:

$$I_1(x) = \exp(x)\left(1 - 3/(8x)\right)/\sqrt{2\pi x} \quad \text{(F2)}$$

and the integral (F1) transforms to

$$p_1 = \int_0^{k\sqrt{2\text{SNR}}} dr\, f(r) \exp\left[-\left(r - \sqrt{2\,\text{SNR}}\right)^2/2\right] \quad \text{(F3)}$$

where

$$f(r) = r^{1.5}\left(1 - 3/(8\sqrt{2\,\text{SNR}}\,r)\right)/\left[\sqrt{2\pi}\,(2\,\text{SNR})^{3/4}\right]. \quad \text{(F4)}$$

Since the region near $k\sqrt{2\,\text{SNR}}$ determines the value of the integral (F3) we can expand the function $f(r)$ near this point in its Taylor series retaining only two first terms (i.e., the value of the function and its first derivative). This is equivalent to the steepest descent method [25]. Using the asymptotic expansion of the erf (x) function [30]:

$$\begin{aligned}\text{erf}(x) &= 2/\sqrt{\pi}\int_0^x \exp(-t^2)\,dt \\ &= 1 - \exp(-x^2)\left[1 - 1/(2x^2)\right]/(\sqrt{\pi}x)\end{aligned} \quad \text{(F5)}$$

and neglecting the higher order terms we get finally

$$p_1 = \frac{k^{3/2}\exp\left[-\text{SNR}\,(1-k)^2\right]\left\{1 - \left[3/(16k) + 0.5/(1-k)^2 + 0.75/(k(1-k))\right]/\text{SNR}\right\}}{2\sqrt{\pi}\,\text{SNR}\,(1-k)}. \quad \text{(F6)}$$

The integral (F1) was also computed numerically and the error of the above approximation does not exceed 7.5 percent for $k = 0.5$ and SNR > 70. We shall see that this value of k is close to the optimum.

Appendix G

The probability of the error p_1 is given by (22):

$$p_1 = \int_0^{\sqrt{T}} r^3 I_2(\sqrt{2\,\text{SNR}}\,r) \cdot \exp\left[-(r^2/2 + \text{SNR})\right]/(2\,\text{SNR})\,dr. \quad \text{(G1)}$$

Since the value of this integral is determined by the region near $T = k\sqrt{2\,\text{SNR}}$ we can replace the Bessel function by its asymptotic expansion [31]

$$I_2(x) = \exp(x)\left(1 - 15/(8x)\right)/\sqrt{2\pi x} \quad \text{(G2)}$$

and the integral (G1) transforms itself to

$$p_1 = \int_0^{k\sqrt{2\text{SNR}}} dr\, f(r) \exp\left[-\left(r - \sqrt{2\,\text{SNR}}\right)^2/2\right] \quad \text{(G3)}$$

where

$$f(r) = r^{2.5}\left(1 - 15/(8\sqrt{2\,\text{SNR}}\,r)\right)/\left[\sqrt{2\pi}\,(2\,\text{SNR})^{5/4}\right]. \quad \text{(G4)}$$

Since the region near $k\sqrt{2\,\text{SNR}}$ determines the value of the integral (G3) we can expand the function $f(r)$ near this point in its Taylor series as it was done for the $\{2 \times 2\}$ receiver. Using again the asymptotic expansion of the erf function and neglecting the higher order terms we get finally

$$p_1 = \frac{k^{5/2}\exp\left[-\text{SNR}\,(1-k)^2\right]\left\{1 - \left[15/(16k) + 0.5/(1-k)^2 + 1.25/(k(1-k))\right]/\text{SNR}\right\}}{2\sqrt{\pi}\,\text{SNR}\,(1-k)}. \quad \text{(G5)}$$

The integral (G1) was also computed numerically and the error of the above approximation does not exceed 4 percent for $k = 0.5$ and SNR > 70. We shall see that this value of k is close to the optimum.

Appendix H

Turning the coordinates by 45°, as it was done in Appendix B, we may express the value of p_1 as

$$p_1(\alpha) = 1/(2\pi)\int_{-T/\sqrt{2}}^{T/\sqrt{2}}\int_{-T/\sqrt{2}}^{T/\sqrt{2}} \cdot \exp\left[-\left(\left(x - \sqrt{2\,\text{SNR}}\cos\alpha\right)^2 + \left(y - \sqrt{2\,\text{SNR}}\sin\alpha\right)^2\right)/2\right]dx\,dy. \quad \text{(H1)}$$

Here $T = k\sqrt{2\,\text{SNR}}$. The above integral reaches a maximum at the points $\alpha = 45°$ (135°, 225°, 315°). The maximum is very sharp for SNR $\gg 1$ and the region near these points determines the value of the integral $p_{1m} = 1/(2\pi)\int p_1(\alpha)\,d\alpha$. Thus, we can replace with negligible errors the integral (D1) by its asymptotic expansion. In this way

$$p_{1m} = 2/\pi\int_0^{\pi/2} 1/(2\pi) \cdot \exp\left[-\text{SNR}\left(1 + k^2 - \sqrt{2}k(\sin\alpha + \cos\alpha)\right)\right]/\left[\text{SNR}\left|(k - \sqrt{2}\cos\alpha)(k - \sqrt{2}\sin\alpha)\right|\right]d\alpha. \quad \text{(H2)}$$

Using the steepest descent method [25] to this integral we get finally

$$p_{1m} = \exp\left[-\text{SNR}\,(1-k)^2\right] / \left[\pi\,\text{SNR}\,(1-k)^2\sqrt{\pi k\,\text{SNR}}\right]. \qquad \text{(H3)}$$

Appendix I

It is easy to see that each (except for $\{3 \times 3\}$ phase diversity receiver) integral expression for BER is a sum of one or a few integrals of the form

$$\int_0^\infty x^m \exp(-x^2)\, I_n(bx)\, dx. \qquad \text{(I1)}$$

These may be expressed in terms of the confluent hypergeometric functions [30], [31], which can be reduced in our case to elementary functions using the Kummer transformation [30], [31].

Let us consider in detail the case of the $\{2 \times 2\}$ phase diversity receiver. Integrals in the other cases can be computed in the same way. We have then from (47), (51), and [31]:

$$\begin{aligned} \text{BER} &= \exp(-2\,\text{SNR}) \int_0^\infty y \\ &\quad \cdot \exp(-y^2)\, I_0(2\sqrt{\text{SNR}}\, y)\, dy \\ &= 0.5 \exp(-2\,\text{SNR})\, {}_1F_1(1, 1, \text{SNR}). \qquad \text{(I2)} \end{aligned}$$

Here ${}_1F_1(x, y, z)$ is the confluent hypergeometric function given by [31]:

$${}_1F_1(x, y, z) = 1 + \frac{(x)_1 z}{1!(y)_1} + \frac{(x)_2 z^2}{2!(y)_2} + \cdots \qquad \text{(I3)}$$

Here $(x)_n = x(x+1)(x+2)\cdots(x+n-1)$ and $(x)_0 = 1$.

The confluent hypergeometric function may be reduced in our case to the elementary functions using the Kummer transformation [31]:

$${}_1F_1(x, y, z) = {}_1F_1(y-x, y, -z)\exp(z). \qquad \text{(I4)}$$

Since from (I3) ${}_1F_1(0, 1, \text{SNR}) = 1$ we get finally the required result

$$\text{BER} = 0.5 \exp(-\text{SNR}). \qquad \text{(I5)}$$

In the case of the $\{2 \times 2\}$ and $\{3 \times 3\}$ phase and polarization diversity receivers a simpler derivation, avoiding the use of the hypergeometric function, is possible. This is due to one of the referees. Let us consider for instance the $\{2 \times 2\}$ phase and polarization diversity receiver. Using (48) and (52) the integral to be expressed analytically is

$$\int_0^\infty (1 + y^2/2)\, y^2 I_1(2\sqrt{\text{SNR}}\, y) \cdot \exp\left[-(y^2 + 2\,\text{SNR})\right] / \left[2\sqrt{\text{SNR}}\right] dy. \qquad \text{(I6)}$$

Changing the variable of integration $y = x/\sqrt{2}$ we get

$$0.25 \exp(-\text{SNR}) \int_0^\infty (1 + x^2/4)\, p_4(x)\, dx \qquad \text{(I7)}$$

$p_4(x)$ being the pdf of the modulus of a four-dimensional vector whose components are independent Gaussian variables with unit variances, i.e., $p_4(x)$ is given by (22) with $N = 4$ and noncentral parameter 2 SNR. The integral (I7) can then be expressed as

$$0.25 \exp(\text{SNR})\left(1 + 0.25\langle x^2 \rangle\right). \qquad \text{(I8)}$$

$\langle\ \rangle$ denoting the expected value. This can be immediately obtained $\langle x^2 \rangle = 2\,\text{SNR} + 4$, which gives

$$\text{BER} = 0.5 \exp(-\text{SNR})(1 + \text{SNR}/4). \qquad \text{(I9)}$$

Acknowledgment

The authors are grateful to the referees for many helpful comments. In particular, a simpler derivation of (56) and (57) is due to one of the referees.

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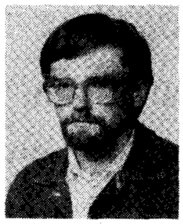

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