Performance Analysis on Phase-Encoded OCDMA Communication System in Dispersive Fiber Medium

C. H. Chua, F. M. Abbou, H. T. Chuah, and S. P. Majumder

***Abstract*—The performance of an asynchronous phase-encoded optical code-division multiple-access system is evaluated in a dispersive fiber medium. We derive an approximate analytical expression for the root mean square width of the phase-encoded signal (pseudorandom optical signal with low intensity) propagating in linear dispersive fibers. Bit-error rate analysis of the system is performed in the case of both ordinary single-mode fiber and dispersion-shifted fiber (DSF). The numerical results demonstrate that even though system performance improves due to the smaller width of initial Gaussian optical pulse, the effect from dispersion is higher. Larger code length reduces the effect of dispersion and the use of DSF greatly increases the transmission distance.**

***Index Terms*—Dispersion, optical code-division multiple-access (OCDMA), phase-encoded.**

I. Introduction

THE PHASE-ENCODED optical code-division multiple-access (OCDMA) communication system has been the subject of recent research [1]–[3]. Such a system imposes less stringent requirements on light sources compared to other implementation schemes such as direct time spread and frequency hopping OCDMA systems. Spectral phase encoding–decoding of the optical signal can be performed using fiber Bragg gratings [2] or high-resolution arrayed-waveguide grating [3]. The performance of such a system has been analyzed by Ma *et al.* [4] without considering the effect of the dispersive nature of the fiber channel. In that analysis, the phase-encoded optical signal was treated as a random process, with a variance, which was found to be inversely proportional to code length. The root mean square (rms) width of the phase-encoded optical signal was derived to be proportional to the width of initial optical pulse and code length. Thus, in order to achieve better system performance, one needs to have as large a code length and as short an initial optical pulse as possible. In this letter, we incorporate second- and third-order fiber dispersion effects of the channel into the performance analysis. Fiber dispersion causes spreading of an optical pulse, which in turn degrade system performance due to increased intersymbol interference and reduced received optical peak power.

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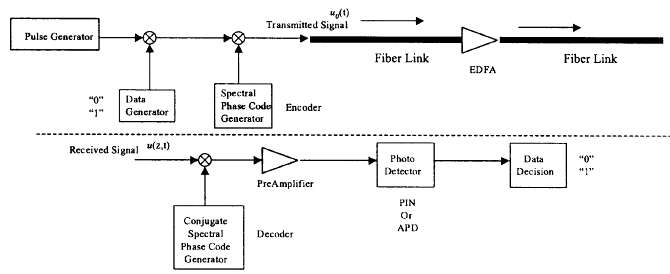

Fig. 1. OCDMA communication system.

II. Model of OCDMA System

Fig. 1 shows a simplified block diagram of an OCDMA communication system, with a pair of transmitter and receiver. The transmitter consists of a pulse generator, a data generator, and a spectral phase encoder.

The phase-encoded signal is transmitted over a fiber channel, which is loss-compensated by a series of erbium-doped fiber amplifiers. We will use a Gaussian pulse shape for the pulse generator for the analysis and assume that the spectral phase encoder appends a random phase (zero or π) to different spectral components of the pulse. The output phase-encoded signal will then appear as a pseudorandom noise signal of low intensity. The receiver consists of a conjugate spectral phase encoder, an optical preamplifier, a photodetector, and data decision circuit. The pulse generator generates an initial Gaussian optical pulse

$$f(t) = e^{-\left(t^2/2T_0^2\right)}. \tag{1}$$

The appendix phase to the spectrum by the spectral phase-encoder can be expressed in the temporal domain as an encoding signal, $m(t)$ given by

$$m(t) = \sum_{j=0}^{F-1} C_j \frac{\delta\omega}{2\pi} Sa\left(\frac{\delta w}{2}t\right) e^{iv(j)\delta\omega t}. \tag{2}$$

In (2), $v(j) = j - ((F-1)/2)$, where F is the code length, and $\delta\omega = \Omega/F$, where Ω is the spectral range encoded by the code sequence C_j.

The resultant phase-encoded optical signal $u_0(t)$ is then given by

$$\begin{aligned} u_0(t) &= \int_{-\infty}^{+\infty} m(\tau) f(t-\tau) d\tau \\ &\approx \frac{\delta\omega}{\sqrt{2\pi}} T_o \sum_{j=1}^{F} C_j Sa\left(\frac{\delta\omega}{2}t\right) e^{iv(j)\delta\omega t}. \end{aligned} \tag{3}$$

At the receiver, if the encoded signal encounters a matched decoder, the original optical pulse is recovered. Otherwise, only a low intensity noise-like signal is obtained.

III. Theoretical Analysis of Dispersion of Pseudorandom Optical Signal

Anderson and Lisak [5] have shown that for an optical pulse propagating along an optical fiber, the rms width varies parabolically with distance, irrespective of initial pulse form and frequency chirp variation. This result is true to any dispersive order and is a very useful tool. We shall use this method to find an analytical approximation to the rms width of the pseudorandom optical signal after undergoing dispersion. This is done without resorting to numerical methods. We can write the moments of the signal as

$$I_n(x) \equiv \langle T^n \rangle = \int_{-\infty}^{+\infty} u^*(t,z) t^n u(t,z) dt \tag{4}$$

where $u(t,z)$ is the wave envelope of the signal after traveling a distance z. The following lowest order moments can then be inferred [5]

$$\begin{aligned} I_0 &= \int_{-\infty}^{+\infty} u^*(t,x)u(t,x)dt = \int_{-\infty}^{+\infty} |u_0(t,x)|^2 dt \\ I_1 &= \int_{-\infty}^{+\infty} u^*(t,x)tu(t,x)dt = a_0 + a_1 z \\ I_2 &= \int_{-\infty}^{+\infty} u^*(t,x)t^2u(t,x)dt = b_0 + b_1 z + b_2 z^2. \end{aligned} \tag{5}$$

The coefficients a_j and b_j of (5) can be derived by knowing only the initial signal $u_0(t)$. We first make the following function approximation in (3) to simplify our analysis:

$$Sa\left(\frac{\delta\omega t}{2}\right) \approx e^{-(c\delta\omega t)^2/2}. \tag{6}$$

We choose $c = 0.304$ so that the functions have the same widths at $1/e$ intensity. The initial phase-encoded optical signal can then be rewritten as

$$u_0(t) = \frac{\delta\omega}{\sqrt{2\pi}} T_o \sum_{j=1}^{F} C_j e^{-(c\delta\omega t)^2/2} e^{iv(j)\delta\omega t}. \tag{7}$$

We can now derive, to third dispersive order, all the coefficients in (5) after making some assumptions and approximations

$$\begin{aligned} a_0 &= 0 \\ a_1 &= \beta_2\left(\frac{-F\sqrt{\pi}}{c}\right) + \frac{\beta_3}{2}\left(\frac{F^3}{12c}\sqrt{\pi}\delta w\right) \\ b_0 &= \frac{F\sqrt{\pi}}{2(c\delta w)^3} \\ b_1 &= 0 \\ b_2 &= \frac{(\beta_2)^2}{12}\left(\frac{F^3}{c}\sqrt{\pi}\delta w\right) + \frac{\beta_2\beta_3}{8}\left(\frac{F^3}{c}\sqrt{\pi}(\delta w)^2\right) \\ &\quad + \frac{(\beta_3)^2}{320}\left(\frac{F^5\sqrt{\pi}(\delta w)^3}{c}\right). \end{aligned} \tag{8}$$

The coefficients a_0, b_0, and b_1 are easily obtained. To obtain the other two coefficients, we ignore all interference and take only the terms with the highest power F. We next find the rms width of the initial phase-encoded signal. In doing so, we also obtain verification if our approximation of the function in (6) is acceptable

$$T_\sigma(0) = \left\{\frac{b_0 - a_0^2}{I_0}\right\}^{1/2} = \frac{1}{\sqrt{2}(c\delta w)} = \frac{T_0 F}{4c}. \tag{9}$$

The obtained expression approximates quite closely the rms width of the phase-encoded optical signal obtained in [4], which has the original $Sa()$ function. Next, we will obtain the moments and rms width $T_\sigma(z)$ of the optical signal at the receiver end located at a distance of z

$$\langle T \rangle = \frac{I_1}{I_0} = \left[\beta_2(-\delta w) + \frac{\beta_3}{24}F^2(\delta w)^2\right] z \tag{10}$$

$$\begin{aligned} \langle T^2 \rangle = \frac{I_2}{I_0} &= \frac{1}{2(c\delta w)^2} \\ &\quad + \left[\frac{(\beta_2)^2}{12}F^2(\delta w)^2 + \frac{\beta_2\beta_3}{8}F^2(\delta w)^3 + \frac{(\beta_3)^2}{320}F^4(\delta w)^4\right] \\ &\quad \cdot z^2 \end{aligned} \tag{11}$$

$$\begin{aligned} T_\sigma &= \left[\langle T^2 \rangle - \langle T \rangle^2\right]^{1/2} \\ &= T_\sigma(0)\left[\begin{matrix} 1 + \frac{32c^2}{3}\left(\frac{\beta_2 z}{T_0^2 F}\right)^2 + \frac{320c^2}{3\sqrt{2}}\left(\frac{\beta_2\beta_3 z^2}{T_0^5 F^3}\right) \\ + \frac{64c^2}{45}\left(\frac{\beta_3 z}{T_0^3 F}\right)^2 \end{matrix}\right]^{1/2}. \end{aligned} \tag{12}$$

We now calculate the peak power $P(z)$ of the correctly decoded pulse by using the conservation of pulse energy and obtain $P(0)T_\sigma(0) = P(z)T_\sigma(z)$. In our case, $P(0) = 1$. We complete our analysis of the dispersion effects by noting that an incorrectly decoded pulse will remain as a low intensity pseudorandom signal. This is due to the fact that the decoder mismatch would randomize the spectral phase relations of the pulse and dispersion would not have extraneous effects on this randomization.

IV. Performance Analysis

In this section, we calculate the bit-error probability of the system operating at 1 Gb/s in three different fiber configurations—the case of normal single-mode fiber (SMF) with $D = 16$ ps/km/nm and $S = 0.08$ ps/km/(nm)^2, dispersion-shifted fiber (DSF) with $D = 3$ ps/km/nm and $S = 0.08$ ps/km/(nm)^2, and a fully dispersion compensated fiber link, at $\lambda = 1550$ nm. It has been shown that the intensity of the pseudorandom signal has an exponential distribution [1], [4]. When the number of interfering users n become large, the probability distribution function of the multiple access noise at the receiver reaches a Gaussian distribution $N(n\sigma^2, n\sigma^4)$, where σ is given as [4] $\sigma = 1.263\sqrt{\frac{T_0}{T_\sigma(0)}}$. We ignore the effects of shot and thermal noise at the receiver. With N users in the system, duty cycle of p, and receiver threshold θ, the bit-error probability P_b is then computed as

$$\begin{aligned} P_b &= \frac{1}{2}\sum_{i=1}^{N-1}\binom{N-1}{i}p^i(1-p)^{N-1-i}Q\left(\frac{\theta - \mu_{i0}}{\sigma_{i0}}\right) \\ &\quad + \frac{1}{2}\sum_{i=1}^{N-1}\binom{N-1}{i}p^i(1-p)^{N-1-i}Q\left(\frac{\mu_{i1} - \theta}{\sigma_{i1}}\right) \end{aligned} \tag{13}$$

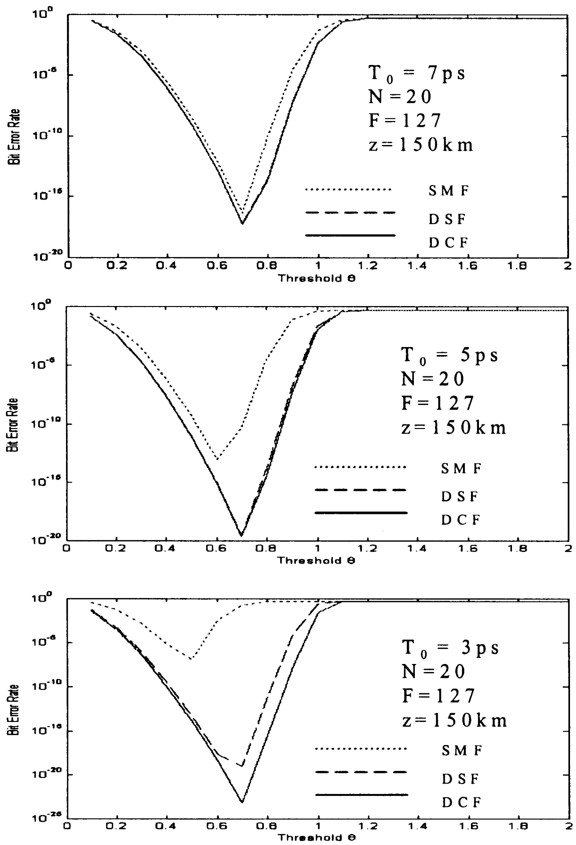

Fig. 2. Bit-error probability versus threshold θ with different T_0.

where

$$\begin{aligned} \sigma_{i0}^2 &= \sigma_{i1}^2 = i\sigma^4 \\ \mu_{i0} &= i\sigma^2 \\ \mu_{i1} &= i\sigma^2 + P(z). \end{aligned} \tag{14}$$

The Q function is defined as

$$Q(\xi) = \frac{1}{\sqrt{2\pi}} \int_{\xi}^{\infty} e^{-x^2/2} dx. \tag{15}$$

V. Results and Discussions

Fig. 2 shows the curves of bit-error rate (BER) versus threshold θ of the receiver when the initial optical pulsewidth T_0 is reduced. We show the curves for cases of SMF, DSF, and dispersion compensating fiber. Fig. 3 shows the curves of BER versus threshold θ when we increase the code length F. From the curves in Fig. 2, we can clearly see that the dispersive effect of the fiber on BER become more apparent as the value of T_0 becomes smaller. So, even though the performance of the system should improve as T_0 becomes narrower, this is countered by the dispersion effects which become worse. We can see from the curve in Fig. 3 that the effect of dispersion is much smaller in the case of increasing code length. So, increasing code length not only improves overall system performance but also reduces the effect of dispersion. Fig. 4 shows the curves of BER versus transmission distance for different pulsewidth of the initial optical pulse. We can clearly see that making the pulsewidth narrower increases performance of the system initially at short distances, but becomes a problem when distance becomes longer. The DSF link achieves more than five times longer transmission compared to the SMF link.

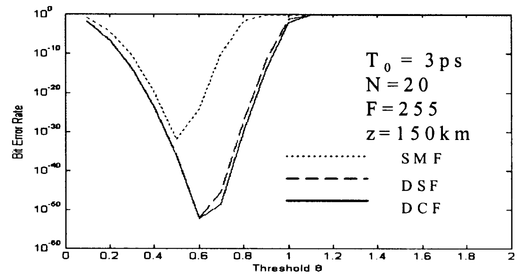

Fig. 3. Bit-error probability versus threshold θ with increased F.

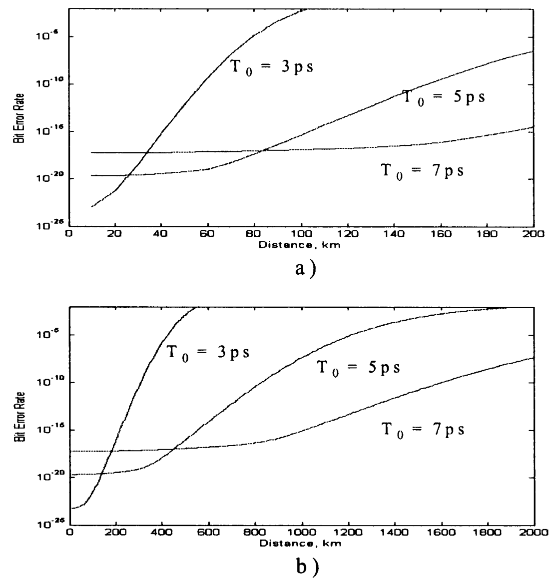

Fig. 4. Bit-error probability versus distance for (a) SMF (b) DSF.

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