PART 2 : BALANCED HOMODYNE DETECTION

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OUTLINE

PART 1

1. Noise Properties of Photodetectors

2. Quantization of Light

3. Direct Photodetection and Photon Counting

PART 2

4. Balanced Homodyne Detection

5. Ultrafast Photon Number Sampling

PART 3

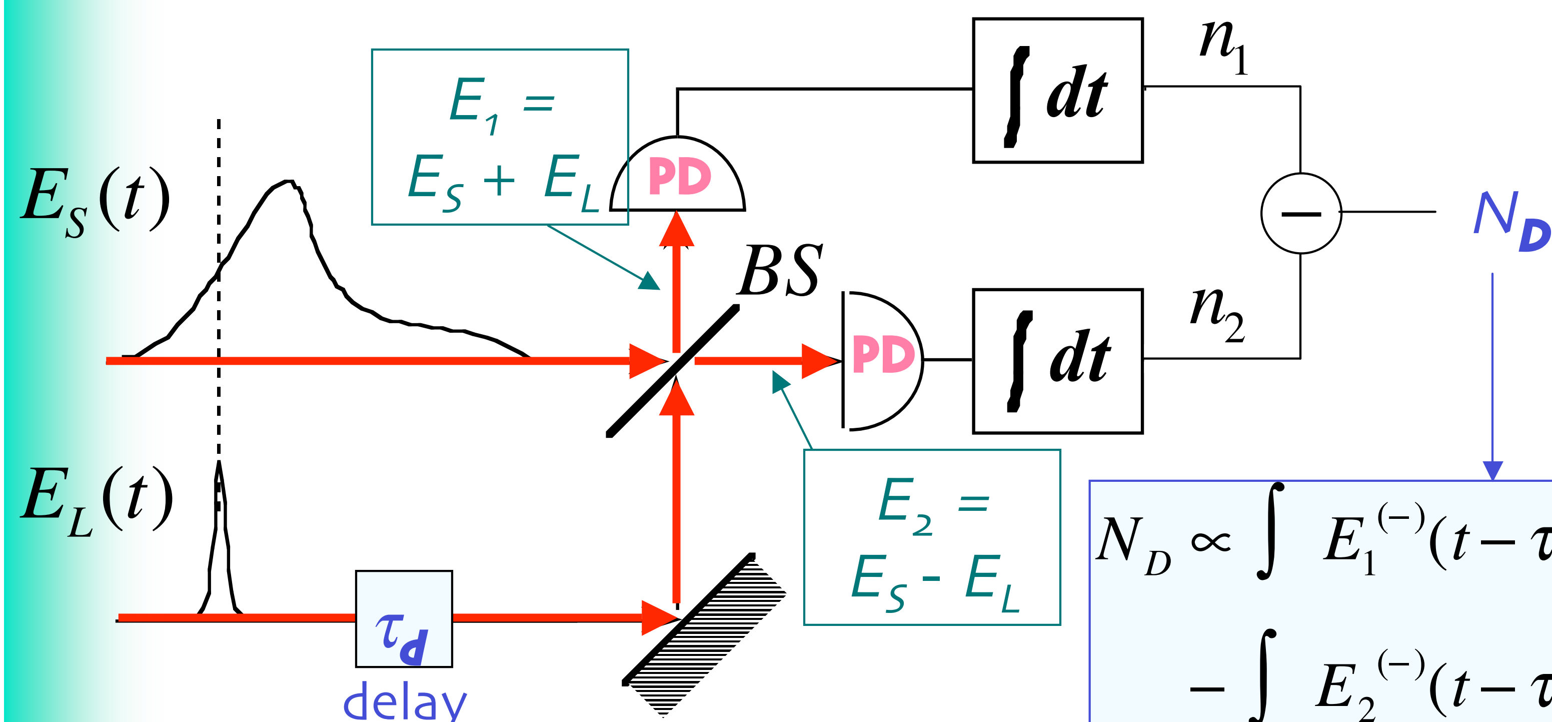
6. Quantum State Tomography

DC-BALANCED HOMODYNE DETECTION I

Goal -- measure quadrature amplitudes with high *Q.E.* and temporal-mode selectivity

E_S = signal field (ω_O), 1 - 1000 photons

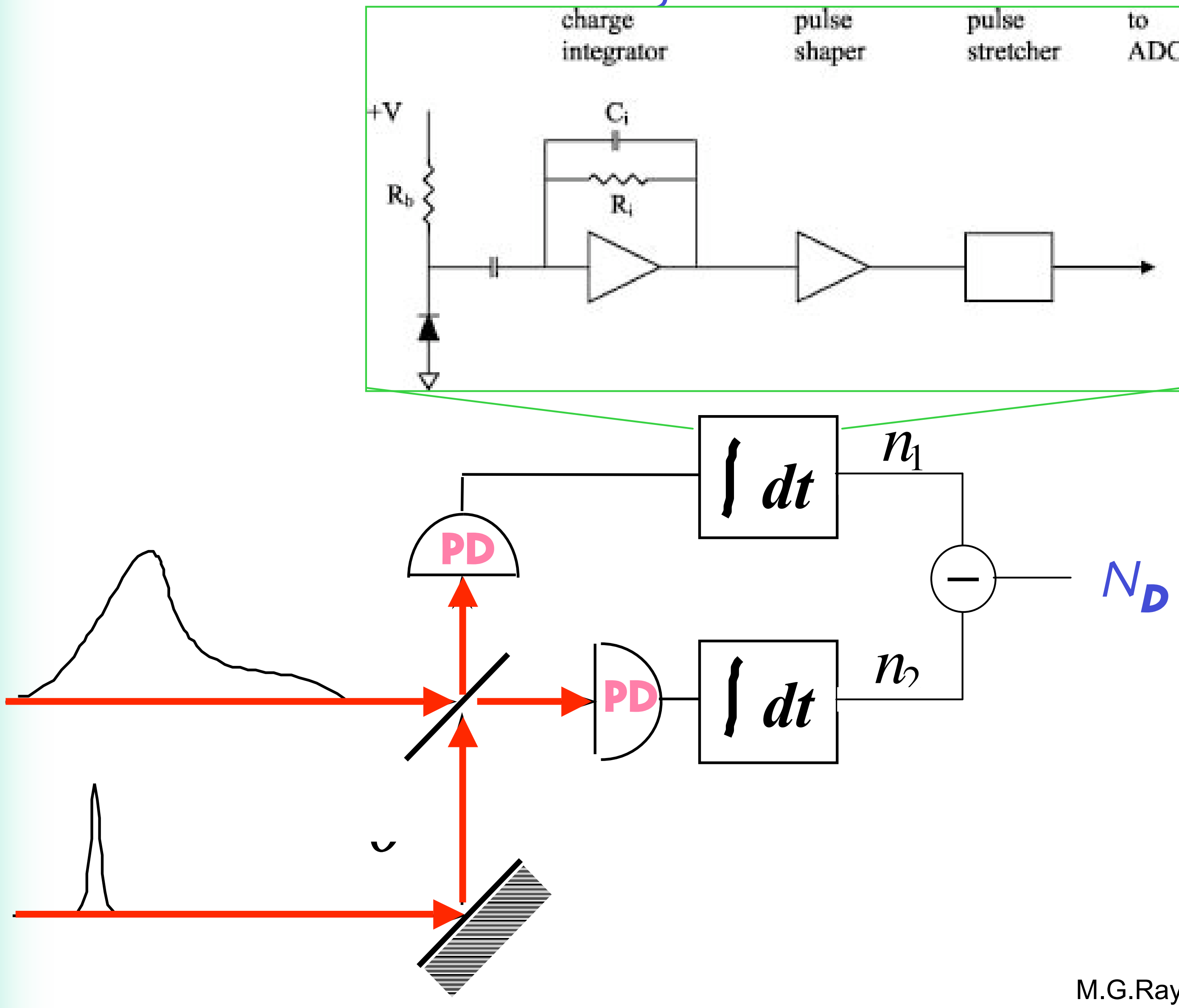
E_L = laser reference field (local oscillator) (ω_O), 10^6 photons



$$N_D \propto \int E_1^{(-)}(t-\tau_d)\,E_1^{(+)}(t)\,dt - \int E_2^{(-)}(t-\tau_d)\,E_2^{(+)}(t)\,dt$$

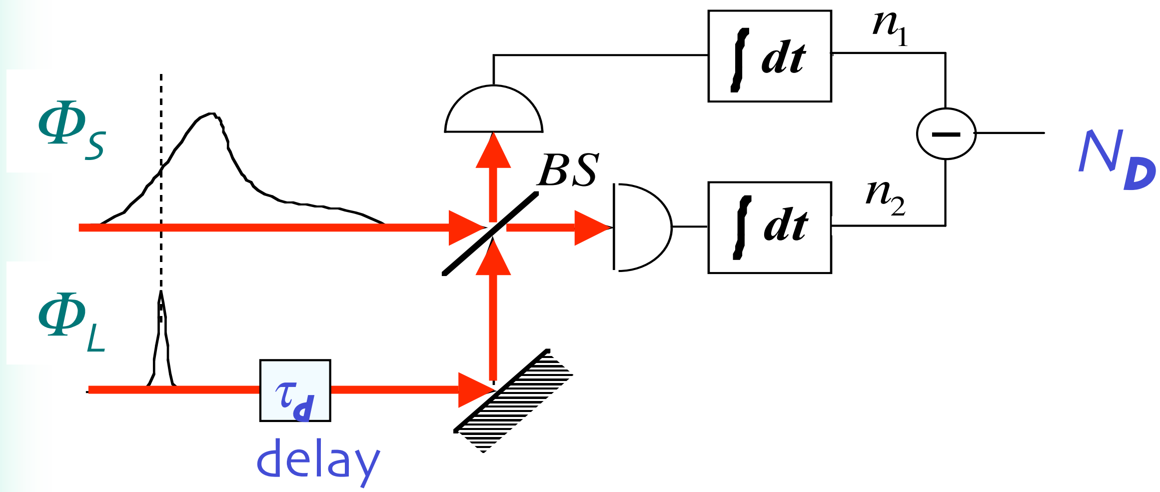
DC-BALANCED HOMODYNE DETECTION II

integrator circuit



DC-BALANCED HOMODYNE DETECTION III

Φ_S = signal amplitude; Φ_L = laser reference amplitude



$$\hat{N}_D = \int_0^T dt \int_{Det} d^2x \; \hat{\underline{\Phi}}_L^{(-)}(\underline{x},0,t-\tau_d)\cdot\hat{\underline{\Phi}}_S^{(+)}(\underline{x},0,t) \; + \; h.c.$$

overlap integral

$$\hat{\underline{\Phi}}_S^{(+)}(\underline{r},t) = i\sqrt{c}\sum_k \hat{a}_k \; \underline{v}_k(\underline{r},t)$$

$$\underline{v}_k(\underline{r},t) = \sum_j C_{kj}\; \underline{u}_j(\underline{r}) \exp(-i\omega_j t)$$

$$c\int_0^T dt \int_{Det} d^2x \; \underline{v}^*_k(\underline{x},0,t)\cdot\underline{v}_m(\underline{x},0,t) = \delta_{km}$$

wave-packet modes

DC-BALANCED HOMODYNE DETECTION IV

$$\hat{N}_D \propto \int_0^T dt \int_{Det} d^2x \; \underline{\hat{\Phi}}_L^{(-)}(\underline{x},0,t-\tau_d) \cdot \sum_k \hat{a}_k \; \underline{v}_k(\underline{x},0,t) \quad + \quad h.c.$$

wave-packet modes

Assume that the LO pulse is a *strong coherent state* of a particular localized wave packet mode:

LO phase

$$\underline{\hat{\Phi}}_L^{(+)}(\underline{r},t) \propto |\alpha_L| exp(i\theta) \, \underline{v}_L(\underline{r},t) \; + \; vacuum$$

$$\hat{N}_D(\theta) = |\alpha_L| (\hat{a} \, e^{-i\theta} + \hat{a}^\dagger e^{i\theta})$$

$$\hat{a} = \sum_k \hat{a}_k \, c \int_0^T dt \int_{Det} d^2x \; \underline{v}^*_L(\underline{x},0,t-\tau_d) \cdot \underline{v}_k(\underline{x},0,t) \; = \; \hat{a}_{k=L}$$

The signal field is spatially and temporally gated by the LO field, which has a controlled shape. Where the LO is zero, that portion of the signal is rejected. Only a single temporal-spatial wave-packet mode of the signal is detected.

DC-BALANCED HOMODYNE DETECTION V

signal : $$\underline{\hat{\Phi}}_S^{(+)}(\underline{r},t) \propto \hat{a}\ \underline{v}_L(\underline{r},t) + \sum_k \hat{a}_k\ \underline{v}_k(\underline{r},t)$$

wave-packet modes

quadrature operators:

$$\hat{q} = (\hat{a} + \hat{a}^\dagger)/2^{1/2}$$

$$\hat{p} = (\hat{a} - \hat{a}^\dagger)/i2^{1/2}$$

detected quantity:

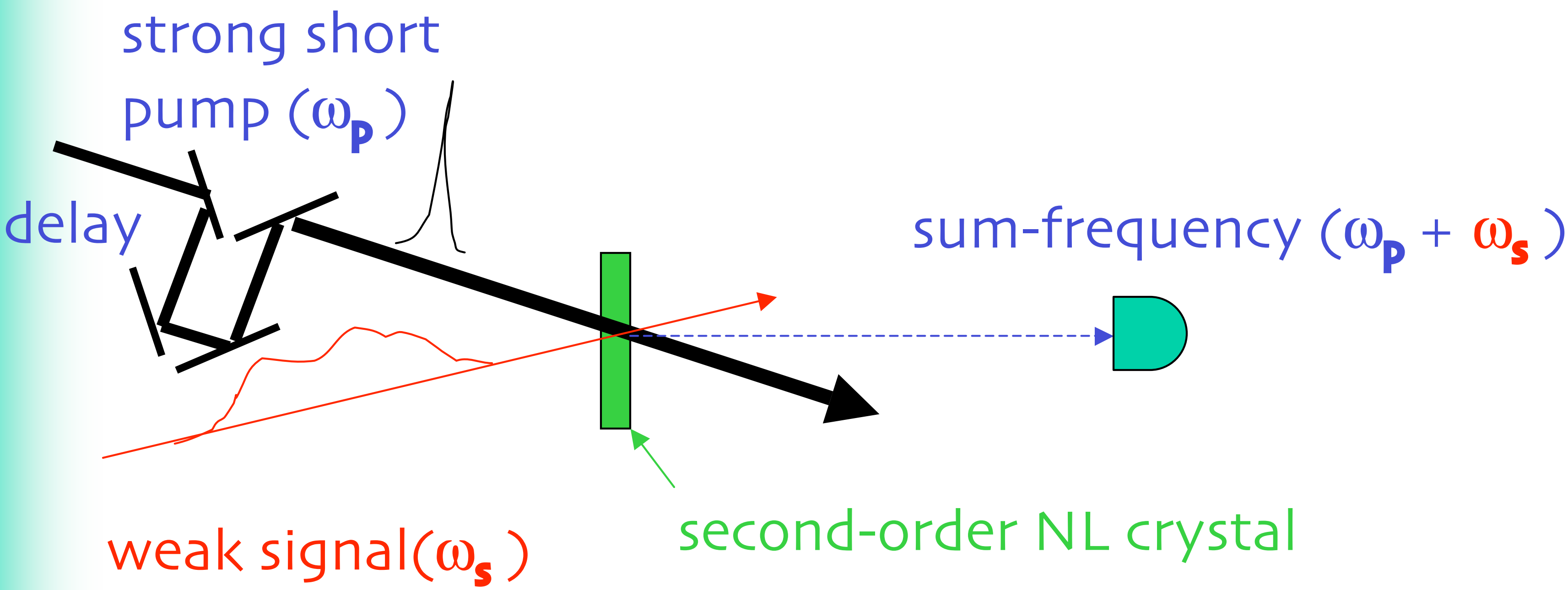
$$\hat{q}_\theta \equiv \frac{\hat{N}_D(\theta)}{|\alpha_L|\sqrt{2}} = \frac{\hat{a}\,e^{-i\theta} + \hat{a}^\dagger\,e^{i\theta}}{\sqrt{2}}$$

LO phase

$$\hat{q}_\theta \equiv \frac{\hat{N}_D(\theta)}{|\alpha_L|\sqrt{2}} = \hat{q}\cos\theta + \hat{p}\sin\theta$$

$$\begin{pmatrix} \hat{q}_\theta \\ \hat{p}_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{q} \\ \hat{p} \end{pmatrix}$$

Conventional Approach:
Ultrafast Time Gating of Light Intensity by NON-LINEAR OPTICAL SAMPLING



LINEAR OPTICAL SAMPLING I

BHD for Ultrafast Time Gating of Quadrature Amplitudes

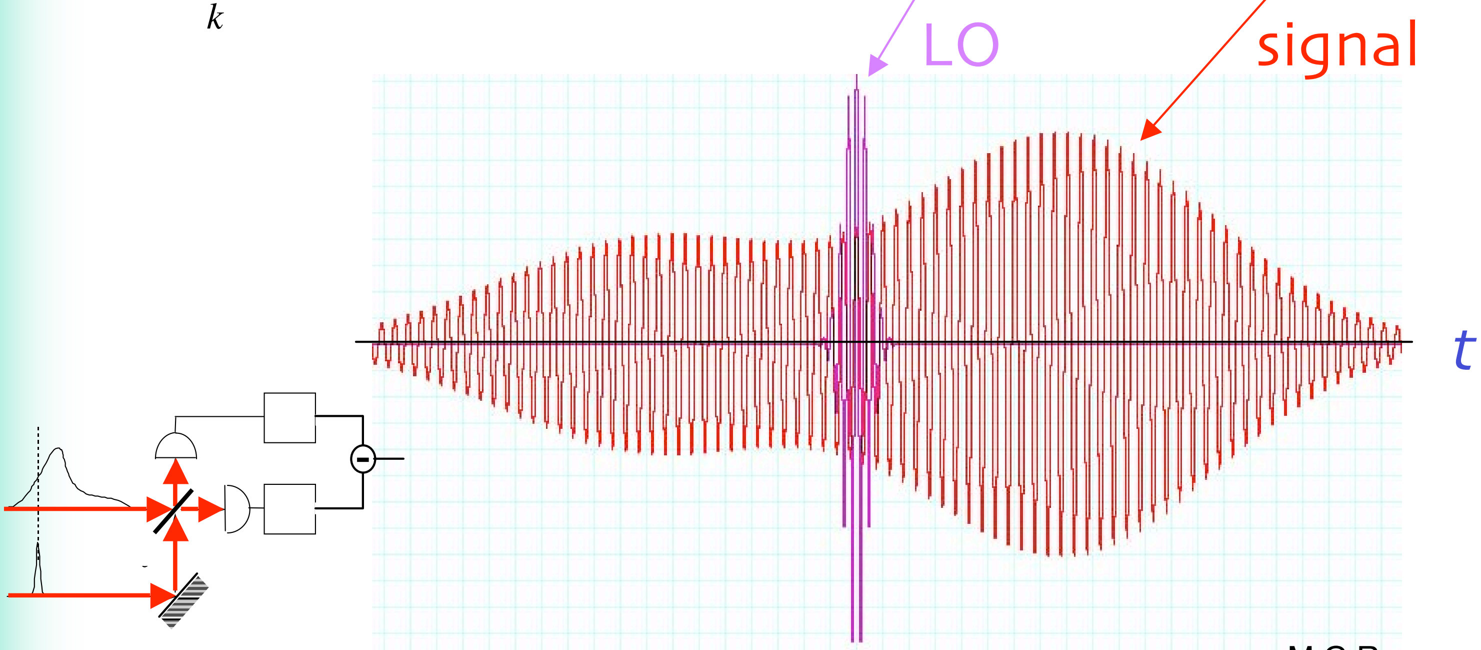
detected quantity:

$$\hat{q}_\theta \equiv \frac{\hat{N}_D(\theta)}{|\alpha_L|\sqrt{2}} = \hat{q}\cos\theta + \hat{p}\sin\theta$$

LO phase

$$\hat{q} = (\hat{a} + \hat{a}^\dagger)/2^{1/2} \qquad \hat{p} = (\hat{a} - \hat{a}^\dagger)/i2^{1/2}$$

$$\underline{\hat{a}} = \sum_k \hat{a}_k \, c \int_0^T dt \int_{Det} d^2x \; \underline{v}^*_L(\underline{x},0,t-\tau_d) \cdot \underline{v}_k(\underline{x},0,t) = \hat{a}_{k=L}$$



LINEAR OPTICAL SAMPLING II

Ultrafast Time Gating of Quadrature Amplitudes

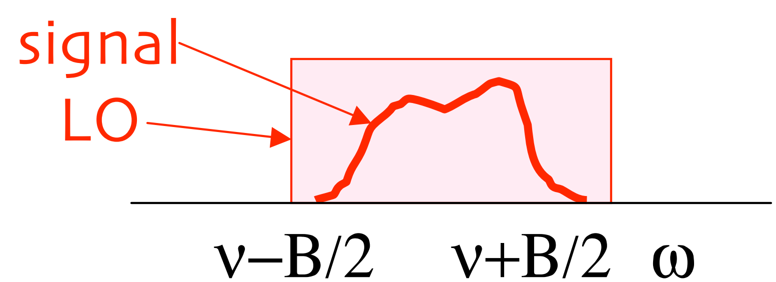
LO mode: $\underline{v}_L(\underline{x},0,t) \propto \alpha_L \; \underline{v}_L(\underline{x}) \; f_L(t-\tau_d)$

$$\Rightarrow \quad \hat{N}_D(\tau_d) = -i\sqrt{c}\,\alpha_L^* \int_0^T dt\, f_L^*(t-\tau_d)\, \phi_S(t) \;+\; h.c.$$

$$\phi_S(t) = \int_{Det} d^2x \;\underline{v}_L{}^*(\underline{x}) \cdot \hat{\underline{\Phi}}_S^{(+)}(\underline{x},0,t)$$

if signal is band-limited and LO covers the band, e.g.

$$f_L(t) \propto (1/t)\sin(B\,t/2)$$



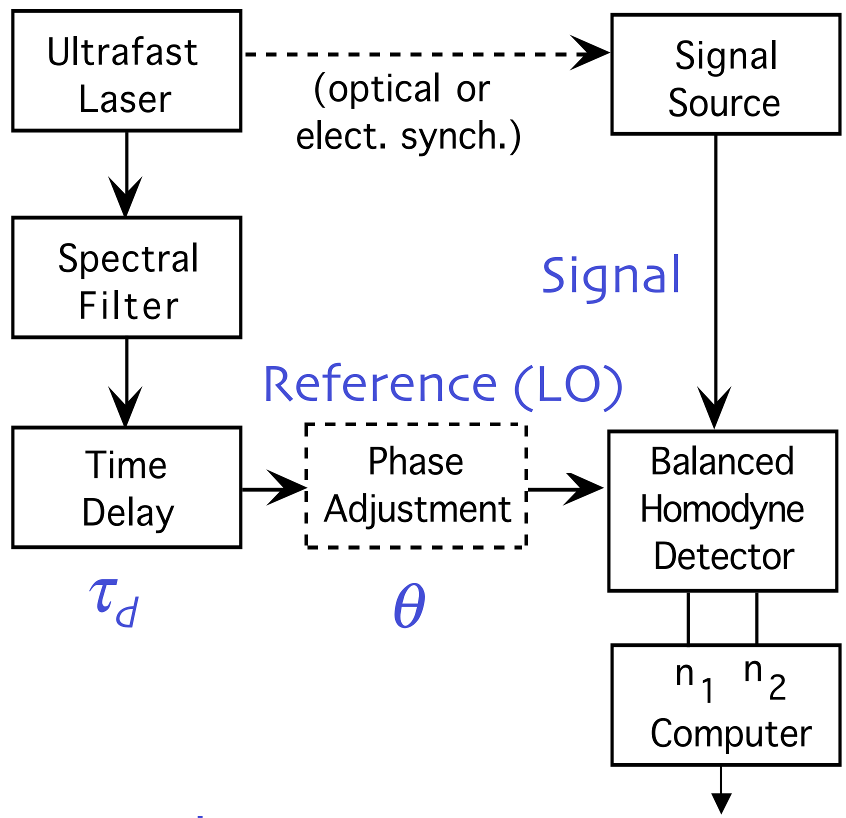
$$\hat{N}_D(\tau_d) \propto \alpha_L^* \tilde{f}_L^*(\nu) \int_{\nu-B/2}^{\nu+B/2} \frac{d\omega}{2\pi} \exp(-i\omega\tau_d)\, \tilde{\phi}_S(\omega) \;+\; h.c.$$

$$\propto \alpha_L^* \tilde{f}_L^*(\nu)\, \underline{\phi_S(\tau_d)} \;+\; h.c.$$

exact sampling

LINEAR OPTICAL SAMPLING III

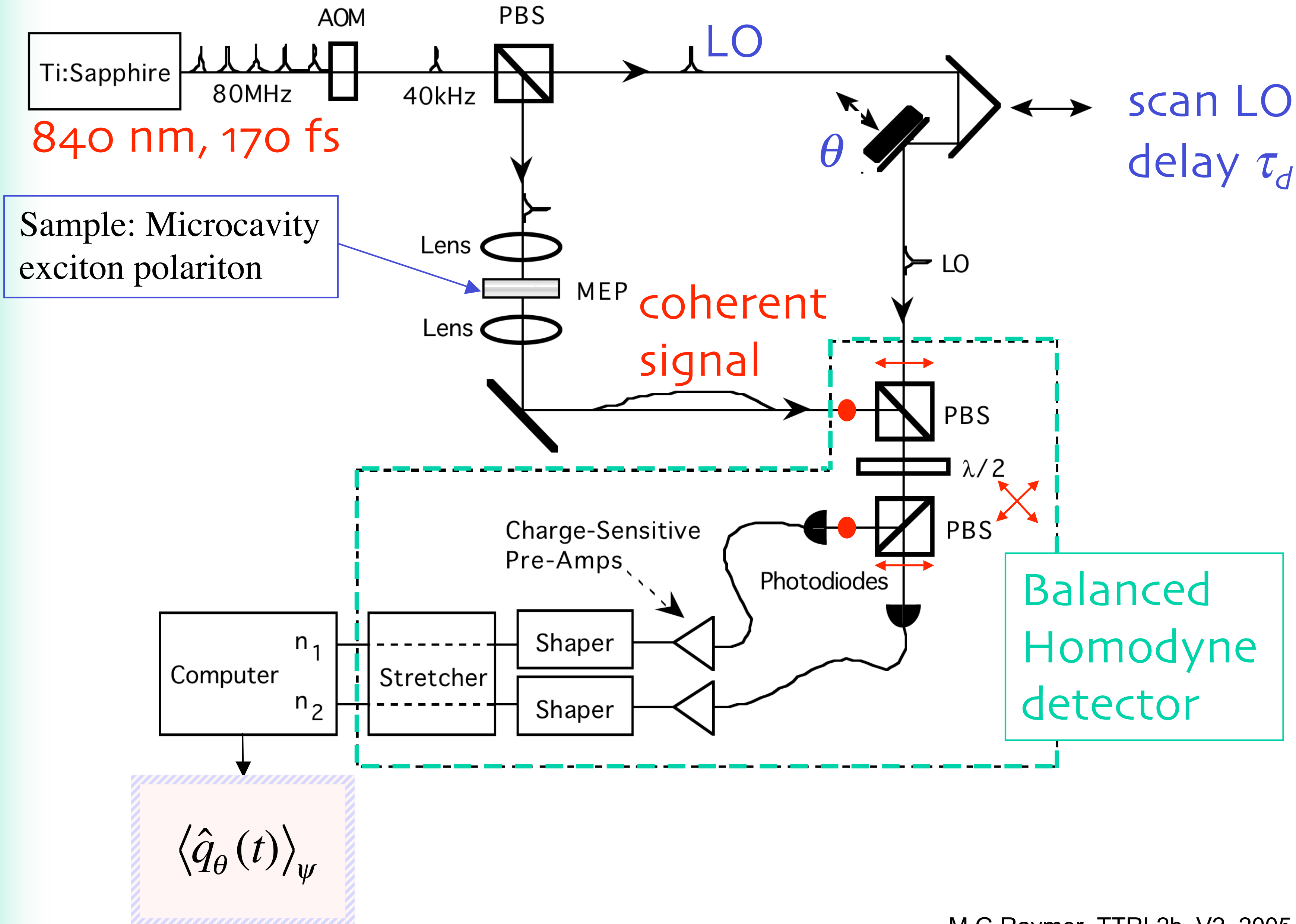
M. E. Anderson, M. Munroe, U. Leonhardt, D. Boggavarapu, D. F. McAlister and M. G. Raymer, Proceedings of Generation, Amplification, and Measurment of Ultrafast Laser Pulses III, pg 142-151 (OE/LASE, San Jose, Jan. 1996) (SPIE, Vol. 2701, 1996).



mean quadrature amplitude in sampling window at time t

$$\langle \hat{q}_\theta (t) \rangle_\psi$$

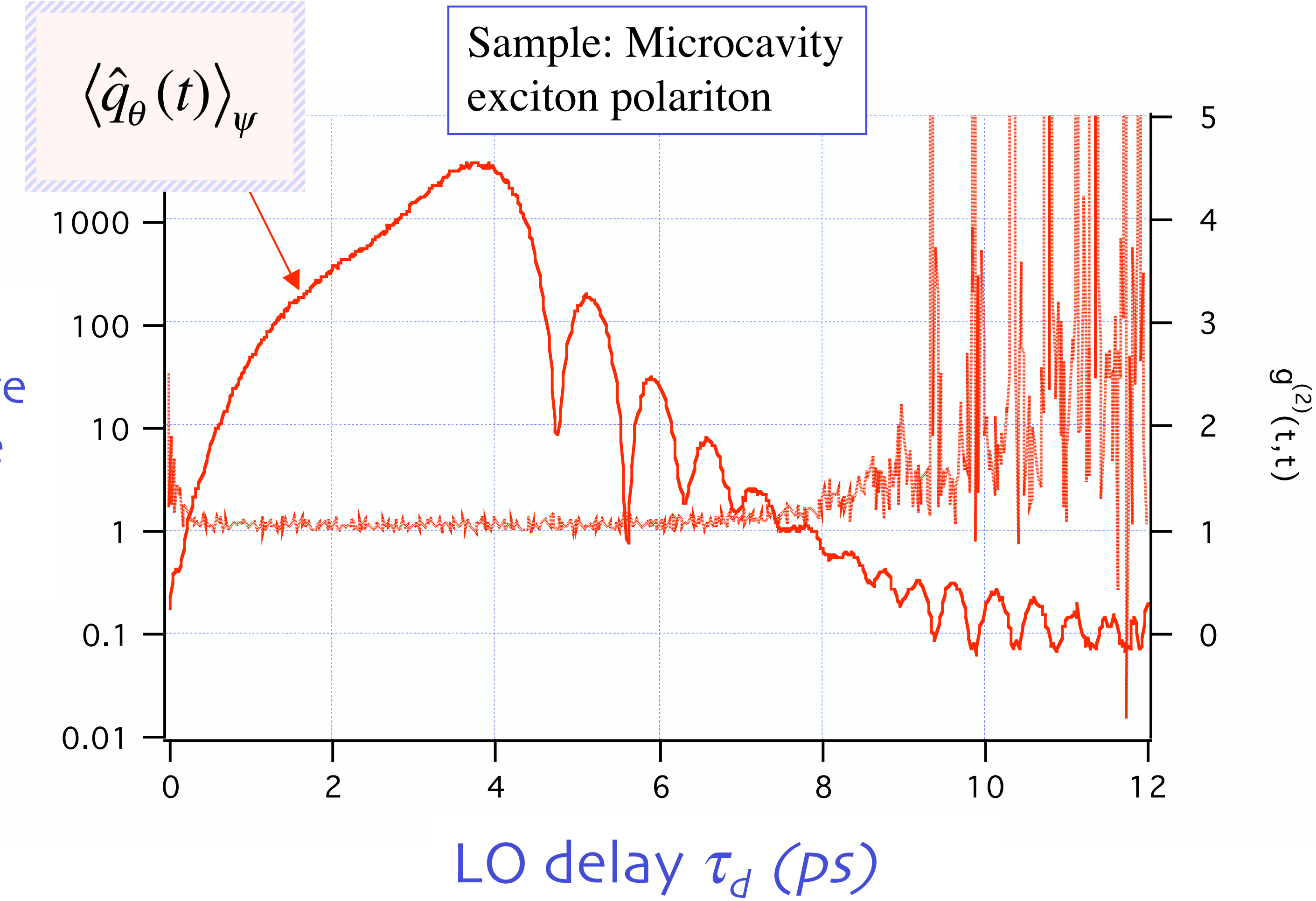
LINEAR OPTICAL SAMPLING IV



LINEAR OPTICAL SAMPLING V

Mean Quadrature Measurement - sub ps Time Resolution

mean quadrature amplitude <q> at time t



coherent field --> $\langle \hat{q}_{\theta+\pi/2}(t) \rangle_\psi = \langle \hat{p}_\theta(t) \rangle_\psi \cong 0$

LINEAR OPTICAL SAMPLING VI

Phase Sweeping for Indirect Sampling of Mean Photon Number and Photon Number Fluctuations

detected quantity: $\hat{q}_\theta \equiv \dfrac{\hat{N}_D(\theta)}{|\alpha_L|\sqrt{2}} = \hat{q}\cos\theta + \hat{p}\sin\theta$ (θ = LO phase)

Relation with photon-number operator:

$$\hat{n} = \hat{a}^\dagger \hat{a} = \frac{1}{2}(\hat{q} - i\hat{p})(\hat{q} + i\hat{p}) = \hat{q}^2 + \hat{p}^2 + \frac{1}{2}$$

Phase-averaged quadrature-squared:

$$\left\langle \hat{q}_\theta{}^2 \right\rangle_\theta = \frac{1}{\pi}\int_0^\pi \hat{q}_\theta{}^2\, d\theta = \frac{1}{\pi}\int_0^\pi (\hat{q}\cos\theta + \hat{p}\sin\theta)^2\, d\theta = \frac{1}{2}\left(\hat{q}^2 + \hat{p}^2\right)$$

$$\hat{n} = \left\langle \hat{q}_\theta{}^2 \right\rangle_\theta - \frac{1}{2} \quad \xrightarrow[\text{average}]{\text{ensemble}} \quad \left\langle \hat{n}(t) \right\rangle_\psi = \left\langle \left\langle \hat{q}_\theta{}^2(t) \right\rangle_\theta \right\rangle_\psi - \frac{1}{2}$$

works also for incoherent field (no fixed phase)

LINEAR OPTICAL SAMPLING VII

Phase Sweeping --> Photon Number Fluctuations

detected quantity: $\hat{q}_\theta \equiv \dfrac{\hat{N}_D(\theta)}{|\alpha_L|\sqrt{2}} = \hat{q}\cos\theta + \hat{p}\sin\theta$

Richter's formula for Factorial Moments:

$$\left\langle n^{(r)} \right\rangle_\psi = \sum_{n=0}^{\infty} [n(n-1)\ldots(n-r+1)]\; p(n) = \left\langle (\hat{a}^\dagger)^r (\hat{a})^r \right\rangle_\psi$$

$$= \frac{(r!)^2}{2^r (2r)!} \int_0^{2\pi} \frac{d\theta}{2\pi} \langle H_{2r}(\hat{q}_\theta) \rangle_\psi$$

Hermite Polynomials: $H_0(x) = 1,\ H_1(x) = 2x,\ H_3(x) = 4x^2 - 2$

$$\left\langle n^{(1)} \right\rangle = \left\langle \hat{a}^\dagger \hat{a} \right\rangle = \frac{1}{4} \int_0^{2\pi} \frac{d\theta}{2\pi} \left\langle 4\hat{q}_\theta{}^2 - 2 \right\rangle_\psi \longrightarrow \langle \hat{n}(t) \rangle_\psi = \left\langle \left\langle \hat{q}_\theta{}^2(t) \right\rangle_\theta \right\rangle_\psi - \frac{1}{2}$$

$$\left\langle n^{(2)} \right\rangle = \left\langle \hat{a}^{\dagger 2} \hat{a}^2 \right\rangle = \int_0^{2\pi} \frac{d\theta}{2\pi} \left\langle \frac{2}{3}\hat{q}_\theta{}^4 - 2\hat{q}_\theta{}^2 + \frac{1}{2} \right\rangle_\psi$$

LINEAR OPTICAL SAMPLING VIII

Phase Sweeping --> Photon Number Fluctuations

Variance of Photon Number in Sampling Time Window: var(n)=< n^2 > - < n >^{2}

$$\mathrm{var}(n) = \int_0^{2\pi} \frac{d\theta}{2\pi} \left[\left\langle \frac{2}{3} \hat{q}_\theta{}^4 \right\rangle - \left\langle \hat{q}_\theta{}^2 \right\rangle - \left\langle \hat{q}_\theta{}^2 \right\rangle^2 + \frac{1}{4} \right]$$

Second-Order Coherence of Photon Number in Sampling Time Window:

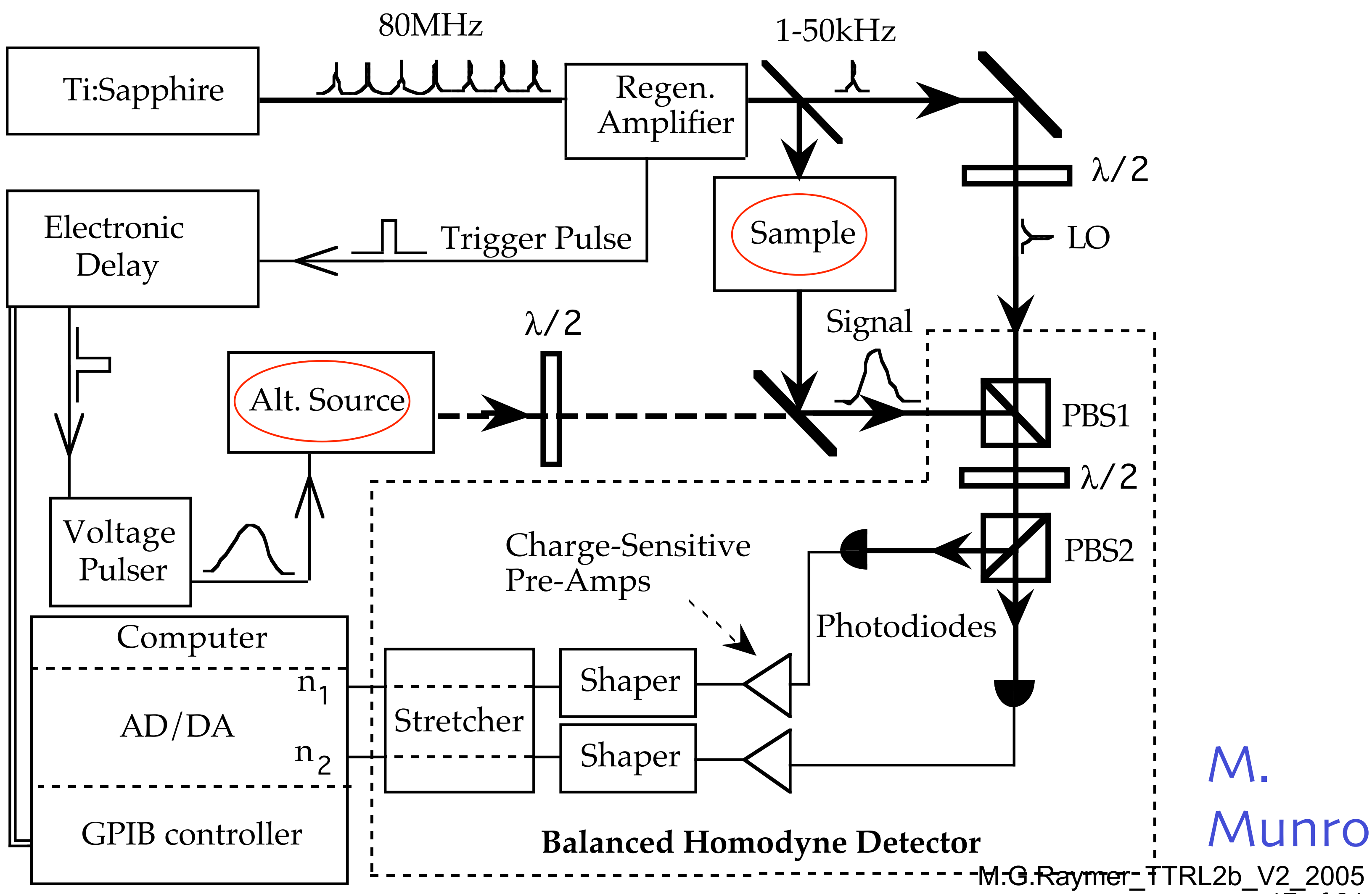
$$g^{(2)}(t,t) = [< n^2 > - < n >]/< n >^2$$

$g^{(2)}(t,t) = 2$ corresponds to thermal light, i.e. light produced primarily by spontaneous emission.

$g^{(2)}(t,t) = 1$ corresponds to light with Poisson statistics, i.e., light produced by stimulated emission in the presence of gain saturation.

LINEAR OPTICAL SAMPLING IX
Photon Number Fluctuations

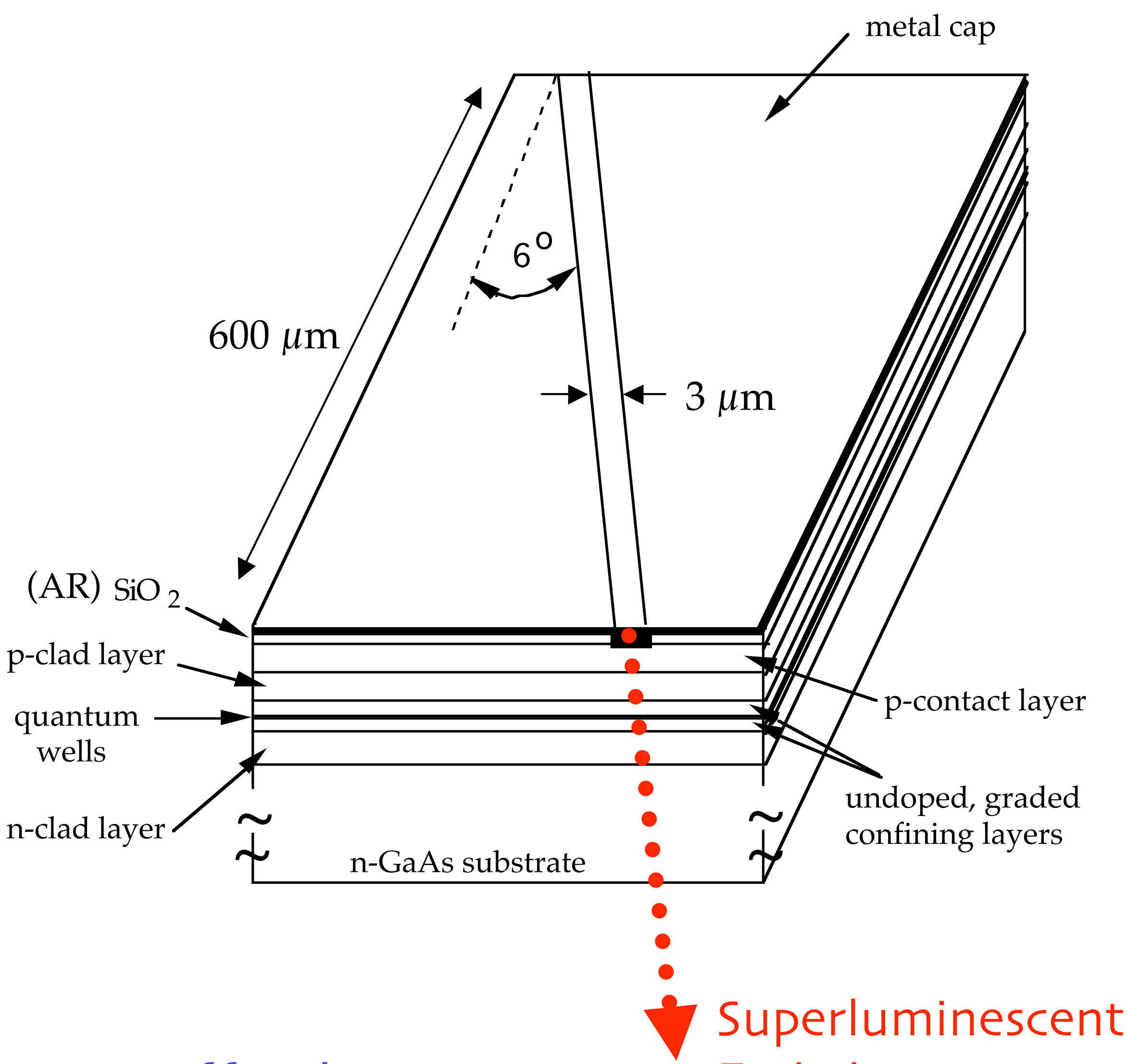
if the signal is incoherent, no phase sweeping is required



M. Munroe

LINEAR OPTICAL SAMPLING X

Superluminescent Diode (SLD) Optical Amplifier

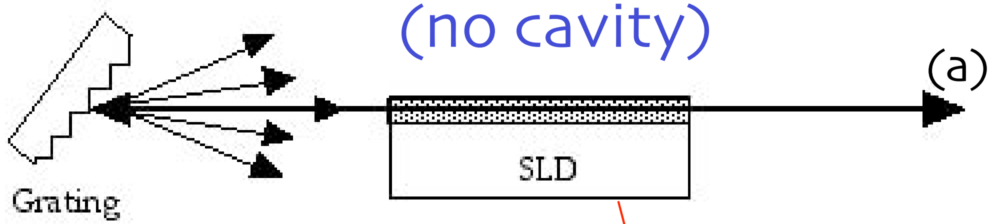


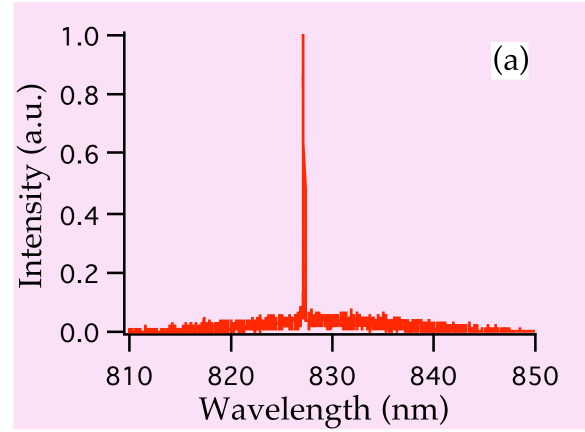
(Sarnoff Labs)

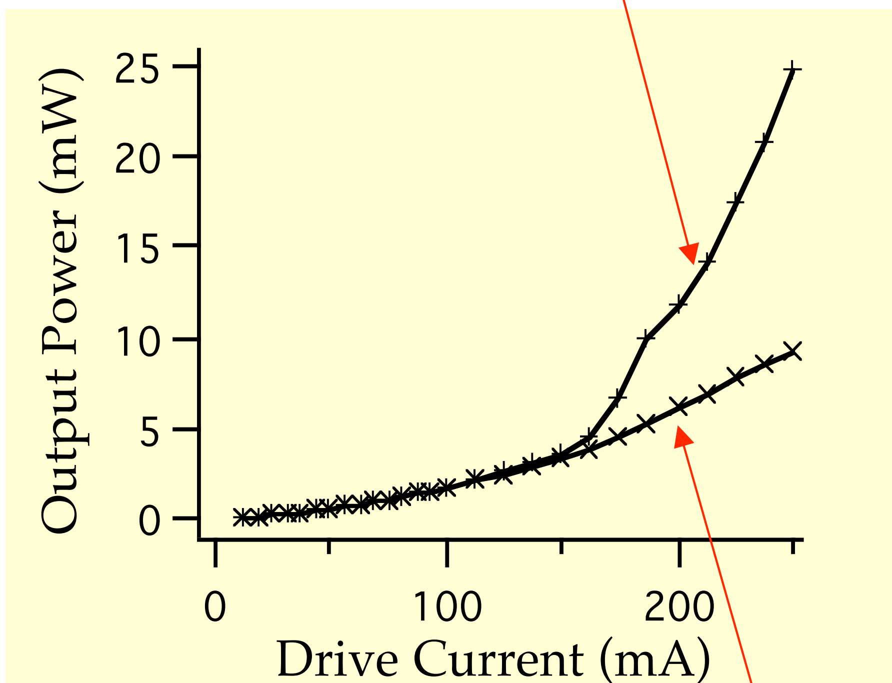
M. Munroe

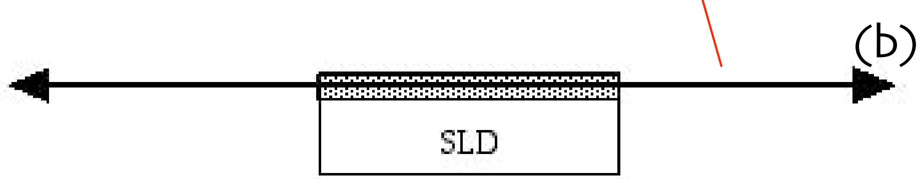
LINEAR OPTICAL SAMPLING XI

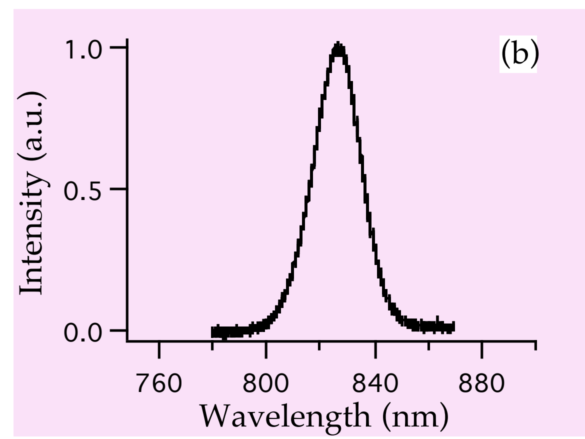
(no cavity)





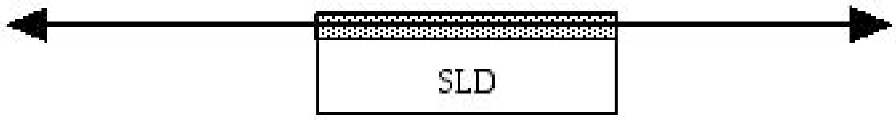


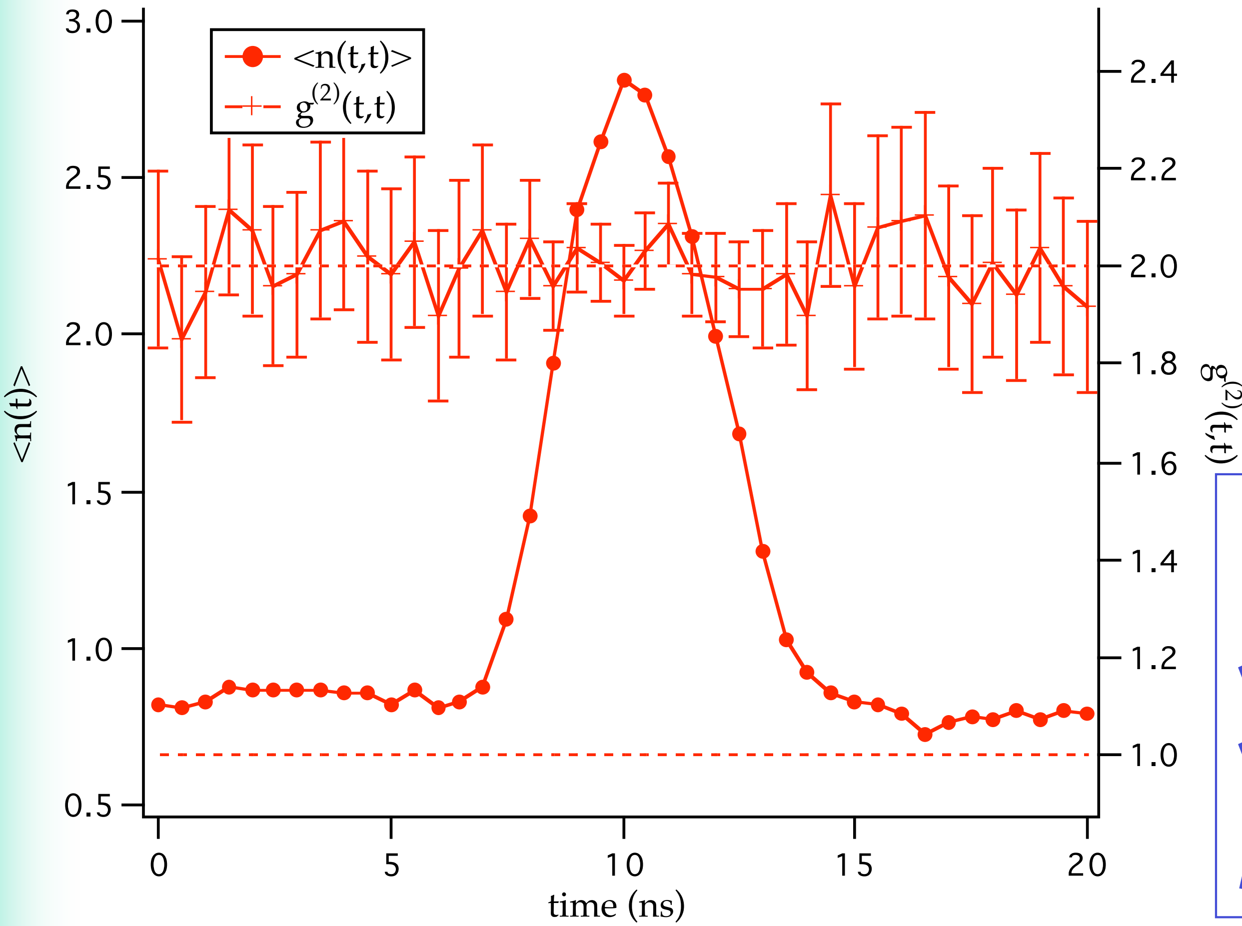




M. Munroe

LINEAR OPTICAL SAMPLING XII
SLD in the single-pass configuration



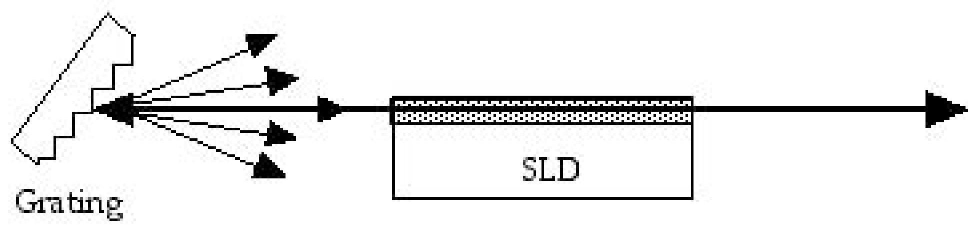


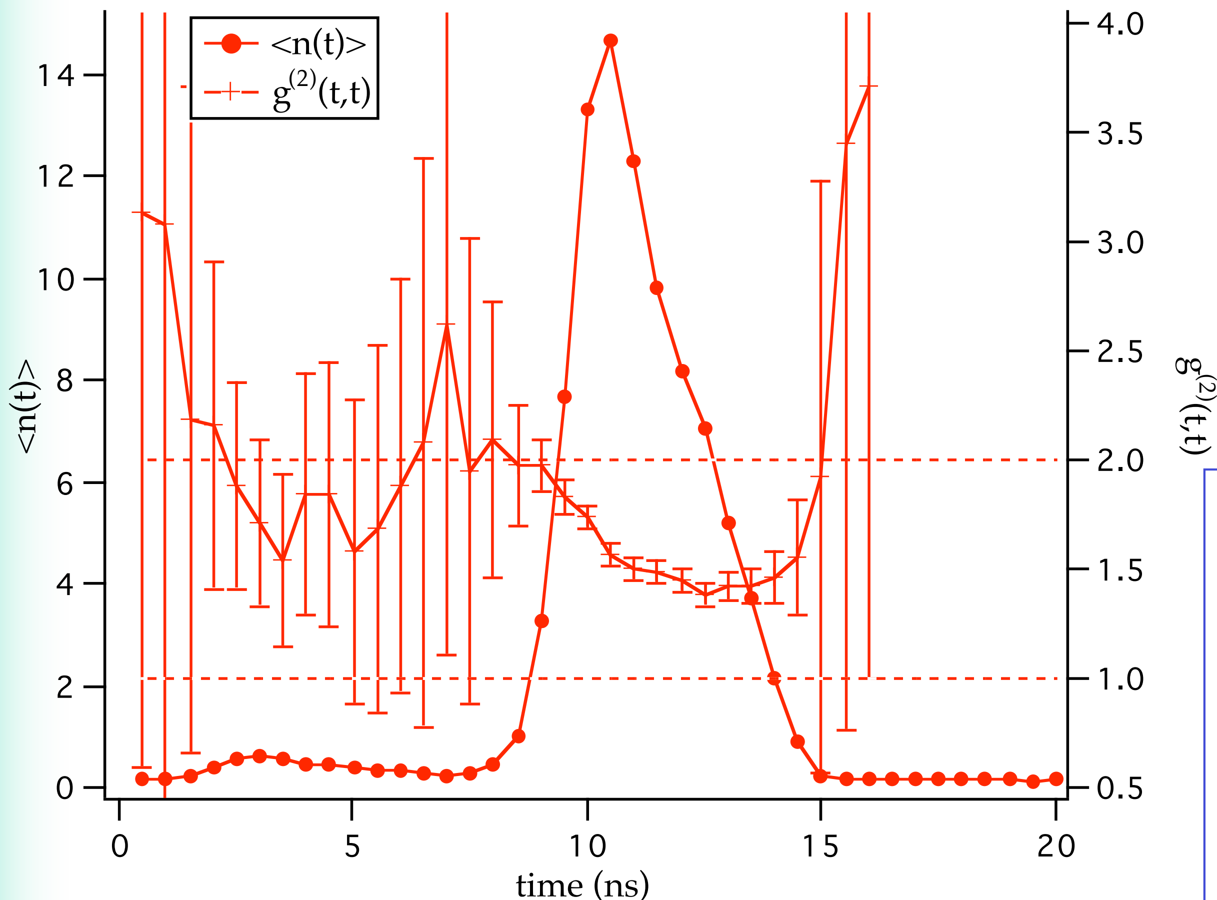
Photon Fluctuation is Thermal-like, within a single time window (150 fs)

M. Munroe

LINEAR OPTICAL SAMPLING XIII

SLD in the double-pass with grating configuration



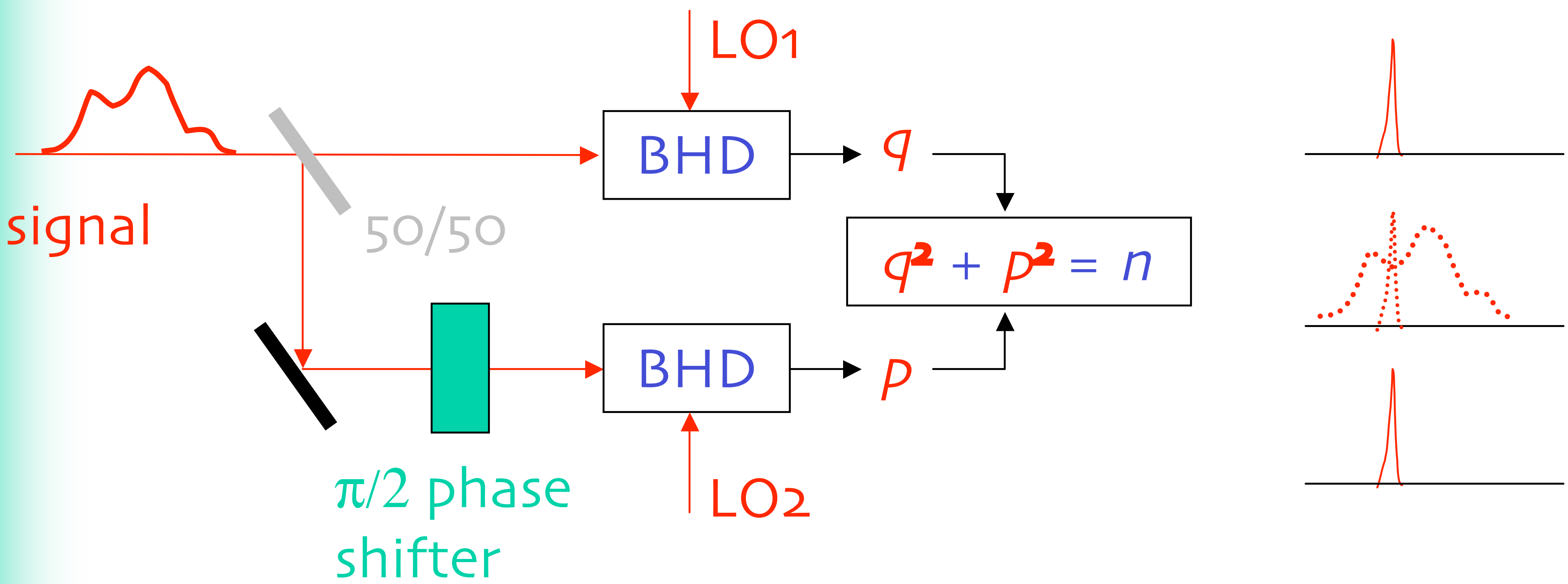


Photon Fluctuation is Laser-like, within a single time window (150 fs)

M. Munroe

Single-Shot Linear Optical Sampling I

-- Does not require phase sweeping.
Measure both quadratures simultaneously.
Dual- DC-Balanced Homodyne Detection



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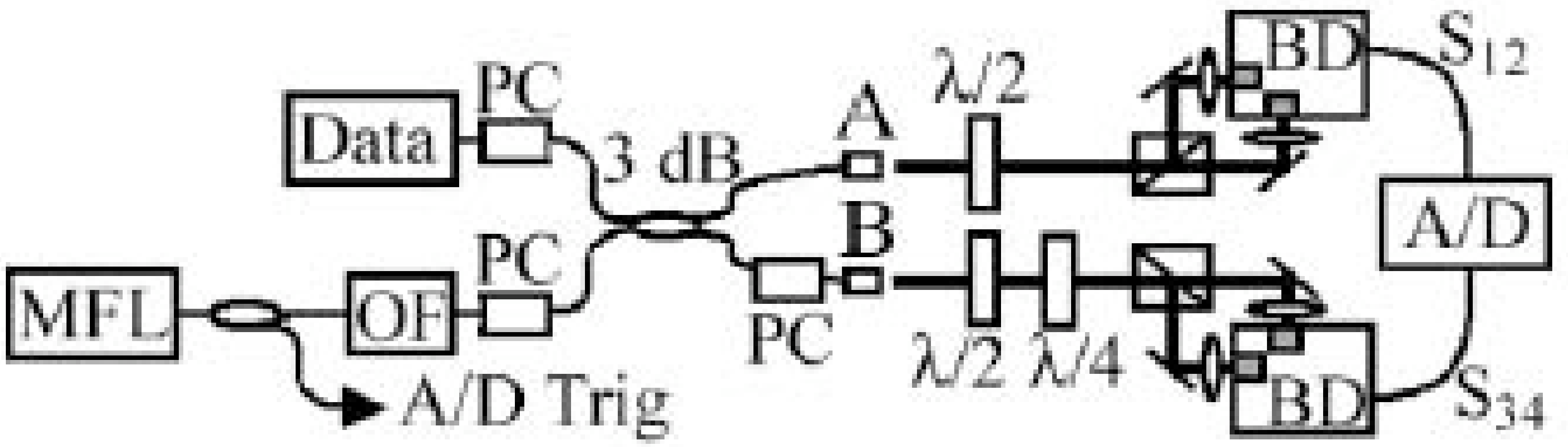
Linear Optical Sampling

C. Dorrer, D. C. Kilper, H. R. Stuart, G. Raybon, and M. G. Raymer

Fiber Implementation of Single-shot Linear Optical Sampling Of Photon Number



MFL: mode-locked Erbium-doped fiber laser. OF: spectral filter.
PC: polarization controller. BD: balanced detector.

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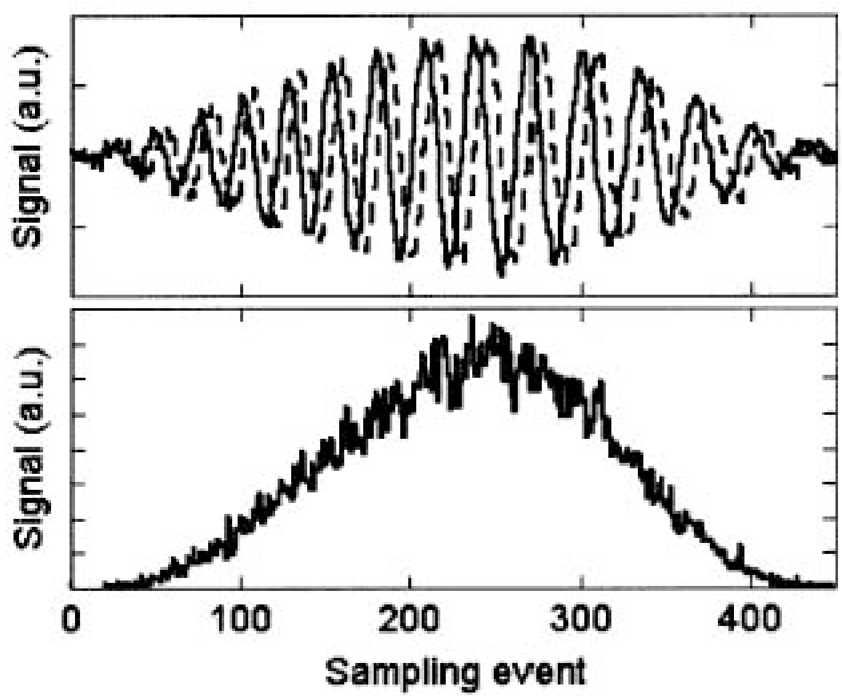
Linear Optical Sampling

C. Dorrer, D. C. Kilper, H. R. Stuart, G. Raybon, and M. G. Raymer



Measured quadratures (continuous and dashed line) on a 10-Gb/s pulse train.

Waveform obtained by postdetection squaring and summing of the two quadratures.

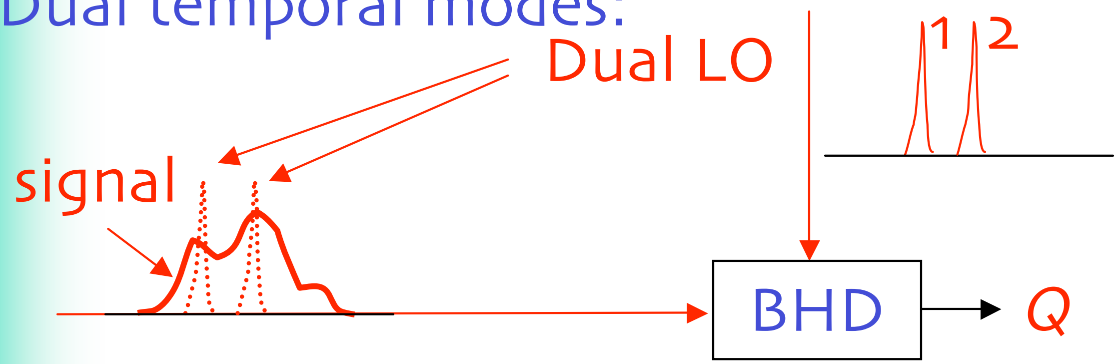
Two-Mode DC-HOMODYNE DETECTION I

LO is in a Superposition of two wave-packet modes, 1 and 2

$$\hat{\underline{\Phi}}_L^{(+)}(\underline{r},t) = i\sqrt{c}\,|\alpha_L|\exp(i\theta)\left[\underline{v}_1(\underline{r},t)\cos\alpha + \underline{v}_2(\underline{r},t)\exp(-i\zeta)\sin\alpha\right]$$

Dual temporal modes:

(temporal, spatial, or polarization)



$$\beta = \theta - \zeta$$

$$\hat{Q} = \cos(\alpha)\underbrace{[\hat{q}_1\cos\theta + \hat{p}_1\sin\theta]}_{\hat{q}_{1\theta}} + \sin(\alpha)\underbrace{[\hat{q}_2\cos\beta + \hat{p}_2\sin\beta]}_{\hat{q}_{2\beta}}$$

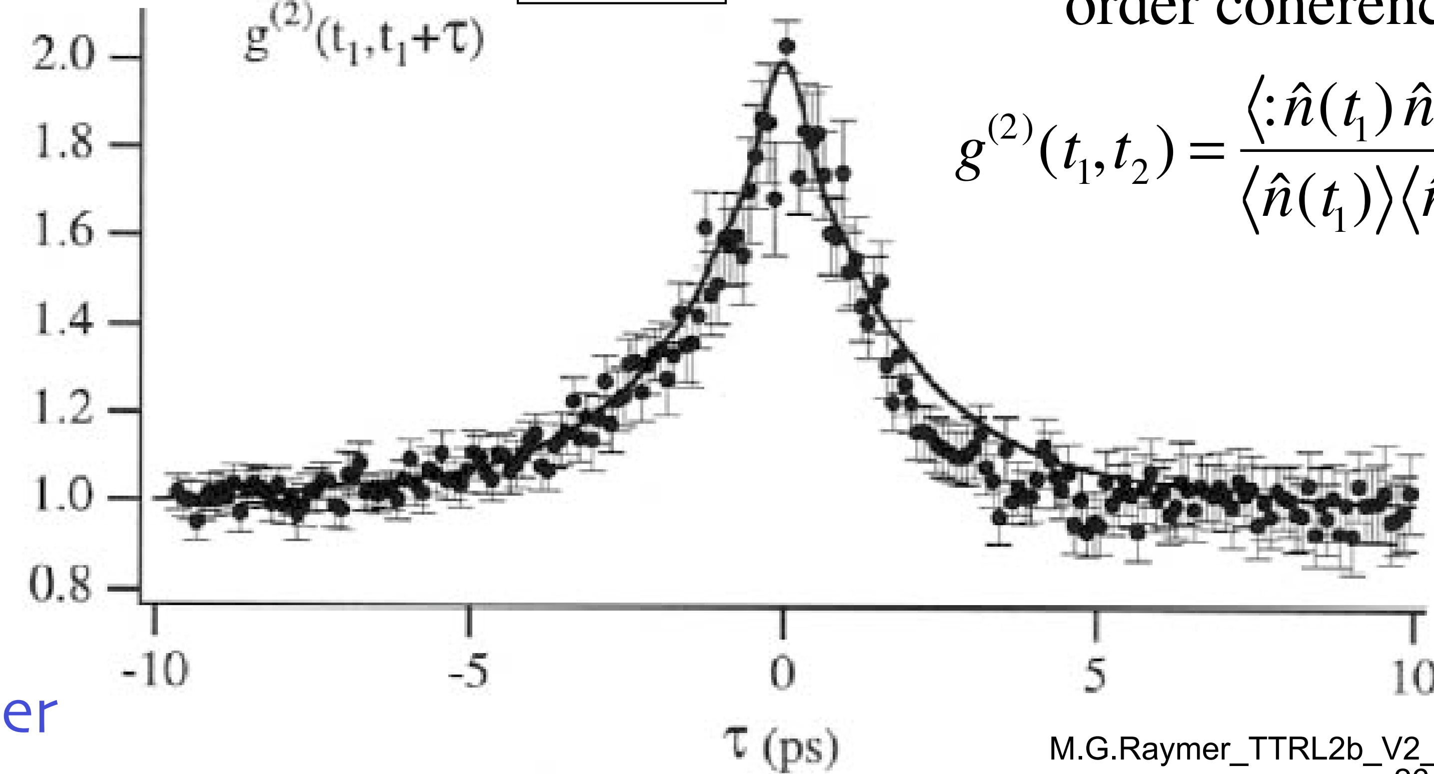
quadrature of mode 1 quadrature of mode 2

Two-Mode DC-HOMODYNE DETECTION II

ultrafast two-time number correlation measurements using dual-LO BHD; super luminescent laser diode (SLD)

two-time second-order coherence

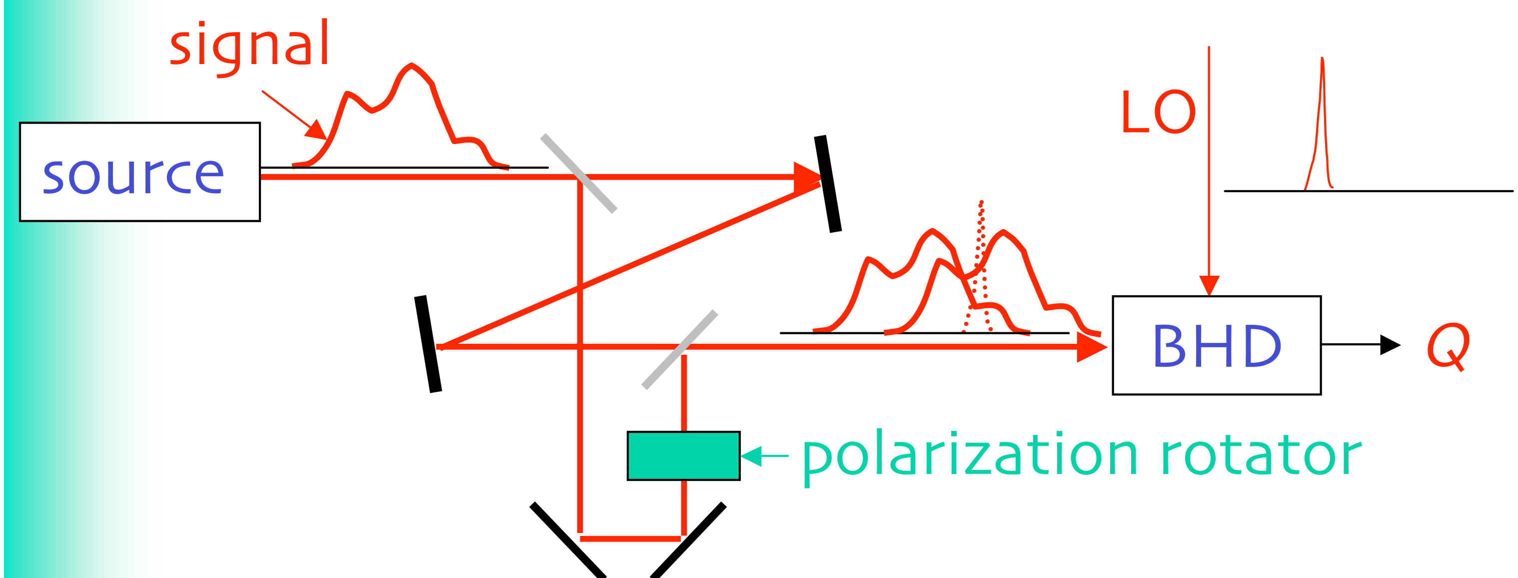
$$g^{(2)}(t_1,t_2) = \frac{\langle : \hat{n}(t_1)\,\hat{n}(t_2) : \rangle}{\langle \hat{n}(t_1)\rangle \langle \hat{n}(t_2)\rangle}$$



D. McAlister

Two-Mode DC-HOMODYNE DETECTION III

Alternative Method using a Single LO.
Signal is split and delayed by different times.
Polarization rotations can be introduced.



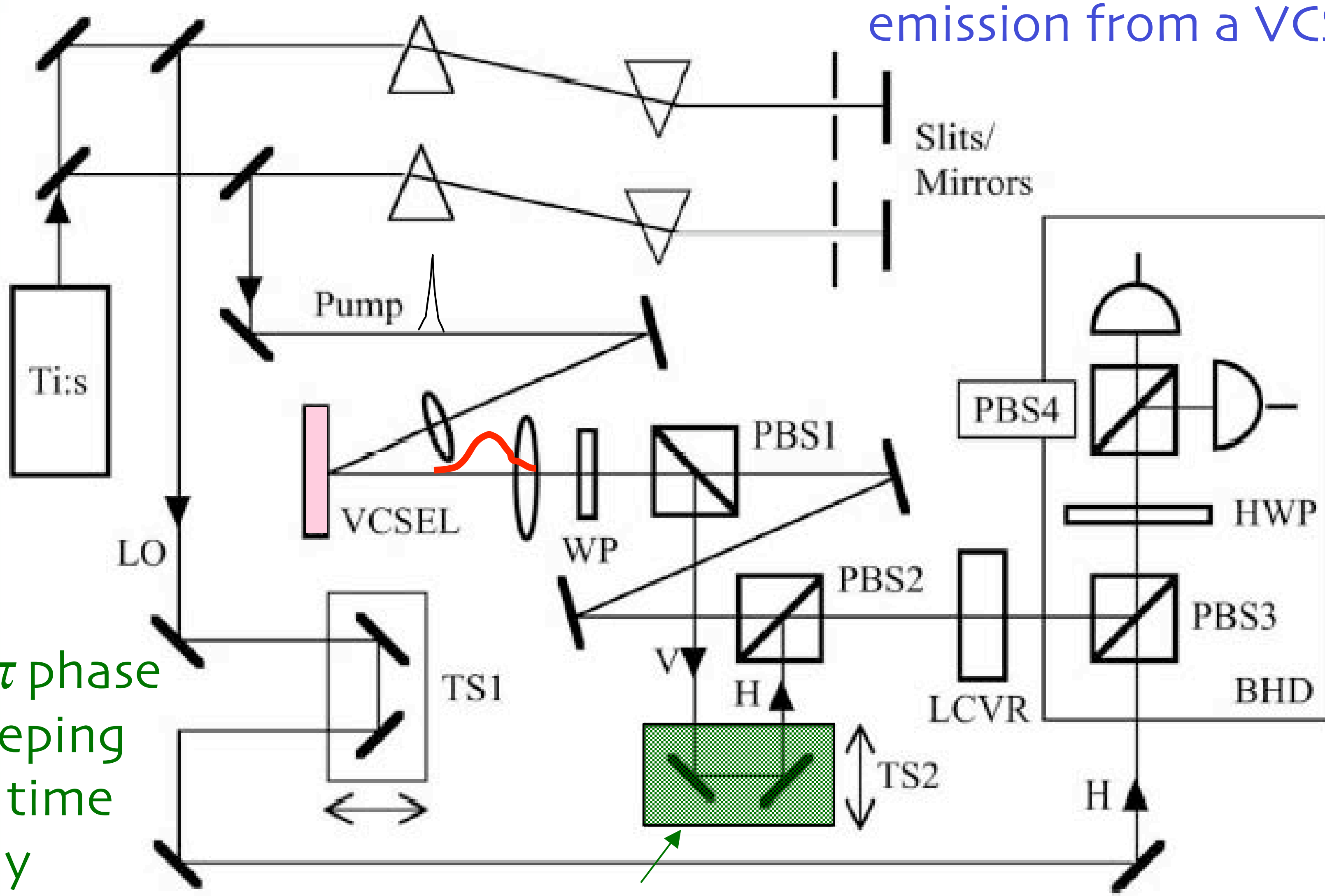
two-pol., two-time
second-order
coherence

$$g^{(2)}_{i,j}(t_1,t_2) = \frac{\langle :\hat{n}_i(t_1)\,\hat{n}_j(t_2): \rangle}{\langle \hat{n}_i(t_1)\rangle \langle \hat{n}_j(t_2)\rangle}$$

A. Funk

Two-Mode DC-HOMODYNE DETECTION IV

Single-time, two-polarization correlation measurements on emission from a VCSEL



0-2π phase sweeping and time delay

0-2π relative phase sweeping

E. Blansett

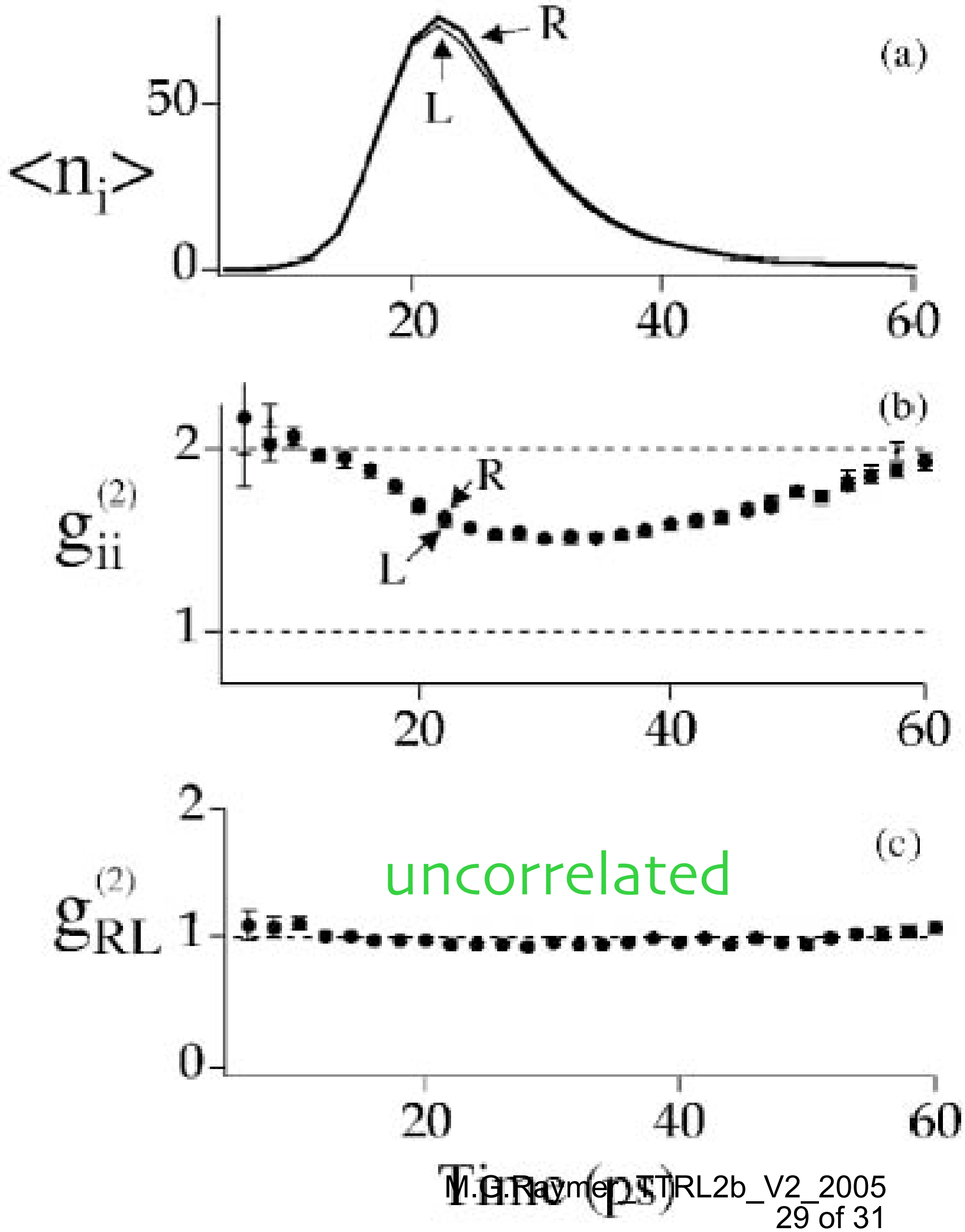
Two-Mode DC-HOMODYNE DETECTION V

Single-time, two-polarization correlation measurements on emission from a VCSEL

at low temp. (10K)

$$g^{(2)}_{i,i}(t_1,t_2)=\frac{\langle :\hat{n}_i(t_1)\,\hat{n}_i(t_2): \rangle}{\langle \hat{n}_i(t_1)\rangle\langle \hat{n}_i(t_2)\rangle}$$

$$g^{(2)}_{i,j}(t_1,t_2)=\frac{\langle :\hat{n}_i(t_1)\,\hat{n}_j(t_2): \rangle}{\langle \hat{n}_i(t_1)\rangle\langle \hat{n}_j(t_2)\rangle}$$



E. Blansett

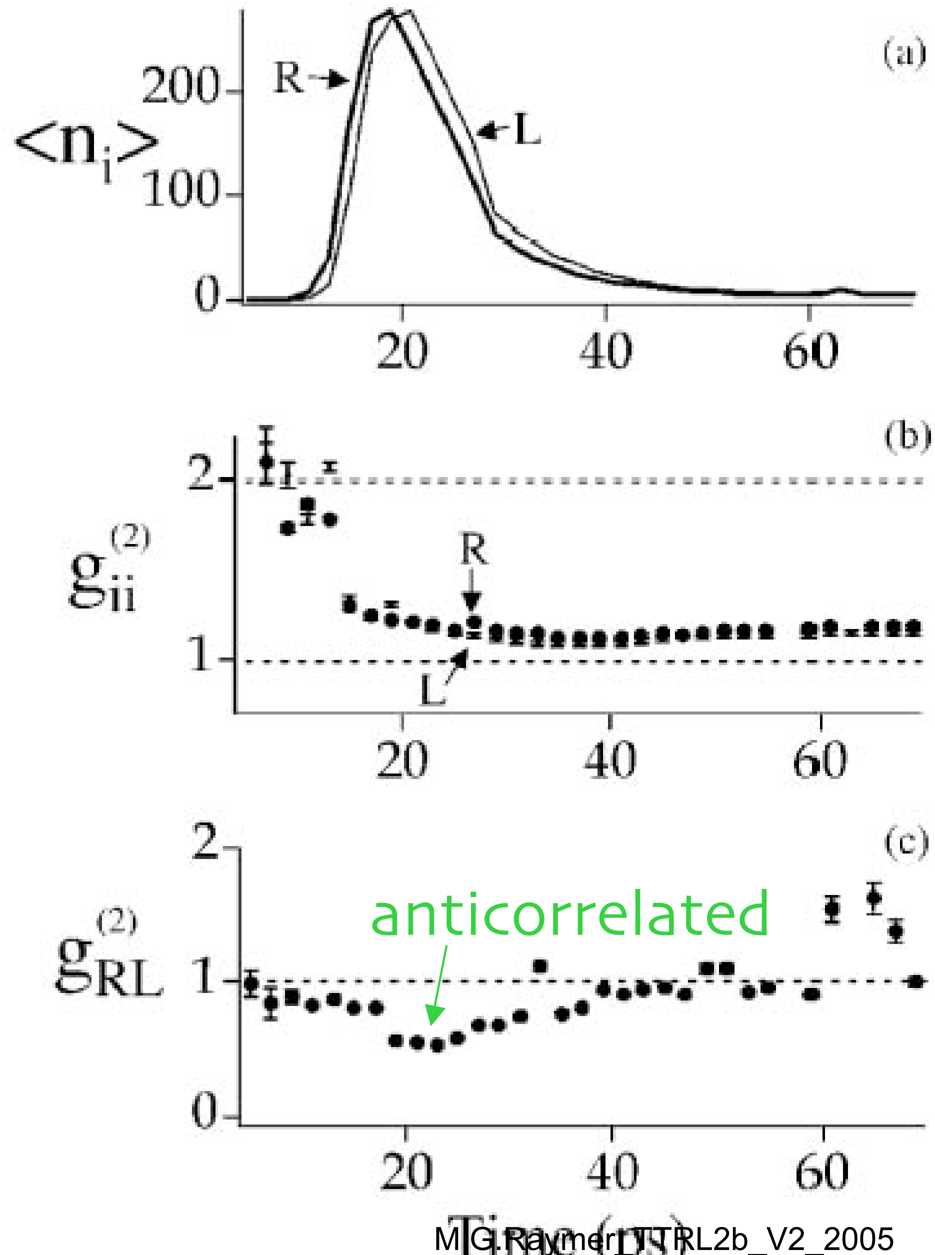
Two-Mode DC-HOMODYNE DETECTION VI

Single-time, two-polarization correlation measurements on emission from a VCSEL

at room temp.

$$g_{i,i}^{(2)}(t_1,t_2)=\frac{\langle :\hat{n}_i(t_1)\,\hat{n}_i(t_2): \rangle}{\langle \hat{n}_i(t_1)\rangle\langle \hat{n}_i(t_2)\rangle}$$

$$g_{i,j}^{(2)}(t_1,t_2)=\frac{\langle :\hat{n}_i(t_1)\,\hat{n}_j(t_2): \rangle}{\langle \hat{n}_i(t_1)\rangle\langle \hat{n}_j(t_2)\rangle}$$



Spin-flip --> gain competition

SUMMARY: DC-Balanced Homodyne Detection

1. BHD can take advantage of: high QE and ultrafast time gating.

2. BHD can provide measurements of photon mean numbers, as well as fluctuation information (variance, second-order coherence).

3. BHD can selectively detect unique spatial-temporal modes, including polarization states.