MAI-free MC-CDMA Systems Based on Hadamard-Walsh Codes

Shang-Ho Tsai, Yuan-Pei Lin and C.-C. Jay Kuo

Abstract

It is known that MC-CDMA systems suffer from multiaccess interference (MAI) when the channel is frequency-selective fading. In this paper, we propose a Hadamard-Walsh code based MC-CDMA system that achieves zero MAI over frequency-selective fading channel. In particular, we will use appropriately chosen subsets of Hadamard-Walsh code as codewords. For a multipath channel of length L, we partition a Hadamard-Walsh code of size N into G subsets, where G is a power of 2 with $G \geq L$. We will show that the N/G codewords in any of the G subsets yields an MAI-free system. That is, the number of MAI-free users for each codeword subset is N/G. Furthermore, the system has the additional advantage that it is robust to carrier frequency offset (CFO) in a multipath environment. It is also shown that the MAI-free property allows us to estimate the channel of each user separately and the system can perform channel estimation much more easily. Owing to the MAI-free property, every user can enjoy a channel diversity gain of order L to improve the bit error performance. Finally, we discuss a code priority scheme for a heavily-loaded system. Simulation results are given to demonstrate the advantages of the proposed code and code priority schemes.

Index Terms—interference free, MAI-free, MC-CDMA, MAI-free, Hadamard-Walsh code, LAS code, carrier frequency offset (CFO), multiuser detection (MUD).

I. Introduction

Multiaccess interference (MAI) or multiuser interference (MUI) is a major impairment that limits the performance of Code Division Multiple Access (CDMA)-based systems. In a synchronous CDMA

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(S-CDMA) system where user's timing is aligned within a fraction of a chip-time interval, MAI can be reduced via the use of orthogonal codewords [21]. S-CDMA can be used in downlink transmission in large cells such as those for the digital cellular IS-95 standard and in both downlink and uplink transmissions in micro cells such as those for the personal communication services (PCS) system [21]. However, orthogonality of these codewords could be destroyed in a multipath environment. For downlink transmission in large cells, the multipath length is often longer than the duration of several chips and the induced MAI will limit the system performance. Even in micro cells, the multipath effect could be serious in an urban area [18]. Multiuser detection (MUD) [25] or related signal processing techniques have been developed to mitigate MAI. However, their complexity is usually high and this imposes a computational burden on the receiver. Moreover, the channel information is needed for the application of the MUD scheme so that effective channel estimation plays an essential role in the system [23].

Recently, multicarrier CDMA (MC-CDMA) has been proposed as a promising multiaccess technique. MC-CDMA systems can be divided into two types [10]. For the first type, one symbol is transmitted per time slot. The input symbol is spread into several chips, which are then allocated to different subchannels. The number of subchannels is equal to the number of chips [8], [27]. For the second type, a vector of symbols is formed via the serial-to-parallel conversion, and each symbol is spread into several chips. The chips corresponding to the same symbol are allocated to the same subchannel [13], which is often called MC-DS CDMA. When compared with conventional CDMA systems, MC-CDMA can combat inter-symbol-interference (ISI) more effectively. Moreover, the frequency diversity gain can be fully exploited if the maximum ratio combing (MRC) technique [10], [18] is used at the receiver in MC-CDMA systems. Despite the above advantages, the performance of MC-CDMA systems is still limited by MAI in a multipath environment. Even though MAI can be reduced by MUD [25] and other signal processing [10] techniques, the diversity gain provided by multipath channels could be sacrificed since the received chips are no longer optimally combined under MRC. Furthermore, channel status information is needed for MRC and MUD. In a multi-user environment, multiuser channel estimation is more complicated and its accuracy degrades as the number of users increases [24], which will in turn degrade the system performance.

In this work, we approach the MAI reduction problem for MC-CDMA systems from another angle. That is, we investigate a novel way to select a set of "good" spreading codes so as to completely eliminate the MAI effect while keeping the transceiver structure simple and the computational burden low. Some earlier work has been done along this direction. For conventional CDMA systems, Scaglione *et al.* [19] used a code to reduce MAI in a multipath environment. However, since the performance curves in [19] have a slope similar to the OFDMA (orthogonal frequency division multiple access) system, this code design does not offer a full diversity gain. Oppermann *et al.* [16] examined several code sequences and selected some codewords to reduce MAI by experiments with little theoretical explanation of the MAI reduction performance. Chen *et al.* [5] proposed a code scheme based on the complementary code to achieve an MAI-free CDMA system in a flat fading channel. Even though the number of supportable users is much less than the codeword length, this scheme can achieve higher spectrum efficiency than conventional CDMA system with a successive transmitting structure. In a multipath environment, this scheme is no longer MAI-free, and a recursive receiver structure is demanded for symbol detection. A large area synchronized (LAS) code was proposed by LinkAir and was examined in [22] to design a code that has an area with zero off-peak autocorrelation and zero crosscorrelation for CDMA. This code scheme has zero ISI and MAI in a multipath environment. The number of supportable users to achieve ISI- and MAI-free conditions depends on the multipath length. The LAS code is generalized to MC-DS-CDMA systems in [26]. As for MC-CDMA systems, Shi and Latva-aho [20] proposed a code scheme for downlink MC-CDMA with little theoretical analysis. Moreover, this scheme is not optimal in minimizing the bit error probability in both uplink and downlink directions. Cai *et al.* [4] proposed a group-orthogonal (GO-) MC-CDMA scheme. By assigning only one user to each group, this scheme can be MAI-free and with a maximum channel diversity gain. Moreover, in a heavily loaded situation, the required computation for MUD to achieve the MAI-free property is small. However, in a CFO environment which causes MAI, relatively complicated multiuser CFO estimation methodology is demanded to estimate every user's CFO.

In this work, a code design based on Hadamard-Walsh codes is proposed to achieve the MAI-free property in a synchronous MC-CDMA system [8], [27]. To be more specific, let N and L denote, respectively, the spreading factor and the multipath length. The $N = 2^{n_s}$ Hadamard-Walsh codewords are partitioned judiciously into G subsets, where $G = 2^{n_g}$ with $n_s > n_g \geq 1$ and $G \geq L$. Then we can obtain an MAI-free system and each user can fully exploit the diversity gain provided by the multipath channel using any subset of codewords in frequency-selective channels. The number of supportable MAI-free users in each codeword subset is N/G. It is worthwhile to point out that, under the same multipath length L and DFT/IDFT size N, the number of supportable MAI-free users in GO-MC-CDMA [4] is exactly the same as the proposed code scheme. Using the proposed code scheme, we also show a procedure to estimate the channel information for individual users under an MAI-free environment. Moreover, we consider the performance of the proposed code scheme in a carrier frequency offset (CFO) environment. It is shown that the proposed code scheme can reduce the CFO-induced MAI effect to a negligible amount under an interested CFO level. Some codewords can even achieve MAI-free in a CFO environment. Furthermore we study the relationship between the multipath length L and the number of allowed users to maintain the MAI-free property and the full diversity gain with the proposed code. Finally, a code priority scheme is presented for a heavily-loaded system.

The rest of this paper is organized as follows. The system model is presented in Sec. II. The MAI effect and the code scheme to achieve an MAI-free system are discussed in Sec. III. Using the proposed code design, we discuss an effective way to estimate the channel status under an MAI-free environment in Sec. IV. Then, we examine the performance of the proposed code scheme in the presence of CFO in Sec. V. Simulation results are provided in Sec. VI to corroborate analytical results. Practical considerations about the relationship between the multipath length and the number of users to maintain the MAI-free property and the full diversity gain with respect to the proposed code is discussed in Sec. VII. Finally, concluding remarks are given in Sec. VIII.

II. System Model

The block diagram of the MC-CDMA system in uplink direction (from the mobile station to the base station) is shown in Fig. 1, where the signal path demonstrates a signal transmitted by user i and detected by user i. Note that the analysis is conducted in the uplink direction because it is a more general case, where the channel fading of individual users are different. (However, the analytical results can be adapted to the downlink direction as well. To obtain the analysis for the downlink direction, we can simply set the channel fading of every user to be the same.) At each time slot, the input is a data symbol. Suppose that there are T users. Let the symbol from user i be x_i. In the first stage, x_i is spread by N chips to form an $N \times 1$ vector, denoted by $\mathbf{y}_i$. Let the kth element of $\mathbf{y}_i$ be $y_i[k]$. The relation between $y_i[k]$ and x_i is given by

$$y_i[k] = w_i[k]x_i, \quad 0 \leq k \leq N-1, \tag{1}$$

where $w_i[k]$ is the kth element of the ith orthogonal code. Note that we consider the short code scenario here, where the spreading code for a target user is the same for any time slot. After spreading, $\mathbf{y}_i$ is passed through the $N \times N$ IDFT matrix. Then, the output is parallel-to-serial (P/S) converted and the cyclic prefix (CP) of length $L-1$ is added to combat the inter-symbol-interference (ISI), where L is the considered maximum delay spread.

At the receiver side, the receiver removes CP and passes each block of size N through the $N \times N$ DFT matrix. Since there are T users, the kth element of the DFT output $\hat{\mathbf{y}}$ can be written as [3], [7]

$$\hat{y}[k] = \sum_{j=0}^{T-1} \lambda_j[k] y_j[k] + e[k], \tag{2}$$

where $\lambda_j[k]$ is the kth component of the N-point DFT of user j's channel impulse response, and $e[k]$ is the received noise after DFT. Based on $\hat{\mathbf{y}}$, we will detect symbols for T users. As shown in Fig. 1, to detect symbols transmitted by the ith user, $\hat{\mathbf{y}}$ is multiplied by $w_i^*[k]$ and frequency gain $\lambda_i^*[k]$, where $*$ denotes the complex-conjugate operation. Here, the channel information $\lambda_j[k]$ of every user is assumed to be known to the receiver. (The estimation of channel information under an MAI-free environment will be described in Sec. IV.) After being multiplied with the frequency gains, N chips are summed up to form reconstructed symbol $\hat{x}_i$. Using (1) and (2) $\hat{x}_i$ is given by

$$\begin{aligned}
\hat{x}_i &= \sum_{k=0}^{N-1} \lambda_i^*[k] w_i^*[k] \hat{y}[k] \\
&= \sum_{k=0}^{N-1} \lambda_i^*[k] w_i^*[k] \left(\sum_{j=0}^{T-1} \lambda_j[k] y_j[k] + e[k] \right) \\
&= \underbrace{x_i \sum_{k=0}^{N-1} |\lambda_i[k]|^2}_{\text{multipath effect}} + \sum_{j=0, j\neq i}^{T-1} \underbrace{x_j \sum_{k=0}^{N-1} \lambda_i^*[k] w_i^*[k] \lambda_j[k] w_j[k]}_{MAI_{i\leftarrow j}} + \sum_{k=0}^{N-1} \lambda_i^*[k] w_i^*[k] e[k],
\end{aligned} \tag{3}$$

where $MAI_{i\leftarrow j}$ denotes the MAI from user j to user i. Note that, when the channel noise $e[k]$ is AWGN, the process from $\hat{y}[k]$ to $\hat{x}_i$ is called the maximum ratio combining (MRC) technique [10], which ensures the minimum bit error probability for detected symbols [18], or the maximum achievable diversity gain provided by multipath channels [17].

For any target user i, if $MAI_{i\leftarrow j} = 0$, the reconstructed symbol $\hat{x}_i$ will be affected only by his/her own transmitted symbols x_i and the corresponding channel response $\lambda_i[k]$. Thus, this allows the system to use some simple detection schemes without involving multiuser detection. When the channel has flat fading, $\lambda_i[k]$ and $\lambda_j[k]$ are independent of k and $MAI_{i\leftarrow j} = 0$ if orthogonal codes such as the Hadamard-Walsh codes are used. However, in practical situations, the channel environment is usually frequency-selective and the orthogonality of orthogonal codes will be lost under MRC.

In downlink transmission, the signal for every user experiences the same fading. In this situation, MAI-free can be achieved using orthogonality restoring combining (ORC) [10], *i.e.* the combining gain is $\lambda_j^{-1}[k]$ instead of $\lambda_j^*[k]$ in (3). However, for subchannels with serious fading, ORC tends to amplify the noise in these subchannels. Thus, the performance will degrade dramatically. That is, the use of ORC may lead to the loss of the diversity gain from multipath channels. In the following sections, we will design $w_i[k]$ such that $MAI_{i\leftarrow j} = 0$ under the multipath environment. Moreover, the proposed code design allows MRC to be used in both uplink and downlink transmissions. Thus, a full diversity gain from the multipath channel can be achieved.

III. MAI ANALYSIS OVER FREQUENCY-SELECTIVE FADING

Let $\mathbf{F}$ be the $N \times N$ DFT matrix with the element at the kth row and the nth column given by $[\mathbf{F}]_{k,n} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}kn}$ and the maximum length of channel impulse response be L, *i.e.* $h_i(n) = 0$, for $n > L - 1$. The MAI term in (3) can be expressed using matrix representation as

$$MAI_{i\leftarrow j} = x_j \mathbf{h}_i^\dagger \underbrace{\mathbf{F}_0^\dagger \mathbf{W}_i^* \mathbf{W}_j \mathbf{F}_0}_{\mathbf{A}_{i,j}} \mathbf{h}_j, \tag{4}$$

where

$$\mathbf{h}_i = \begin{pmatrix} h_i(0) \\ \vdots \\ h_i(L-1) \end{pmatrix}, \quad \mathbf{F}_0 = \mathbf{F} \begin{pmatrix} \mathbf{I}_L \\ \mathbf{0} \end{pmatrix}_{N\times L}, \quad \mathbf{W}_i = \text{diag}(w_i[0] \cdots w_i[N-1]),$$

superscript † denotes the Hermitian operation [11], and diag(.) is the function that puts the elements along the diagonal.

To have zero MAI for a frequency-selective fading channel, we need to have $MAI_{i\leftarrow j} = 0$ for all nonzero $\mathbf{h}_i$ and $\mathbf{h}_j$. This means that $\mathbf{A}_{i,j}$ in (4) should be the $L \times L$ zero matrix for all $i \neq j$. It is clear that

$$\mathbf{A}_{i,j} = \begin{pmatrix} \mathbf{I}_L & \mathbf{0} \end{pmatrix} \mathbf{F}^\dagger \mathbf{R}_{i,j} \mathbf{F} \begin{pmatrix} \mathbf{I}_L \\ \mathbf{0} \end{pmatrix}, \tag{5}$$

where $\mathbf{R}_{i,j} = \mathbf{W}_i^* \mathbf{W}_j$, and that matrix $\mathbf{R}_{i,j}$ is diagonal, *i.e.*

$$\mathbf{R}_{i,j} = \text{diag}\left(r_{i,j}[0] \cdots r_{i,j}[N-1]\right),$$

with

$$r_{i,j}[k] = w_i^*[k] w_j[k].$$

Let $\mathbf{B}_{i,j} = \mathbf{F}^\dagger \mathbf{R}_{i,j} \mathbf{F}$. Then, it is well known that $\mathbf{B}_{i,j}$ is a circulant matrix [9]. That is, the first column of $\mathbf{B}_{i,j}$, $(b_{i,j}(0) \cdots b_{i,j}(N-1))^T$, is the N-point IDFT of $\mathbf{r}_{i,j}$, where $\mathbf{r}_{i,j} = (r_{i,j}[0] \cdots r_{i,j}[N-1])^T$. Matrix $\mathbf{A}_{i,j}$ is an $L \times L$ upper left submatrix of $\mathbf{B}_{i,j}$, *i.e.*

$$\mathbf{A}_{i,j} = \begin{pmatrix} b_{i,j}(0) & b_{i,j}(N-1) & \ldots & b_{i,j}(N-L+1) \\ b_{i,j}(1) & b_{i,j}(0) & & \vdots \\ \vdots & & \ddots & \\ b_{i,j}(L-1) & & \ldots & b_{i,j}(0) \end{pmatrix}. \tag{6}$$

To have $\mathbf{A}_{i,j} = \mathbf{0}$ means that $b_{i,j}(0) = \cdots = b_{i,j}(L-1) = 0$ and $b_{i,j}(N-L-1) = \cdots = b_{i,j}(N-1) = 0$. That is, the first L samples and the last $L-1$ samples of the IDFT of $\mathbf{r}_{i,j}$ are zeros. Hence, we have

$$\begin{cases} b_{i,j}(n) = 0, & 0 \leq n \leq L-1 \\ b_{i,j}(N-n) = 0, & 1 \leq n \leq L-1 \end{cases}. \tag{7}$$

Lemma 1: Suppose the channel length is L and the spreading gain is N. To achieve MAI-free property, N should be greater or equal to $2L$.

Proof: From (7), there should be at least $2L-1$ elements for the codewords. However, if $N = 2L-1$, all elements of the codewords are zeros. Therefore, $N \geq 2L$. ■

Note that Lemma 1 holds for both real and complex code design. In what follows, we show how to achieve the MAI-free conditions in (7) using the Hadamard-Walsh codes. Before proceeding, let us recall a well known property of the Hadamard matrix [2]. An $N \times N$ Hadamard matrix $\mathbf{H}_N$ with $N = 2^p$, $p = 1, 2, \cdots$, can be recursively defined using the Hadamard matrix of order 2, *i.e.*

$$\mathbf{H}_N = \mathbf{H}_2 \otimes \mathbf{H}_{N/2} = \begin{pmatrix} \mathbf{H}_{N/2} & \mathbf{H}_{N/2} \\ \mathbf{H}_{N/2} & -\mathbf{H}_{N/2} \end{pmatrix}, \tag{8}$$

where $\otimes$ is the Kronecker product [2], [11] and

$$\mathbf{H}_2 = \begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix}.$$

Our proposed code scheme is stated below. Suppose $N = 2^{n_s}$ and $G = 2^{n_g}$, where $n_s > n_g \geq 1$. The columns of an $N \times N$ Hadamard matrix $\mathbf{H}_N$ form the N Hadamard-Walsh codes. We divide the N codewords into G subsets. Each subset has N/G codewords. That is, the gth subset, denoted by G_g, has codewords $\left\{\mathbf{w}_{\frac{N}{G}g}, \cdots, \mathbf{w}_{\frac{N}{G}(g+1)-1}\right\}$, where $\mathbf{w}_i$ is the ith column of $\mathbf{H}_N$ and $0 \leq g \leq G-1$. For instance. Let $N = 8$ and $G = 2$. Then, G_0 contains codewords $\{\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ and G_1 contains codewords $\{\mathbf{w}_4, \mathbf{w}_5, \mathbf{w}_6, \mathbf{w}_7\}$.

Lemma 2: Let $\mathbf{r}_{i,j}$ be an $N \times 1$ vector with the kth element be $r_{i,j}[k] = w_i^*[k]w_j[k]$. For $\mathbf{w}_i$ and $\mathbf{w}_j$ that belong to the same subset, $\mathbf{r}_{i,j}$ is equal to one of the codewords in G_0 excluding codeword $\mathbf{w}_0$.

Proof: Let us first prove that for $\mathbf{w}_i$ and $\mathbf{w}_j \in G_0$, $\mathbf{r}_{i,j}$ is again a codeword within G_0. According to (8), the $N/G \times N/G$ upper left submatrix of $\mathbf{H}_N$ is a $N/G \times N/G$ Hadamard matrix. Thus, the product of any two columns of this submatrix is again a column of this submatrix (see [12]). Since the codewords in subset 0 are the first N/G columns of $\mathbf{H}_N$, which is obtained by repeating the $N/G \times N/G$ submatrix by G times. Hence, for $\mathbf{w}_i$ and $\mathbf{w}_j$ in subset 0, $\mathbf{r}_{i,j}$ is a codeword in subset 0.

Now, let us consider $\mathbf{r}_{i,j}$ for $\mathbf{w}_i$ and $\mathbf{w}_j$ that are in the same subset other than subset 0. Recall that $w_i[k]$ is the kth element of the ith codeword. It can also be used to denote the kth element of the ith column of $\mathbf{H}_N$. According to (8), for $0 \leq i \leq N/2-1$, we have the following property

$$\begin{cases} w_i[k] = w_{i+N/2}[k], & 0 \leq k \leq N/2-1, \\ w_i[k] = -w_{i+N/2}[k], & N/2 \leq k \leq N-1. \end{cases} \tag{9}$$

We see from (9) that the product of any two columns in the last half $N/2$ columns is equal to that of the two corresponding columns in the first half $N/2$ columns, *i.e.*

$$w_i[k]w_j[k] = w_{i+N/2}[k]w_{j+N/2}[k], \quad 0 \leq i,j \leq N/2-1. \tag{10}$$

Suppose that we divide the N codewords into two sets, denoted by S_0 and S_1, respectively. The first $N/2$ half codewords form S_0 while the last $N/2$ half codewords form S_1. Hence, as proved in the beginning of the lemma that for $\mathbf{w}_i$ and $\mathbf{w}_j$ in S_0, $\mathbf{r}_{i,j}$ is again a codeword in S_0. For $\mathbf{w}_i$ and $\mathbf{w}_j$ in S_1, $\mathbf{r}_{i,j}$ is equal to a codeword in S_0 based on (10). Using a similar procedure, we can divide S_0 into 2 sets, S_{00} and S_{01}. Thus, for $\mathbf{w}_i$ and $\mathbf{w}_j$ in S_{00}, $\mathbf{r}_{i,j}$ is a codeword in S_{00}. Now, we prove that for $\mathbf{w}_i$ and $\mathbf{w}_j$ in S_{01}, $\mathbf{r}_{i,j}$ is a codeword in S_{00}. From (8), for $0 \leq i \leq N/4-1$, we have the following property

$$\begin{cases} w_i[k] = w_{i+N/4}[k], & 0 \leq k \leq N/4-1 \quad \text{or} \quad N/2 \leq k \leq 3N/4-1, \\ w_i[k] = -w_{i+N/4}[k], & N/4 \leq k \leq N/2-1 \quad \text{or} \quad 3N/4 \leq k \leq N-1. \end{cases} \tag{11}$$

We see from (11) that the product of any two columns in the second quarter is equal to the product of the two corresponding columns in the first quarter, *i.e.*

$$w_i[k]w_j[k] = w_{i+N/4}[k]w_{j+N/4}[k], \quad 0 \leq i,j \leq N/4-1. \tag{12}$$

From (14), for $\mathbf{w}_i$ and $\mathbf{w}_j$ in S_{01}, $\mathbf{r}_{i,j}$ is again a codeword in S_{00}. Similarly, we can divide S_1 into 2 sets, *i.e.* S_{10} and S_{11}, and show that for $\mathbf{w}_i$ and $\mathbf{w}_j$ in either S_{10} or S_{11}, $\mathbf{r}_{i,j}$ is again a codeword in S_{00}. Using the same procedure, we can continue to divide the codewords until we have G subsets, and show that for $\mathbf{w}_i$ and $\mathbf{w}_j$ in the same subset, $\mathbf{r}_{i,j}$ is a codeword of subset 0. ■

Lemma 3: Let $\tilde{w}_i(n)$, $0 \leq n \leq N-1$ and $1 \leq i \leq N/G-1$, be the N-point IDFT of the codewords in G_0 excluding $\mathbf{w}_0$. Then, $\tilde{w}_i(n)$ has the following property:

$$\begin{cases} \tilde{w}_i(n) = 0, & 0 \leq n \leq G-1, \\ \tilde{w}_i(N-n) = 0, & 1 \leq n \leq G-1. \end{cases} \tag{13}$$

Proof: For $n=0$, it is easy to see $\tilde{w}_i(0) = \sum_{k=0}^{N-1} w_i[k] = 0$ since there are an equal number of $+1$ and -1 for any codeword except $\mathbf{w}_0$. For $n \neq 0$, since $\tilde{w}_i(n)$ is the IDFT of the codewords in G_0, we have

$$\tilde{w}_i(n) = \frac{1}{N}\sum_{m=0}^{N-1} w_i[k]e^{j\frac{2\pi}{N}mn}. \tag{14}$$

Let $m = k + gN/G$, $0 \le k \le N/G - 1$, $0 \le g \le G - 1$, we can rewrite (14) as

$$\tilde{w}_i(n) = \frac{1}{N}\sum_{k=0}^{N/G-1}\sum_{g=0}^{G-1} w_i[k + gN/G]e^{j\frac{2\pi}{N}(k+gN/G)n}. \tag{15}$$

Since codewords $w_i[k]$ in G_0 are the first N/G columns of $\mathbf{H}_N$, they are formed by repeating the upper left $N/G \times N/G$ submatrix of $\mathbf{H}_N$ by G times. Hence, $w_i[k] = w_i[k + gN/G]$, $0 \le k \le N/G - 1$, $0 \le g \le G - 1$. We can rewrite (15) as

$$\tilde{w}_i(n) = \frac{1}{N}\sum_{k=0}^{N/G-1} w_i[k]a_n, \tag{16}$$

where

$$a_n = \sum_{g=0}^{G-1} e^{j\frac{2\pi}{G}gn} = \begin{cases} G, & n = cG \text{ with } c = 0, \pm 1, \pm 2, \cdots, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, we obtain

$$\tilde{w}_i(n) = \begin{cases} \frac{G}{N}\sum_{k=0}^{N/G-1} w_i[k], & n = cG \text{ with } c = 0, \pm 1, \pm 2, \cdots, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

From (17) and $\tilde{w}_i(0) = 0$, we are led to (13). ∎

From (13) and Lemma 2, we have the following property

$$\begin{cases} b_{i,j}(n) = 0, & 0 \le n \le G - 1, \\ b_{i,j}(N-n) = 0, & 1 \le n \le G - 1, \end{cases} \tag{18}$$

where $b_{i,j}(n)$ denotes the nth element of the IDFT of $\mathbf{r}_{i,j}$ within the same subset.

Let us give an example to illustrate Lemma 3. Let $N = 16$, the N-point DFT of the N Hadamard-Walsh codewords are shown in Fig. 2. From this figure, we see that, except the all-one codeword, the IDFT of any codeword has zero at $n = 0$. If $G = 8$, we have 8 subsets and each subset has 2 codewords. From Lemma 3, for $\mathbf{w}_i$ and $\mathbf{w}_j$ in the same subset, $\mathbf{r}_{i,j}$ is equal to $\mathbf{w}_1$. From the figure, the first 8 elements of $\tilde{w}_1(n)$ are zeros. If $G = 4$, then we have 4 subsets and each subset has 4 codewords. Again from Lemma 3, for $\mathbf{w}_i$ and $\mathbf{w}_j$ in the same subset, $\mathbf{r}_{i,j}$ is equal to either $\mathbf{w}_1$, $\mathbf{w}_2$ or $\mathbf{w}_3$. From the figure, the first 4 elements of $\tilde{w}_1(n)$, $\tilde{w}_2(n)$ and $\tilde{w}_3(n)$ are zeros.

Based on the above discussion, we have established one of the main results of this work as stated below.

Theorem 1: Let the channel length be L. We divide the N Hadamard-Walsh codewords into G subsets with $G \geq L$, where N and G are power of 2 and each subset consisting of N/G codewords. Then, using any one of the G subsets of codewords, the corresponding MC-CDMA system is completely MAI-free.

Note that Theorem 1 holds for arbitrary multipath coefficients. Moreover, the maximum number of MAI-free users, T, in each subset depends on the spreading gain N and multipath length L. Hence, the system can be designed accordingly. Different applications may have different concerns. We describe two application scenarios below.

Application Scenario 1. In cellular systems, frequency reuse for different cells is an important issue since improper frequency reuse will lead to significant co-channel interference [18]. The proposed scheme divides the codewords into several subsets to achieve MAI-free property. It is intuitive to use distinct subsets of codewords in neighboring cells to reduce co-channel interference. Let us give an example to illustrate this point. Let $G = L = 4$. Thus, the orthogonal codes are divided into 4 subsets, *i.e.* subsets 0, 1, 2 and 3. Fig. 3 gives an example of frequency planning using the proposed code scheme. For a larger L, G should be increased accordingly to be MAI-free. In this situation, we have more subsets and the distance among the same subset in reuse can be increased to reduce co-channel interference further.

Application Scenario 2. In wireless local area network (WLAN) applications, the distance among cells is not as close as that in cellular systems. Hence, co-channel interference may not be a major concern. According to Theorem 1, the maximum number of users that a cell can support while maintaining the MAI-free property depends on N/G and hence the multipath number L. Thus, a smaller value of L or G enables the system to support more users in one cell. In this situation, N should be much larger than L to support more users. For a fixed sample frequency, this can be done by increasing the OFDM-block duration. Hence, if the complexity is ignored, N can be as large as possible if the duration of one block does not exceed the channel coherent time. Generally speaking, this concept stands in contrast with that in an MC-CDMA system, where N should be chosen to be close to L so that subchannels have less correlation and a more random signature waveform. However, as stated in Theorem 1, when the proposed code design is used, the system is completely MAI-free so that we can choose N that is much larger than L to support more users in WLAN applications.

IV. Channel Estimation Under MAI-free Condition

In the last section, we assume that the channel information $\lambda_i[k]$ for every user is known to the receiver. Without accurate channel information, neither ORC nor MRC can be performed at the receiver end. For non-MAI-free schemes, channel information is needed for the MUD-based technique in the receiver. If channel information is not available, it has to be estimated by some techniques [23], [24]. For uplink transmission, every user experiences a different fading. Thus, multiuser channel estimation is required if the system is not MAI-free. For downlink transmission, although the mixed signal of all users from the base station experiences the same channel fading, orthogonality of users' codes may be destroyed as a result of frequency-selective fading. Unless the base station uses the same training sequence x_i and spreading code $w_i[k]$ for every user at the same time slot, it would be difficult for an individual user to acquire his/her own downlink channel information without extra signal processing techniques. However, this reduces the system flexibility since all users have to be coordinated for training with the same signature waveform at the same time slot. Thus, it is desirable to design a system where channel estimation is conducted under an MAI-free environment. In this section, we will show that the channel information can be obtained in an MAI-free environment if the proposed code scheme is used. Thus, there is no need to do multiuser estimation in the uplink direction and the training procedure is more flexible in the downlink transmission.

To get $\lambda_i[k]$ is equivalent to obtaining its time domain impulse response $h_i(n)$, $0 \leq n \leq L-1$. We will show how to obtain every user's $h_i(n)$ without worrying about MAI. Again, the result derived here is for the more general uplink case. It can be adapted for the downlink case as well. Referring to Fig. 1 and from (2), if the real Hadamard-Walsh code is used, the $N \times 1$ chip vector of user i before gain combining is

$$\hat{\mathbf{z}}_i = x_i \mathbf{F}_0 \mathbf{h}_i + \sum_{j=0, j \neq i}^{T-1} x_j \mathbf{W}_i \mathbf{W}_j \mathbf{F}_0 \mathbf{h}_j + \mathbf{W}_i \mathbf{e}, \tag{19}$$

where $\mathbf{e}$ is the noise vector after DFT. Taking the N-point IDFT of $\hat{\mathbf{z}}_i$ in (19), we have

$$\mathbf{F}^{\dagger} \hat{\mathbf{z}}_i = x_i \begin{pmatrix} \mathbf{I}_L \\ \mathbf{0} \end{pmatrix} \mathbf{h}_i + \sum_{j=0, j \neq i}^{T-1} x_j \underbrace{\mathbf{F}^{\dagger} \mathbf{W}_i \mathbf{W}_j \mathbf{F}_0 \mathbf{h}_j}_{\mathbf{c}_{i,j}} + \mathbf{F}^{\dagger} \mathbf{W}_i \mathbf{e}, \tag{20}$$

where the second term is the interference term from other users. Since the channel path is of length L, if the first L elements of $\mathbf{c}_{i,j}$ are zeros for all $\mathbf{h}_j$, we can obtain channel $\mathbf{h}_i$ without worrying about the interference from other users.

Theorem 2: Suppose that the channel length is equal to L and the code scheme as stated in Theorem 1 is used, where $G \geq L$. Then, if we use any one subset of codewords in the MC-CDMA system, the first L elements of $\mathbf{c}_{i,j}$ are zeros. As a result, we can estimate the channel $\mathbf{h}_i$ in a completely MAI-free environment. That is,

$$z_i(n) = x_i h_i(n) + \tilde{e}_i(n), \quad 0 \leq n \leq L-1, \tag{21}$$

where $z_i(n)$ is the nth element of $\mathbf{F}\hat{\mathbf{z}}_i$ and $\tilde{e}_i(n)$ is the nth element of $\mathbf{F}^{\dagger}\mathbf{W}_i\mathbf{e}$.

Proof: Let us express the DFT matrix $\mathbf{F}$ as $(\ \mathbf{F}_0\ \mathbf{F}_1)$, $\mathbf{c}_{i,j}$ in (20) can be manipulated as

$$\mathbf{c}_{i,j} = \begin{pmatrix} \mathbf{F}_0^\dagger \\ \mathbf{F}_1^\dagger \end{pmatrix} \mathbf{W}_i \mathbf{W}_j \mathbf{F}_0 \mathbf{h}_j = \begin{pmatrix} \mathbf{F}_0^\dagger \mathbf{R}_{i,j} \mathbf{F}_0 \\ \mathbf{F}_1^\dagger \mathbf{R}_{i,j} \mathbf{F}_0 \end{pmatrix} \mathbf{h}_j. \tag{22}$$

From the discussion in Sec. III, $\mathbf{F}_0^\dagger \mathbf{R}_{i,j} \mathbf{F}_0 = \mathbf{0}$ if any one subset of codewords are used. Hence, the first L elements of $\mathbf{c}_{i,j}$ are zeros, and we get (21). ■

According to (21), if x_i is a known training symbol, we can obtain $h_i(n)$, $0 \leq n \leq L-1$, without worrying about the interference from symbols of other users. That is, channel estimation can be done in a completely MAI-free environment.

Discussion on System Parameters and Performance Tradeoff. From the discussion above, when the number of users increases, we may increase the spreading gain N or decrease the number of partitioned subsets G to accommodate more users. The adjustment of parameters N and G dynamically is an interesting problem, which is under our current investigation. When the system is heavily loaded in the sense that the number of active users is approaching N/L, the proposed code design provides a set of optimal codes for the system in terms of MAI reduction and multipath diversity.

Another tradeoff results from the change of the multipath length L. Under the condition $G = L$, if L becomes larger (or smaller), the number of allowed users decreases (or increases). For a fixed N, since the diversity gain of a user is equal to L, there exists a tradeoff between the number of users and the diversity gain [17].

Finally, it is interesting to examine the case where the number of active users exceeds N/L. Under this scenario, to get an MAI-free system, MUD can be used in the uplink direction while the orthogonal resorting combining (ORC) scheme [10] can be performed in the downlink direction. However, the full diversity gain may be lost due to noise amplification by MUD and ORC. Moreover, if no MUD is used, there exist an MAI effect. We observe from computer simulation in Sec. VI that the proposed code scheme still outperforms other codes in terms of MAI reduction (even though the system is not completely MAI-free in this case). Thus, the proposed code scheme is still a preferred choice.

V. Performance of the Proposed Code Design in the Presence of CFO

In this section, we consider the CFO effect and show that it can be handled by the use of the proposed code design. In particular, we show that the MAI due to the CFO effect can be reduced to zero or a negligible amount. Consider the kth chip of the received vector after DFT in a CFO environment, *i.e.*

$$\hat{y}[k] = \sum_{j=0}^{T-1} r_j[k] + e[k], \tag{23}$$

where $e[k]$ is the received noise after DFT, and $r_j[k]$ is the received signal due to channel fading and the CFO effect. Suppose the jth user has a normalized CFO ϵ_j, which is the actual CFO normalized by $1/N$ of the overall bandwidth and $-0.5 \le \epsilon_j \le 0.5$. $r_j[k]$ in (23) can be expressed by [14]:

$$\begin{aligned} r_j[k] &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left[\frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \lambda_j[m] y_j[m] e^{j\frac{2\pi}{N}nm} \right] e^{j\frac{2\pi}{N}n\epsilon_j} e^{-j\frac{2\pi}{N}nk} \\ &= \underbrace{\alpha_j \lambda_j[k] y_j[k]}_{r_j^{(0)}[k]} + \underbrace{\beta_j \sum_{m=0,m\neq k}^{N-1} \lambda_j[m] y_j[m] \frac{e^{-j\pi\frac{m-k}{N}}}{N \sin\frac{\pi(m-k+\epsilon_j)}{N}}}_{r_j^{(1)}[k]}, \end{aligned} \tag{24}$$

where α_j and β_j are given by

$$\alpha_j = \frac{\sin \pi\epsilon_j}{N \sin\frac{\pi\epsilon_j}{N}} e^{j\pi\epsilon_j \frac{N-1}{N}} \text{ and } \beta_j = \sin(\pi\epsilon_j) e^{j\pi\epsilon_j \frac{N-1}{N}}. \tag{25}$$

The first term of Eqn. (24) is the distorted chip and the second term is the ICI caused by the CFO. Note that, when there is no CFO, $r_j[k]$ equals $\lambda_j[k]y_j[k]$ as in (2). From (3) and (23), if the real Hadamard-Walsh code is used, we see that $\hat{x}_i[k]$ under CFO is given by

$$\hat{x}_i = \underbrace{\sum_{k=0}^{N-1} r_i[k]\lambda_i^*[k]w_i[k]}_{s_i} + \sum_{j=0,j\neq i}^{T-1} \underbrace{\sum_{k=0}^{N-1} r_j[k]\lambda_i^*[k]w_i[k]}_{\widetilde{MAI}_{i\leftarrow j}} + \sum_{k=0}^{N-1} e[k]\lambda_i^*[k]w_i[k], \tag{26}$$

where s_i is the desired signal and $\widetilde{MAI}_{i\leftarrow j}$ is the MAI of user i due to the jth user's CFO. Using (24) and (26), it can be shown that the MAI term is given by

$$\widetilde{MAI}_{i\leftarrow j} = MAI_{i\leftarrow j}^{(0)} + MAI_{i\leftarrow j}^{(1)}, \tag{27}$$

where

$$\begin{aligned} MAI_{i\leftarrow j}^{(0)} &= \sum_{k=0}^{N-1} r_j^{(0)}[k]\lambda_i^*[k]w_i[k] \\ &= \alpha_j x_j \sum_{k=0}^{N-1} \lambda_j[k] w_j[k] \lambda_i^*[k] w_i[k], \end{aligned} \tag{28}$$

and

$$\begin{aligned} MAI_{i\leftarrow j}^{(1)} &= \sum_{k=0}^{N-1} r_j^{(1)}[k]\lambda_i^*[k]w_i[k] \\ &= \beta_j x_j \underbrace{\sum_{k=0}^{N-1} \sum_{m=0,m\neq k}^{N-1} \lambda_j[m] w_j[m] \frac{e^{-j\pi\frac{m-k}{N}}}{N \sin\frac{\pi(m-k+\epsilon_j)}{N}} \lambda_i^*[k] w_i[k]}_{\eta_j}. \end{aligned} \tag{29}$$

Note that if there is no CFO for user j, *i.e.* $\alpha_j = 1$ and $\beta_j = 0$, $MAI^{(1)}_{i\leftarrow j} = 0$ and $MAI^{(0)}_{i\leftarrow j}$ is equal to the MAI term defined in (3). This gives us an intuition that $MAI^{(0)}_{i\leftarrow j}$ is the dominating MAI term when the CFO is small. Hence, if we can find a way that makes $MAI^{(0)}_{i\leftarrow j} = 0$, the MAI due to the CFO can be reduced to a negligible amount. According to (3), (28) and Theorem 1, we have the following Lemma.

Lemma 4: Let the channel length be L and the code scheme as stated in Theorem 1 is used. Then, if we use any one of the G subsets of codewords for the MC-CDMA system with $G \geq L$, the dominating MAI term $MAI^{(0)}_{i\leftarrow j}$ in (28) is zero.

Now, let us look at another interference term $MAI^{(1)}_{i\leftarrow j}$, which is called the "residual MAI" for convenience. Define $g_j(p) = \frac{e^{-j\pi\frac{p}{N}}}{N\sin\frac{\pi(p+\epsilon_j)}{N}}$. Then, we have the following Lemma.

Lemma 5: Let the channel length be L and the code scheme as stated in Theorem 1 is used. Then, if we use any one of the G subsets of codewords for the MC-CDMA system with $G \geq L$, the residual MAI term $MAI^{(1)}_{i\leftarrow j}$ in (29) becomes

$$MAI^{(1)}_{i\leftarrow j} = \beta_j x_j \sum_{p=1}^{N-1} g_j(-p)\left\{\left(\mathbf{h}_i^{(p)}\right)^\dagger \underbrace{\mathbf{F}_0^\dagger \mathbf{W}_i^{(p)} \mathbf{W}_j \mathbf{F}_0}_{\mathbf{D}_{i,j}^{(p)}} \mathbf{h}_j\right\}, \tag{30}$$

where

$$\mathbf{W}_i^{(p)} = \mathrm{diag}\left(w_i[p]\cdots w_i[N-1]\ w_i[0]\cdots w_i[p-1]\right), \tag{31}$$

and

$$\mathbf{h}_i^{(p)} = \begin{pmatrix} h_i(0)e^{-j\frac{2\pi}{N}0p} \\ \vdots \\ h_i(L-1)e^{-j\frac{2\pi}{N}(L-1)p} \end{pmatrix}. \tag{32}$$

Proof: The proof is given in appendix.

Since the MAI due to the CFO is divided into two terms, *i.e.* the dominating MAI in (28) and the residual MAI in (29), if we can make both terms equal zero, the system can be MAI-free under a CFO environment. From Lemma 4, $MAI^{(0)}_{i\leftarrow j} = 0$ with the proposed code. Thus, our goal now is to find a way to make $MAI^{(1)}_{i\leftarrow j} = 0$. Let us further manipulate $\mathbf{D}_{i,j}$ in (30) as

$$\mathbf{D}_{i,j}^{(p)} = \begin{pmatrix} \mathbf{I}_L & \mathbf{0} \end{pmatrix} \mathbf{F}^\dagger \mathbf{W}_i^{(p)} \mathbf{W}_j \mathbf{F} \begin{pmatrix} \mathbf{I}_L \\ \mathbf{0} \end{pmatrix}. \tag{33}$$

According to (30) and (33), if $\mathbf{D}_{i,j}^{(p)} = 0$ for all $1 \leq p \leq N-1$, we have $MAI^{(1)}_{i\leftarrow j} = 0$.

Lemma 6: Suppose the codeword set G_0 is used, the two codewords $\mathbf{w}_0$ and $\mathbf{w}_1$ will have zero $MAI^{(1)}_{i\leftarrow j}$ term. That is, $\sum_{j=0,j\neq 0}^{T-1} MAI^{(1)}_{0\leftarrow j} = 0$ and $\sum_{j=0,j\neq 1}^{T-1} MAI^{(1)}_{1\leftarrow j} = 0$.

Proof. For the all-one code $\mathbf{w}_0$, we have

$$\mathbf{W}_0^{(p)}\mathbf{W}_j = \mathbf{W}_j, \quad 1 \le p \le N-1.$$

Hence, we have $\mathbf{D}_{0,j}^{(p)} = \begin{pmatrix} \mathbf{I}_L & \mathbf{0} \end{pmatrix} \mathbf{F}^\dagger \mathbf{W}_j \mathbf{F} \begin{pmatrix} \mathbf{I}_L \\ \mathbf{0} \end{pmatrix}$, for $1 \le p \le N-1$. Since $\mathbf{W}_j$ is a diagonal matrix with diagonal elements drawn from G_0. From the discussion in Sec III, $\mathbf{D}_{0,j}^{(p)} = 0$ for all p. Hence, $\sum_{j=0,j\neq 0}^{T-1} MAI_{0\leftarrow j}^{(1)} = 0$.

For $\mathbf{w}_1$, which is a codeword with a sign change for every consecutive code symbol, *i.e.* $\mathbf{w}_1 = (+1\ -1\ +1\ -1\cdots)^t$, its circulant shift is either $\mathbf{w}_1$ or $(-1\ +1\ -1\ +1\cdots)^t = -\mathbf{w}_1$. Hence, we have

$$\mathbf{W}_1^{(p)}\mathbf{W}_j = \begin{cases} \mathbf{W}_j, & p \text{ even}, \\ -\mathbf{W}_j, & p \text{ odd}, \end{cases} \quad 1 \le p \le N-1.$$

Therefore, $\sum_{j=0,j\neq 0}^{T-1} MAI_{1\leftarrow j}^{(1)} = 0$. ■

Lemma 6 suggests to use the 0th codeword set so that there are two codewords to preserve the MAI-free property under the CFO environment. Since codewords $\mathbf{w}_0$ and $\mathbf{w}_1$ are completely MAI-free under the CFO environment when G_0 is used, we can use them as training sequences to estimate the channel and/or CFO for each user. That is, in uplink direction, every user use these two codewords in turn to acquire his/her own channel and/or CFO information. In the downlink direction, one of these two codewords can be reserved as the pilot signal for CFO estimation. In this case, any single-user CFO estimation algorithm (*e.g.*, the one given in [14]) can be applied while sophisticated MUD or signal processing techniques can be avoided. This result stands in contrast with the CFO estimation for GO-MC-CDMA systems [4], where multiuser estimation is demanded to acquire accurate CFO information.

VI. Simulation Results

Computer simulation results are provided in this section to corroborate theoretical results derived earlier. In the simulation, we considered the performance in the uplink direction so that every user has a different channel fading and CFO value. Note that the Hadamard-Walsh codewords are generated using the Kronecker product in Eqn. (8) so that the codeword indices are adopted based on this fact.

Example 1: Illustration of the MAI-free property.

In this example, we show that MC-CDMA is MAI-free with the proposed code scheme. The simulation was conducted under the following setting: $N = 64$, $G = L = 2$ or 4. The transmit power had an unit variance. The taps of the channel were i.i.d. (independently identically distributed) random variables with an unit variance. We evaluate the $MAI_{i\leftarrow j}$ as given in (3). For $L = 2$, one-realization of $|MAI_{i\leftarrow j}|$ as a function of user indices i and j is shown in Fig. 4 (a). As shown in the figure, there are two zones where the MAI is zero; *i.e.*, the zone with codewords from 1 to 32, and the zone with codewords from 33 to 64. The peak values appear along the diagonal since they correspond to the reconstructed desired signal power for each user. Thus, the system is MAI-free if either one of the two subsets of codewords is in use. For $L = 4$, the performance is shown in Fig. 4 (b). We see 4 zones where the MAI is equal to zero. Hence, if we use either one of these 4 subsets, we can achieve an MAI-free system. These results corroborate the claim in Theorem 1.

Example 2: Illustration of the diversity gain.

In this example, we would like to show that, when the proposed code scheme is used, every user can achieve a low bit error probability to reflect the diversity gain L. The BPSK modulation and Hadamard-Walsh codes of $N = 16$ were adopted. The channel coefficients were i.i.d. complex Gaussian random variables of unit variance. For each individual user, the Monte Carlo method was run for more than 250,000 symbols. The bit error probabilities of two systems were shown in Fig. 5. The solid curve is obtained from a system with flat fading, *i.e.* $L = 1$, with N full codewords used. The dashed curve is resulted from a system of multipath length $L = 2$ and with the proposed $N/2$ Hadamard-Walsh codes G_0. Since there is no MAI, simulation results are consistent with the theoretical results in [1] and [17].

We see that, when L grows form 1 to 2 with the proposed code scheme, the bit error probability improves dramatically due to the increase of the diversity order. Actually, the dashed curve is the same as that for a system with $L = 1$ and two receive antennas with MRC [1]. That is, a diversity order of 2 is achieved via code design in the frequency domain rather than the space domain (see p. 777 in [17]). This example also explains the interplay between the diversity order and the number of users allowed. That is, when L grows, we need to divide N codewords into more subsets to achieve MAI-free. Hence, fewer users can be supported within each cell. However, these users can enjoy a higher diversity order as L increases.

Note that frequency diversity is inherent in MC-CDMA systems. However, without a proper code design, the system has MAI that will degrade the BER performance as the number of users increases. Under this situation, MAI will dominate system performance and increasing diversity gain alone may not necessarily improve overall performance [10]. If the proposed code design is used together with M_r receive antennas, a diversity order of LM_r can be achieved for each individual user.

Example 3: MAI in the presence of CFO.

In this example, we demonstrate that the dominating MAI due to the CFO effect can be completely eliminated by the use of the proposed code design. The system configuration was the same as that in Example 2 with multipath length $L = 2$. Since the simulation was conducted in the uplink direction, every user has his/her own CFO value. Let us consider the worst case, where every user is randomly assigned a CFO either $+\varepsilon$ or $-\varepsilon$. According to (3), when there is no CFO, the desired signal will be scaled by $\sum_{k=0}^{N-1} |\lambda_i[k]|^2$. Thus, we normalize the MAI by $\sum_{k=0}^{N-1} |\lambda_i[k]|^2$ for fair comparison. The dominating total MAI of user i, denoted by $MAI_i^{(0)}$, is obtained by averaging $\left| \frac{1}{\sum_{k=0}^{N-1} |\lambda_i[k]|^2} \sum_{j=0, j\neq i}^{T-1} MAI_{i\leftarrow j}^{(0)} \right|^2$ for more than 250,000 symbols. Similarly, the residual total MAI, denoted by $MAI_i^{(1)}$, is obtained by averaging $\left| \frac{1}{\sum_{k=0}^{N-1} |\lambda_i[k]|^2} \sum_{j=0, j\neq i}^{T-1} MAI_{i\leftarrow j}^{(1)} \right|^2$ for more than 250,000 symbols. To illustrate the MAI effect clearer, we did not add noise in this example.

First, let us consider the fully-loaded case, *i.e.* $T = N = 16$. The slim-triangular curves in Fig. 6 show the dominating and the residual total MAI of each individual user. The bold-diamond curve, denoted by $MAI^{(0)}$, is the averaged dominating total MAI for the 16 slim-triangular curves of $MAI_i^{(0)}$. Note that the 16 slim-triangular curves of $MAI_i^{(0)}$ are tightly clustered and thus overlap with the bold-diamond curve. Similarly, the bold-square curve, denoted by $MAI^{(1)}$, is the averaged residual total MAI for the 16 slim-circle curves of $MAI_i^{(1)}$. We see that $MAI^{(0)}$ is larger than $MAI^{(1)}$ by 5-32 dB. Hence, it confirms that the dominating MAI term defined in (28) is indeed the key MAI impairment, which is due to CFO.

Now, we consider several half-loaded scheme. First, we examine Shi and Latva-aho's scheme [20] for a half-loaded system, *i.e.* $\mathbf{w}_0$, $\mathbf{w}_1$, $\mathbf{w}_6$, $\mathbf{w}_7$, $\mathbf{w}_{10}$, $\mathbf{w}_{11}$, $\mathbf{w}_{12}$ and $\mathbf{w}_{13}$. The MAI performance is shown in Fig. 7. By comparing Fig. 7 with Fig. 6, we see that both the dominating MAI and the residual MAI decrease by only about 3 dB, which shows a reasonable but not satisfactory MAI reduction as the number of users decreases to half in the system.

Next, let us consider the proposed code selection schemes with half-loaded. Since $L = 2$, we divide the codewords into two subsets. G_0 contains the first $N/2$ codewords and G_1 contains the last $N/2$ codewords. The performance is shown in Fig. 8 (a) and (b), respectively. Note that the dominating MAI term $MAI_i^{(0)}$ is equal to zero so that it is not shown here. Moreover, there are only 6 curves in Fig. 8 (a) for $MAI_i^{(1)}$, since the two codewords $\mathbf{w}_0$ and $\mathbf{w}_1$ are completely MAI-free under the CFO environment.

By examining Fig. 6 and Fig. 8 (a) and (b), we see that the dominating MAI can be completely eliminated by the proposed code scheme. In this case, the residual MAI will determine the system performance. Furthermore, the residual MAI decreases around 5 dB. These results show that the MAI due to the CFO effect can be greatly reduced using the proposed code schemes. Moreover, if the codewords of G_0 are used, users of $\mathbf{w}_0$ and $\mathbf{w}_1$ can still have zero MAI under the CFO environment.

Example 4: BER in the presence of CFO.

In this example, we consider the bit error rate (BER) performance in the presence of the CFO effect for several code schemes of MC-CDMA and the GO-MC-CDMA scheme [4]. The parameter setting remains the same as that in Example 3. For MC-CDMA, we consider the proposed schemes G_0 and G_1, Shi and Latva-aho's scheme [20], and the even indexed codewords, *i.e.* $\mathbf{w}_0$, $\mathbf{w}_2$, $\cdots$, $\mathbf{w}_{N-2}$. For GO-MC-CDMA, since $L = 2$, we divide 16 subcarriers into 8 groups, where each group can support 2 users. Since the system is half-loaded, each group has exactly one user so that the system is MAI-free when there is no CFO. We assume that every scheme can accurately estimate individual user's CFO. With G_0, this can be achieved using estimation algorithms for single-user OFDM systems since there are two MAI-free codewords even in a multiuser CFO environment. In contrast, other schemes need to use multiuser CFO estimation. The CFO effect is compensated at the receiver without any feedback. Fig. 9 shows the BER as a function of the CFO with SNR ($= E_b/N_0$) fixed at 15 dB for different schemes. It is clear that the proposed code schemes G_0 and G_1 outperform Shi and Latva-aho's scheme and the set of even-indexed codewords significantly. They also outperform GO-MC-CDMA slightly. We see from this figure that codeword set G_0 slightly outperforms codeword set G_1. This is because that $\mathbf{w}_0$ and $\mathbf{w}_1$ are free from MAI in the presence of CFO.

Example 5: CFO estimation with a single-user algorithm.

It was shown in Example 4 that the GO-MC-CDMA system can achieve comparable performance with the proposed code scheme in a CFO environment. However, this result is obtained under the assumption that every user's CFO can be estimated accurately. For the MC-CDMA system with codeword set G_0, users with codewords $\mathbf{w}_0$ and $\mathbf{w}_1$ do not have MAI from others in a CFO environment and, consequently, accurate CFO can be estimated if each user adopts these codewords to estimate his/her own CFO in turn. In this case, estimation algorithms developed for single-user OFDM can be used for the proposed scheme. In contrast, we need more sophisticated estimation algorithms for multiuser OFDM systems for GO-MC-CDMA since none of the users in GO-MC-CDMA is free from MAI in a CFO environment. In this example, we will evaluate the CFO estimation error for the proposed system and the GO-MC-CDMA system when the single-user CFO estimation algorithm given in [14] is used.

The parameter setting remains the same as that in Example 4. Since both MC-CDMA and GO-MC-CDMA spread one symbol into several chips, the detection output is actually one symbol. Hence, we only need two symbols for CFO estimation. Referring to Fig. 1, we denote the two successive output symbols, *i.e.* the current and the next ones, by $\hat{x}_i$ and $\hat{x}'_i$. The CFO estimation can be obtained via [14]

$$\hat{\epsilon}_i = \frac{1}{2\pi}\tan^{-1}\left[\Im\left\{\hat{x}_i^*\hat{x}'_i\right\}/\Re\left\{\hat{x}_i^*\hat{x}'_i\right\}\right],$$

where $\Re\{x\}$ and $\Im\{x\}$ are the real and the imaginary parts of x. For the proposed scheme, all eight codewords in G_0 are active. For the GO-MC-CDMA, each user occupies one of the eight groups and there are eight users [4]. Without loss of generality, $\mathbf{w}_0$ is used for CFO estimation in the proposed scheme. For GO-MC-CDMA, the user who occupies the 0th and the 8th subcarriers with the all-one codeword is used for CFO estimation. The Monte Carlo method is used to run for more than 20,000 realizations. The estimation mean square errors, *i.e.*, $E\left\{|\epsilon_i - \hat{\epsilon}_i|^2\right\}$, as a function of CFO for both systems are shown in Fig. 10. We see that the estimation error in GO-MC-CDMA increases as the CFO value becomes larger. This is because the CFO-induced MAI increases as the CFO value increases, which deteriorates estimation accuracy. On the other hand, the estimation error in the proposed MC-CDMA system with G_0 remains constant in a multiuser CFO environment. This shows that the use of codeword set G_0 has a better CFO estimation result in a multiuser environment.

Example 6: Code priority of MC-CDMA.

In this example, we consider a code priority scheme for a fully-loaded MC-CDMA system using the proposed code scheme. The system setting is the same as that in Example 4 except that SNR is set to 18 dB while CFO is set to zero. The Monte Carlo method is used with more than 10,000,000 symbols for all users in the simulation.

We first consider two code priority schemes that assign codewords according to the following order:

$$\text{Priority Scheme I} : \mathbf{w_0},\ \mathbf{w_8},\ \mathbf{w_1},\ \mathbf{w_9},\ \mathbf{w_2},\ \mathbf{w_{10}},\ \mathbf{w_3},\ \mathbf{w_{11}},\ \mathbf{w_4},\ \mathbf{w_{12}},\ \mathbf{w_5},\ \mathbf{w_{13}},\ \mathbf{w_6},\ \mathbf{w_{14}},\ \mathbf{w_7},\ \mathbf{w_{15}}. \quad (34)$$

$$\text{Priority Scheme II} : \mathbf{w_0},\ \mathbf{w_9},\ \mathbf{w_2},\ \mathbf{w_{11}},\ \mathbf{w_4},\ \mathbf{w_{13}},\ \mathbf{w_6},\ \mathbf{w_{15}},\ \mathbf{w_1},\ \mathbf{w_8},\ \mathbf{w_3},\ \mathbf{w_{10}},\ \mathbf{w_5},\ \mathbf{w_{12}},\ \mathbf{w_7},\ \mathbf{w_{14}}. \quad (35)$$

Scheme I assigns the next user an even-indexed (or odd-indexed) codeword in G_1 whenever the current user is assigned an even-indexed (or odd-indexed) codeword in G_0. Since even-indexed (or odd-indexed) codewords in G_1 cause more serious MAI to even-indexed (or odd-index) codewords, the first code priority has poor performance. It is adopted as a performance benchmark. Scheme II assigns even- and odd-indexed codewords from G_0 and G_1 alternatively for the first 8 users. It serves as another performance benchmark. Furthermore, we also implement the code priority scheme proposed by Shi and Latva-aho's in [20]. Since this scheme only considers a system up to the half-loaded situation, its performance curve is plotted up to 8 users.

Finally, we consider the proposed code priority scheme, where we first assign 8 codewords in G_0 to the first 8 active users. When the number of active users exceeds 8, we will use codewords in G_1. One such code priority scheme can be written as

$$\text{Proposed Priority Scheme}: \mathbf{w_0},\ \mathbf{w_1},\ \mathbf{w_2},\ \mathbf{w_3},\ \mathbf{w_4},\ \mathbf{w_5},\ \mathbf{w_6},\ \mathbf{w_7},\ \mathbf{w_8},\ \mathbf{w_9},\ \mathbf{w_{10}},\ \mathbf{w_{11}},\ \mathbf{w_{12}},\ \mathbf{w_{13}},\ \mathbf{w_{14}},\ \mathbf{w_{15}}. \tag{36}$$

If there is no CFO, the system is MAI-free using only 8 codewords in either G_0 or G_1 according to Theorem 1. The order of the first 8 codewords can be changed arbitrarily. Also, we can assign codewords all from G_1 first and then from G_0.

The bit error rate is plotted as a function of the number of active users for the above four code priority schemes in Fig. 11. Scheme I has the worst performance as expected. The performance of the proposed code priority stays the same when the number of active users is smaller than 9 due to the MAI-free property. When T exceeds 8, the performance of the proposed scheme degrades dramatically. However, its performance remains at least as good as Schemes I and II. We also see that for Shi and Latva-aho's scheme, it has the same performance as the proposed code priority when the numbers of users are 1, 2, 3 and 5. This is reasonable since codewords of these user numbers fall in the set of G_0 and, hence, they are free from MAI. However, for other number of users, its performance is worse than the proposed code priority scheme. Moreover, in the Shi and Latva-aho's priority, if the number of active users changes, some users will need to change their codewords, which complicates the actual deployment of this scheme.

VII. Practical Considerations on Applicability of the Proposed Scheme

The proposed scheme is applicable to both upper and down links in a multiuser system. The basic assumption of having a synchronous channel holds in the downlink. As to the uplink, we may consider a quasi-synchronous channel, where the time offset is within one chip. Such a channel holds in the uplink direction for a micro cell, *e.g.* see pp. 1179-1195 in [21]. In practice, quasi-synchronism can be achieved by the use of the globe positioning service (GPS). Therefore, there are several systems or code designs based on this assumption; *e.g.*, group-orthogonal (GO-) MC-CDMA system [4], the LV (Lagrange/Vandermonde) code [19], the code scheme for MC-CDMA in [20], and the LAS (large area synchronized) code in [22]. Even if the system is not perfectly synchronized, time delay can still be included in the channel impulse response. In this case, we may have larger L and, hence, the number of supportable users to meet the MAI-free property decreases.

As presented above, there is a close connection between the multipath length, the spreading gain (*i.e.*, the number of subcarriers) and the number of users that can be supported by the proposed technique. In particular, in order to achieve a zero MAI, the system load has to be significantly reduced for a larger multipath length. This is a potential disadvantage for the proposed scheme. In practice, under a reasonable sampling frequency, the multipath length is in general moderate. For example, in an outdoor environment, the most commonly used multipath duration is around 1-3 μ seconds [18]. For the IS-95 standard, the chip rate is 1.2288 M chips/second in the uplink direction. Hence, the resolvable multipath length is around 1-3 taps. In an indoor environment, the maximum multipath duration for an office building is around 0.27 μ seconds. If we take the sampling frequency of WLAN of 20 MHz as an example, the resolvable multipath length is around 5 taps. However, the indoor multipath duration is in general under 0.1 μ seconds. In this case, the resolvable multipath length is around 2-3 taps. It is worthwhile to point out that under a fixed multipath L and DFT/IDFT size N, the GO-MC-CDMA system [4] without MUD supports exactly the same number of MAI-free users as the proposed system. Although the number of MAI-free users in both systems decreases as channel length L increases, every MAI-free user in these systems can enjoy an increased channel diversity order L.

As commented by Chen in [6] "...all existing CDMA systems fail to offer satisfactory performance and capacity, which is usually far less than half of the processing gain of CDMA systems." Hence, the choice of the spreading code to reduce MAI can be a direction for the design of next-generation CDMA systems. We approach the MAI reduction problem from a similar viewpoint. That is, for fixed channel conditions, we attempt to select a subset of codewords that can lead to an MAI-free system and hence provide a high date rate with simple transceiver design. This design concept stands in contrast with that of conventional CDMA systems. For instance, as the number of users increases in IS-95, the achievable data rate decreases in order to support the full-loaded user capacity. If a higher data rate is desired in IS-95, we need to use the more sophisticated MUD, which will increase the transceiver burden.

VIII. CONCLUSION

A code design to achieve MAI-free MC-CDMA systems in both uplink and downlink directions was proposed by properly choosing a subset of codewords from Hadamard-Walsh codes. This method does not destroy the diversity gain of the MC-CDMA system. We also demonstrated how to perform channel estimation for each individual user under an MAI-free environment. As a result, there is no need to use MUD or sophisticated signal processing for symbol detection and channel estimation. Furthermore, we considered the CFO effect for the proposed code and showed that the proposed code scheme can mitigate the MAI effect due to CFO to zero or a negligible amount. Thus, there is no need to perform multiuser estimation to estimate every users' CFO. Note that the number of supportable users to achieve the MAI-free property is less than or equal to the ratio N/L, where N is the spreading gain and L is the multipath length. However, if a fully-loaded MC-CDMA system is demanded, the proposed code scheme can also be used in the design of a code priority scheme.

IX. Appendix: Proof of Lemma 5

Proof: It can be shown that η_j in (29) is given by

$$\eta_j = \sum_{p=1}^{N-1} \left\{ g_j(p) \sum_{q=0}^{N-1-p} \lambda_j[p+q] w_j[p+q] \lambda_i^*[q] w_i[q] + g_j(-p) \sum_{q=0}^{N-1-p} \lambda_j[q] w_j[q] \lambda_i^*[p+q] w_i[p+q] \right\}. \tag{37}$$

For convenience, let us define

$$\mu_j(p) = g_j(p) \sum_{q=0}^{N-1-p} \lambda_j[p+q] w_j[p+q] \lambda_i^*[q] w_i[q]. \tag{38}$$

Since $\sin \pi \frac{-N+p+\epsilon_j}{N} = -\sin \pi \frac{p+\epsilon_j}{N}$ and $e^{j\pi \frac{-N+p}{N}} = -e^{j\pi \frac{p}{N}}$, $1 \leq p \leq N-1$, we have $g_j(-N+p) = g_j(p)$. Hence,

$$\mu_j(p) = g_j(-N+p) \sum_{q=0}^{N-1-p} \lambda_j[p+q] w_j[p+q] \lambda_i^*[q] w_i[q]. \tag{39}$$

Let $p' = N - p$, we can rewrite (39) as

$$\mu_j(p) = g_j(-p') \sum_{q=0}^{p'-1} \lambda_j[N-p'+q] w_j[N-p'+q] \lambda_i^*[q] w_i[q]. \tag{40}$$

Let $q' = N - p' + q$, we can rewrite (40) as

$$\mu_j(p) = g_j(-p') \sum_{q'=N-p'}^{N-1} \lambda_j[q'] w_j[q'] \lambda_i^*[p'+q'-N] w_i[p'+q'-N]. \tag{41}$$

Let $((n))_N$ denote n modulo N [15]. Then, (41) can be rewritten as

$$\mu_j(p) = g_j(-p) \sum_{q=N-p}^{N-1} \lambda_j[q] w_j[q] \lambda_i^*[((p+q))_N] w_i[((p+q))_N]. \tag{42}$$

From (38) and (42), we can rewrite (37) as

$$\begin{aligned} \eta_j = \sum_{p=1}^{N-1} g_j(-p) \Bigg\{ & \sum_{q=N-p}^{N-1} \lambda_j[q] w_j[q] \lambda_i^*[((p+q))_N] w_i[((p+q))_N] \\ & + \sum_{q=0}^{N-1-p} \lambda_j[q] w_j[q] \lambda_i^*[p+q] w_i[p+q] \Bigg\}. \end{aligned} \tag{43}$$

Since $p \leq p+q \leq N-1$ for $0 \leq q \leq N-1-p$, we have $p \leq [((p+q))_N] \leq N-1$ for $0 \leq q \leq N-1-p$. Therefore, (43) can be rewritten as

$$\eta_j = \sum_{p=1}^{N-1} g_j(-p) \left\{ \sum_{q=0}^{N-1} \lambda_j[q] w_j[q] \lambda_i^*[((p+q))_N] w_i[((p+q))_N] \right\}. \quad (44)$$

From (44) and using matrix representation in (4), we can rewrite (29) as that given in (30). ■

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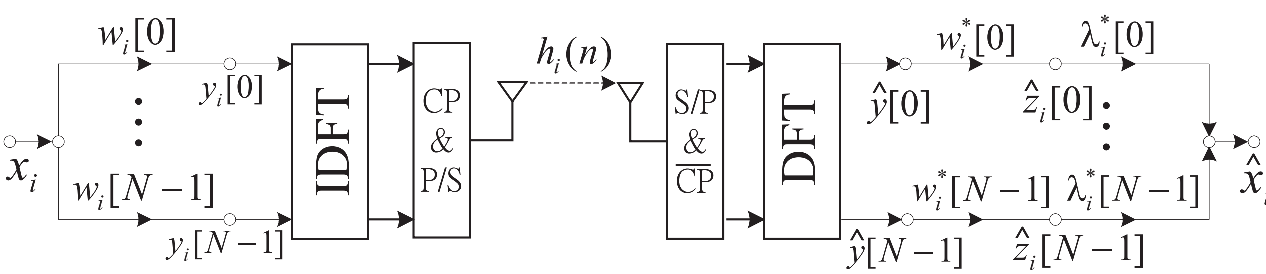


Fig. 1. The block diagram of an MC-CDMA system.

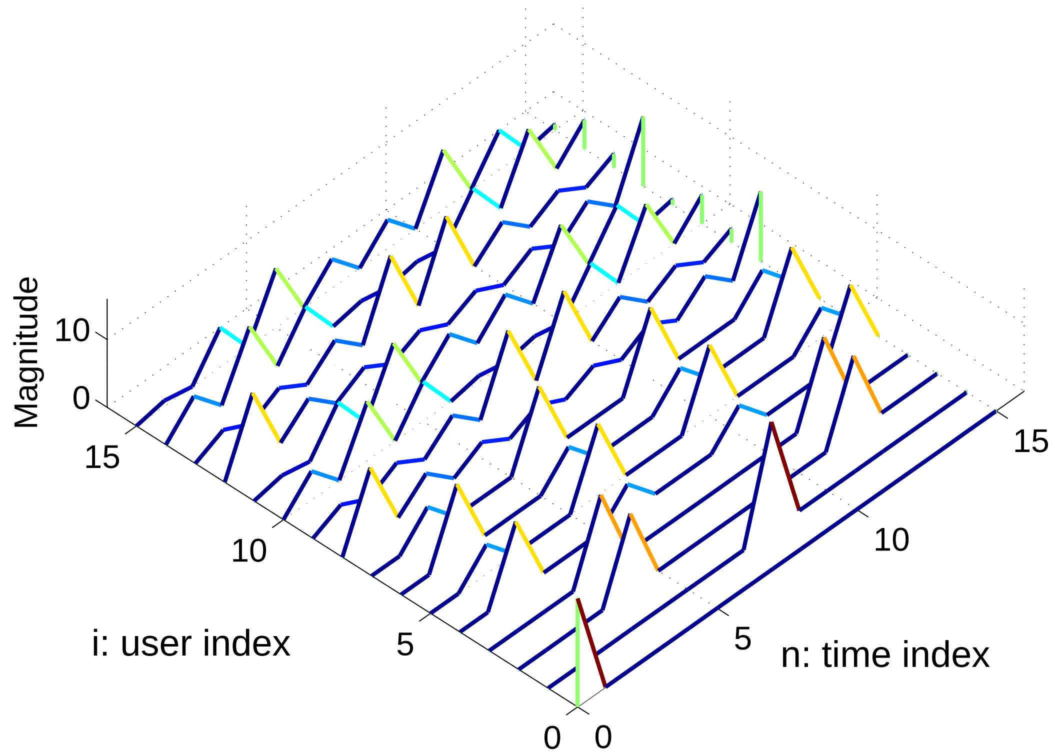


Fig. 2. $|\tilde{w}_i(n)|$ as a function of user index i and time index n with $N = 16$.

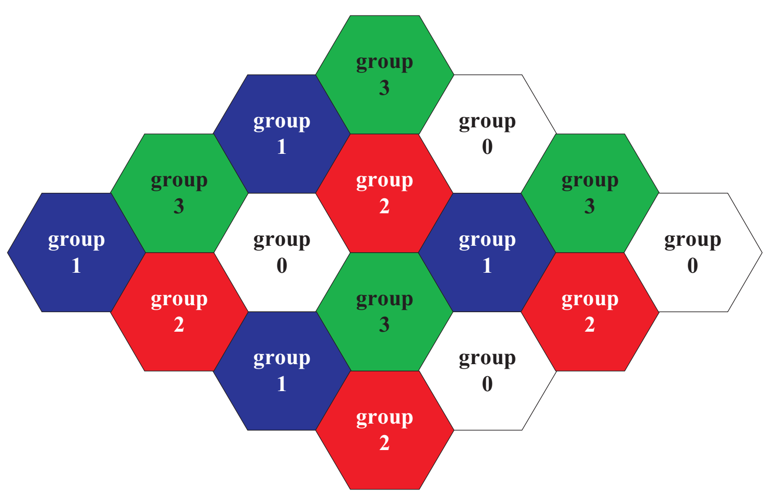
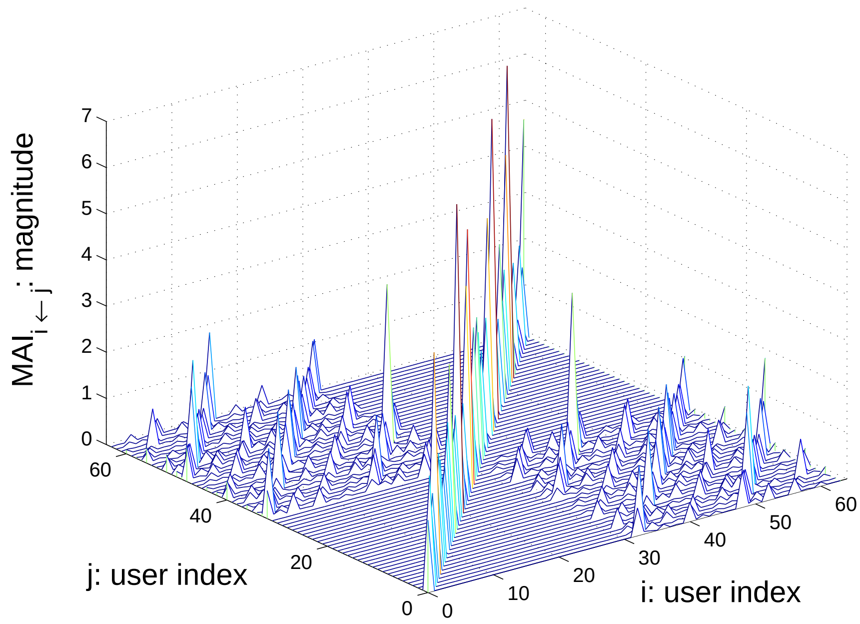
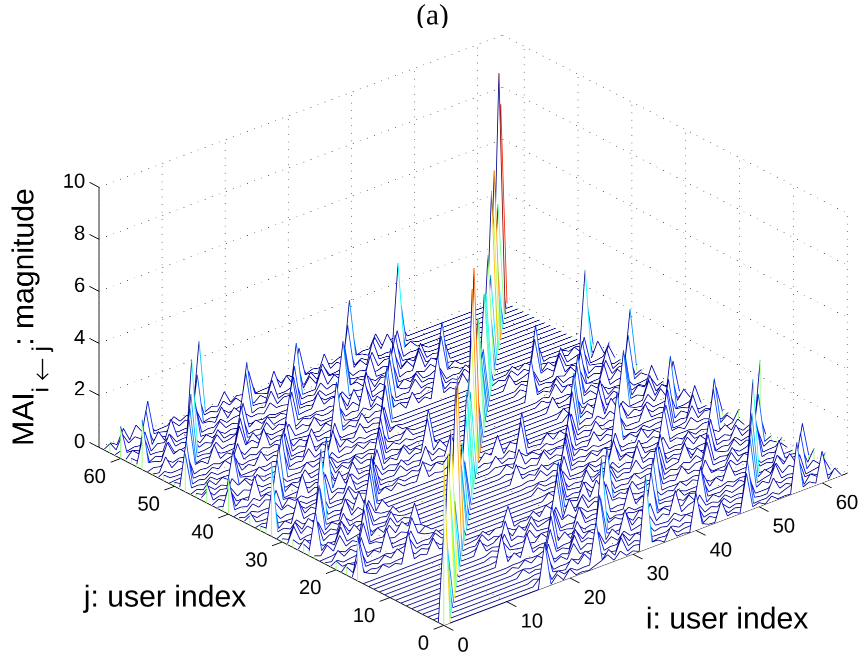


Fig. 3. An example of frequency reuse using the proposed code scheme with $G = L = 4$.



(a)



(b)

Fig. 4. $|MAI_{i \leftarrow j}|$ as a function of user indices i and j with $N = 64$: (a) $L = 2$ and (b) $L = 4$.

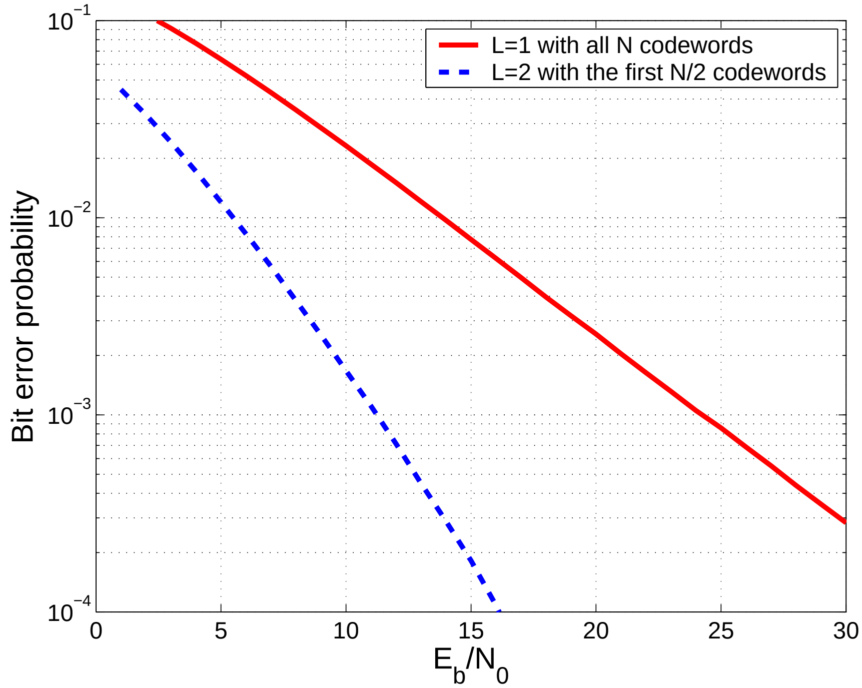


Fig. 5. The bit error rate as a function of E_b/N_0 to illustrate the diversity order of the proposed code scheme.

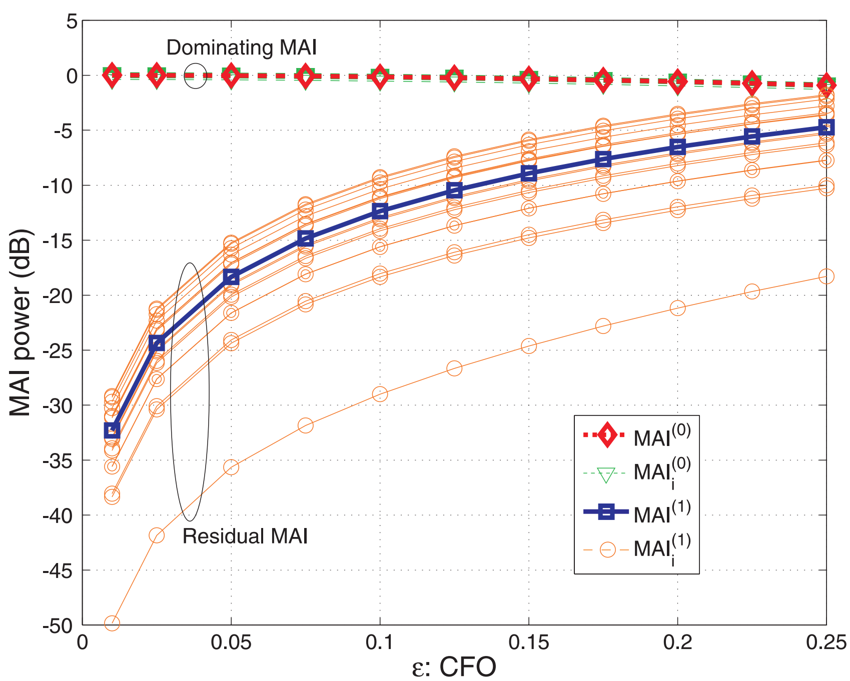


Fig. 6. The dominating and the residual MAI as a function of CFO in a fully-loaded situation.

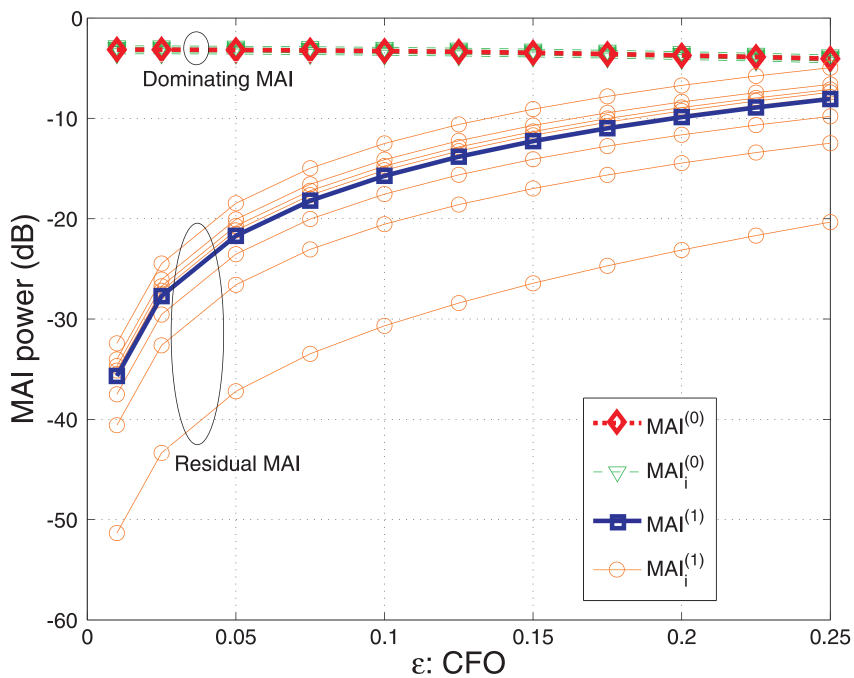
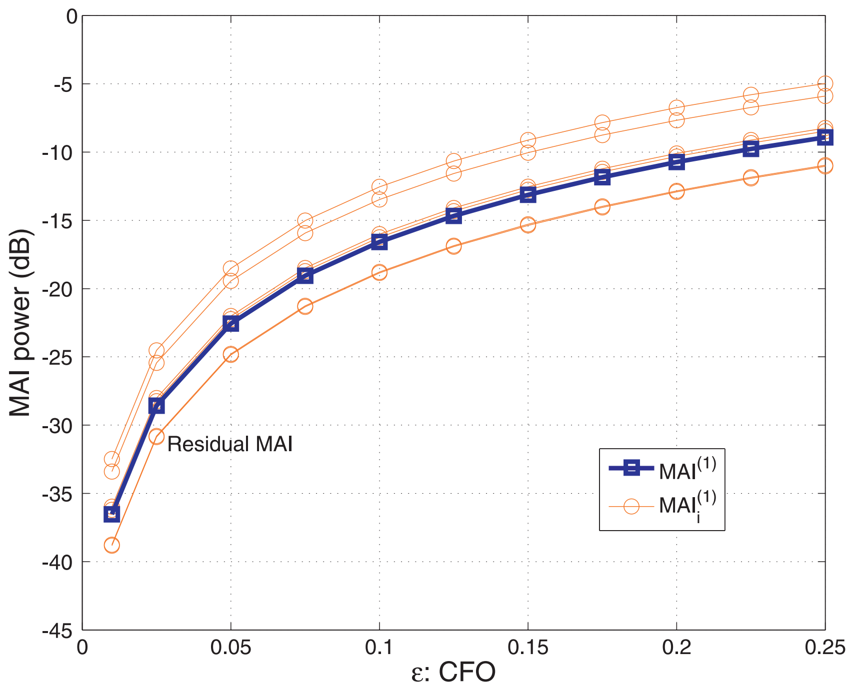
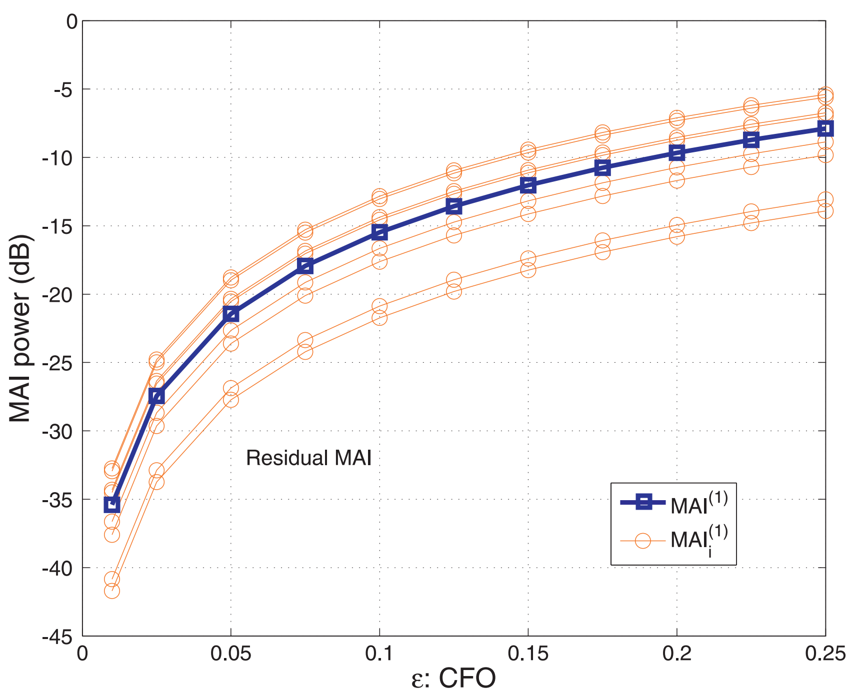


Fig. 7. The dominating and the residual MAI as a function of CFO in a half-loaded situation with Shi and Latva-aho's scheme.



(a)



(b)

Fig. 8. MAI reduction via the proposed code schemes using codewords in (a) G_0 and (b) G_1.

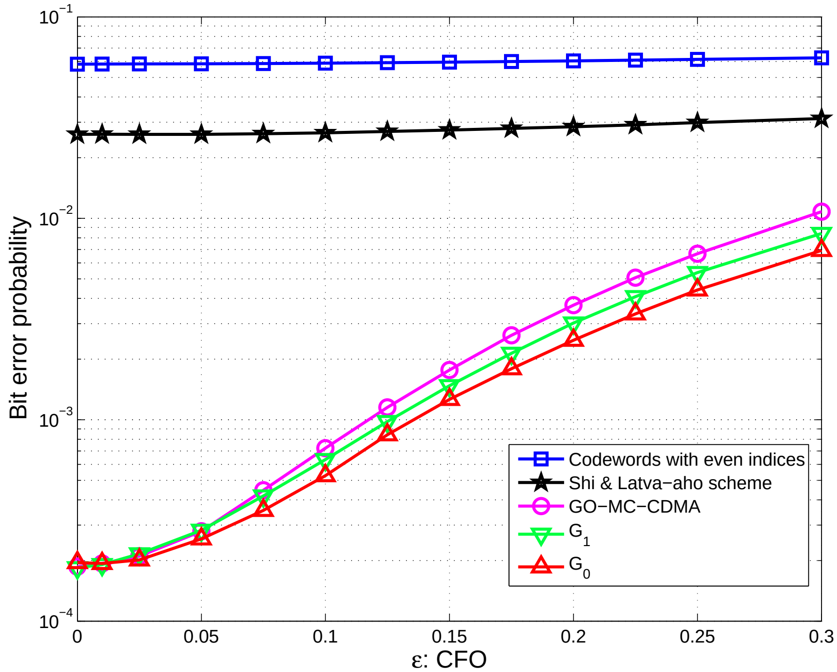


Fig. 9. The bit error rate as a function of CFO (with fixed $E_b/N_0 = 15$ dB).

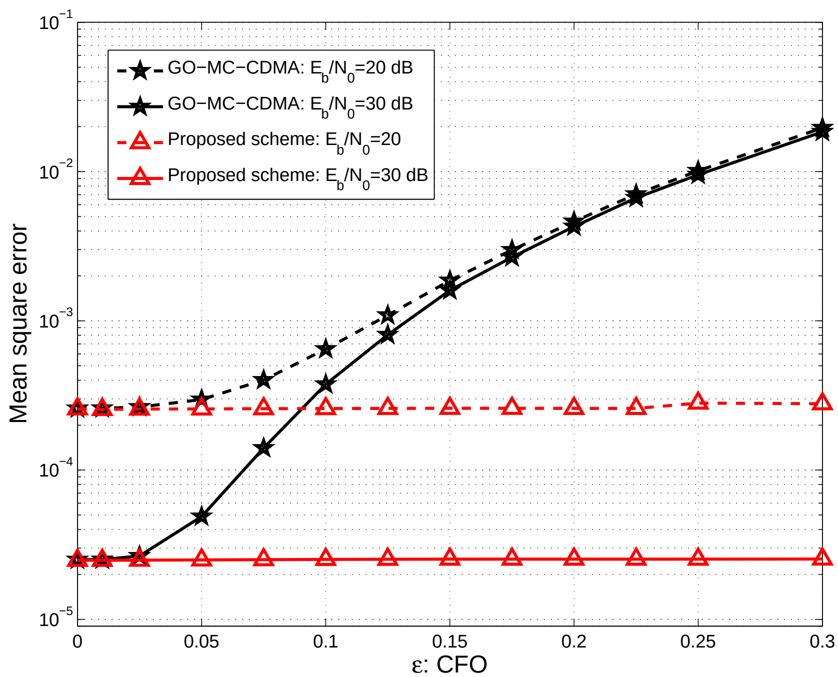


Fig. 10. The mean squared error of CFO estimation as a function of CFO for the proposed and the GO-MC-CDMA schemes.

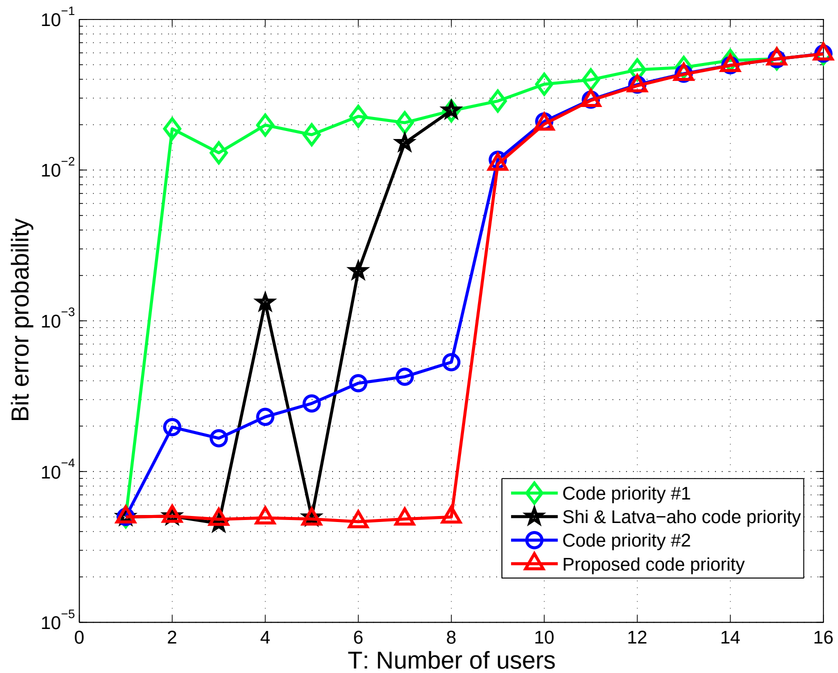


Fig. 11. The bit error rate as a function of the number of users with SNR=18 dB.