

1. Einleitung

2. Darstellung von ultrakurzen Lichtimpulsen

2.1 Prinzip der Modenkopplung

2.2 Komplexe Darstellung ultrakurzer Lichtimpulse

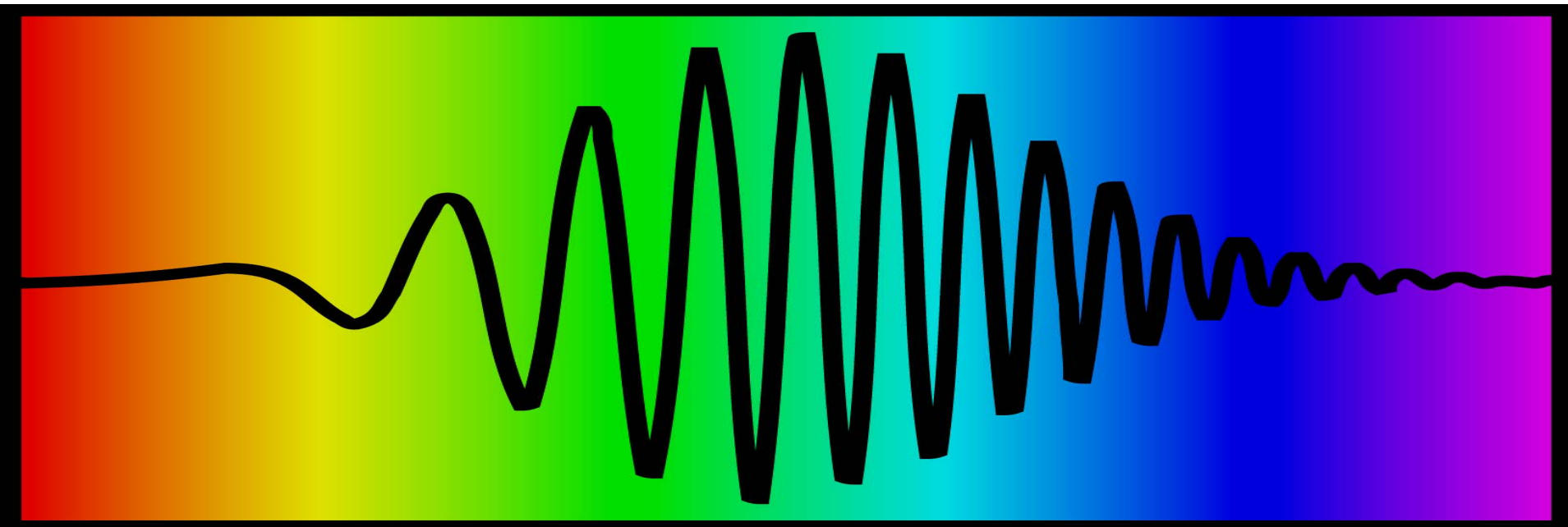
2.2.1 Fourier Transformation

2.2.2 Zeitliche und spektrale Phase

2.2.3 Taylor Entwicklung der Phase

2.2.4 Nützliche Umrechnungen

2.2.5 Ergänzungen zur Pulsbreite



An ultrashort laser pulse has an intensity and phase vs. time.

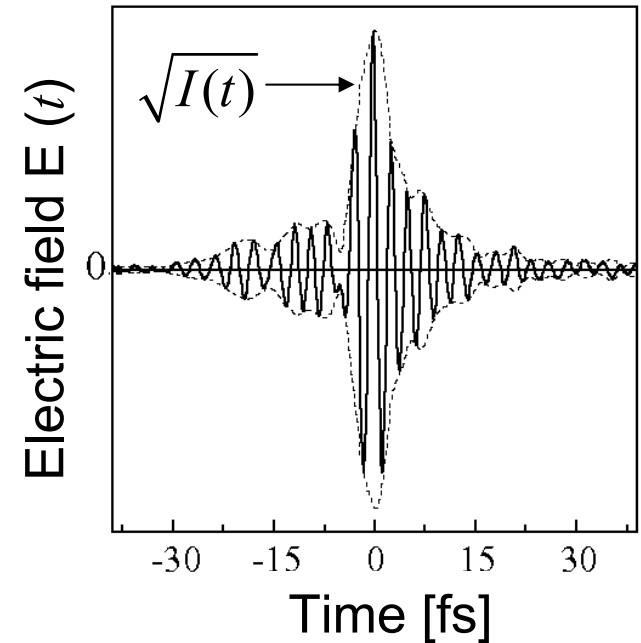
Neglecting the spatial dependence for now, the pulse electric field is given by:

$$E(t) = \frac{1}{2} \sqrt{I(t)} \exp\{i[\omega_0 t - \phi(t)]\} + c.c.$$

Intensity

Carrier
frequency

Phase



A sharply peaked function for the intensity yields an ultrashort pulse. The phase tells us the color evolution of the pulse in time.

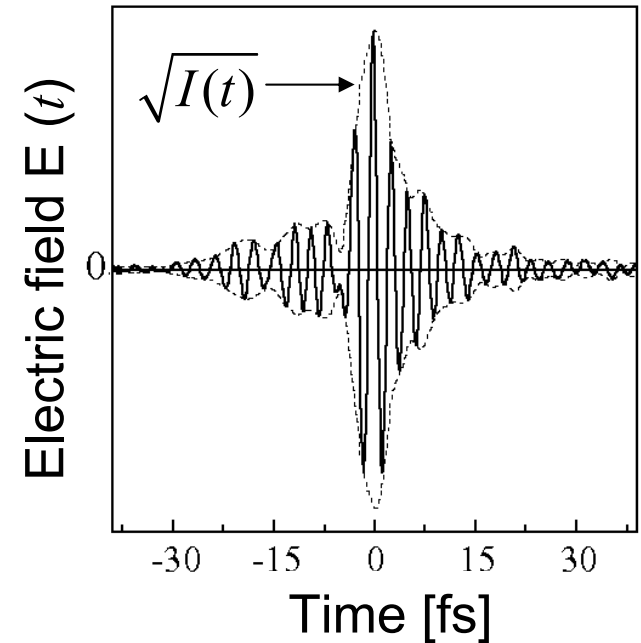
The real and complex pulse amplitudes

Removing the $1/2$, the c.c., and the exponential factor with the carrier frequency yields the *complex amplitude*, $E(t)$, of the pulse:

$$E(t) = \sqrt{I(t)} \exp\{-i\phi(t)\}$$

This removes the rapidly varying part of the pulse electric field and yields a complex quantity, which is actually easier to calculate with.

$\sqrt{I(t)}$ is often called the *real amplitude*, $A(t)$, of the pulse.



Calculating the intensity and the phase

It's easy to go back and forth between the electric field and the intensity and phase.

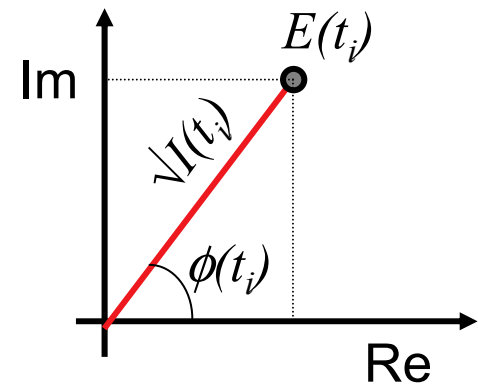
The intensity: $I(t) = |E(t)|^2$

The phase:

$$\phi(t) = \arctan \left\{ \frac{\text{Im}[E(t)]}{\text{Re}[E(t)]} \right\}$$

Equivalently,

$$\phi(t) = \text{Im} \{ \ln[E(t)] \}$$



The Fourier Transform

To think about ultrashort laser pulses, the Fourier Transform is essential.

$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t) \exp(-i\omega t) dt$$

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) \exp(i\omega t) d\omega$$

We always perform Fourier transforms on the real or complex pulse electric field, and not the intensity, unless otherwise specified.

The frequency-domain electric field

The frequency-domain equivalents of the intensity and phase are the spectrum and spectral phase.

Fourier-transforming the pulse electric field:

$$\mathbf{E}(t) = \frac{1}{2} \sqrt{I(t)} \exp\{i[\omega_0 t - \phi(t)]\} + c.c.$$

yields:

Note that ϕ and φ are different!

$$\begin{aligned} \tilde{\mathbf{E}}(\omega) = & \frac{1}{2} \sqrt{S(\omega - \omega_0)} \exp\{-i[\varphi(\omega - \omega_0)]\} + \\ & \frac{1}{2} \sqrt{S(\omega + \omega_0)} \exp\{-i[\varphi(\omega + \omega_0)]\} \end{aligned}$$

The frequency-domain electric field has positive- and negative-frequency components.

The complex frequency-domain pulse field

Since the negative-frequency component contains the same information as the positive-frequency component, we usually neglect it.

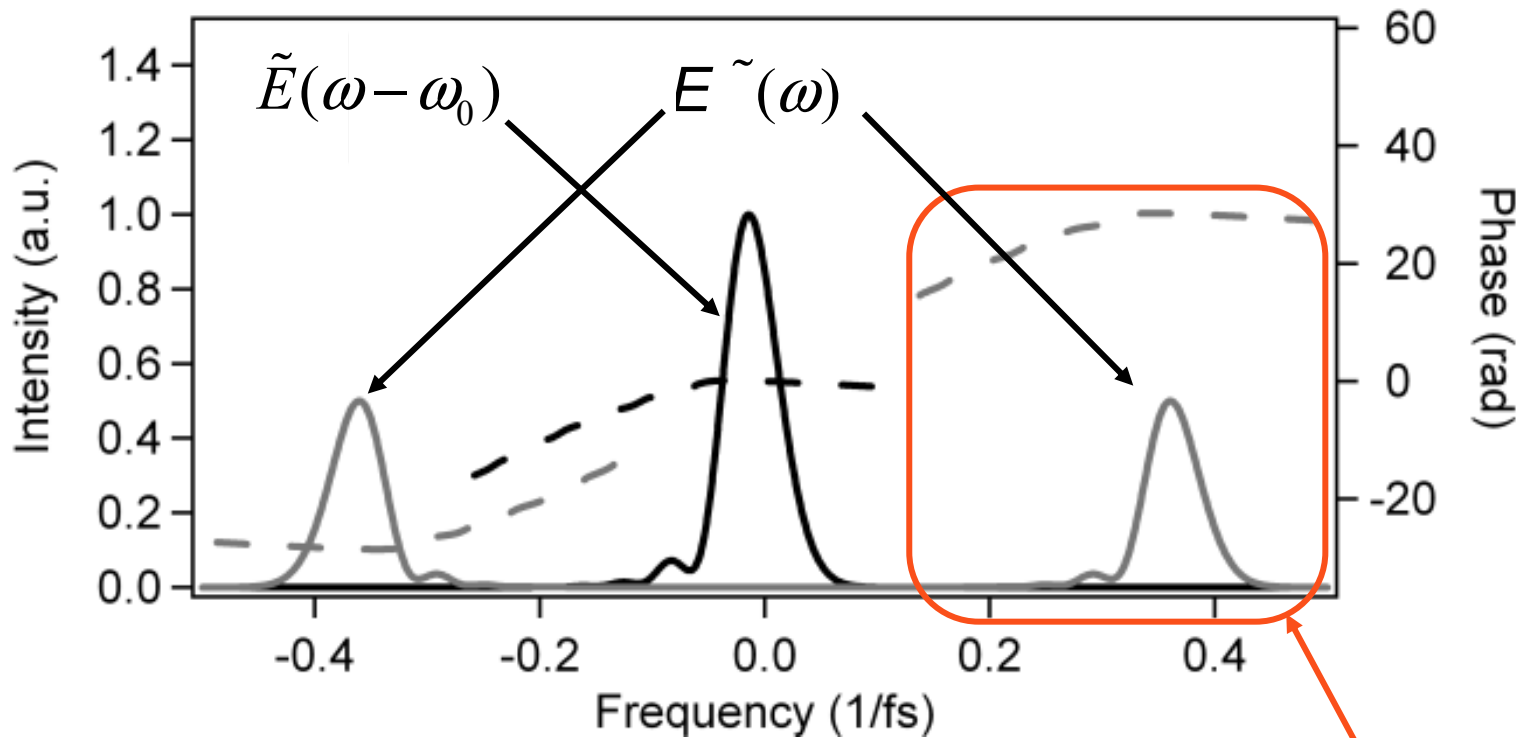
We also center the pulse on its actual frequency, not zero. Thus, the most commonly used complex frequency-domain pulse field is:

$$\tilde{E}(\omega) \equiv \sqrt{S(\omega)} \exp\{-i\varphi(\omega)\}$$

Thus, the frequency-domain electric field also has an intensity and phase. S is the spectrum, and φ is the spectral phase.

The spectrum with and without the carrier frequency

Fourier transforming $E(t)$ and $E(t)$ yield different functions.



We usually use just this component.

The spectrum and spectral phase

The spectrum and spectral phase are obtained from the frequency-domain field the same way as the intensity and phase are from the time-domain electric field.

$$S(\omega) = |\tilde{\mathbf{E}}(\omega)|^2$$

$$\varphi(\omega) = -\arctan \left\{ \frac{\text{Im}[\tilde{\mathbf{E}}(\omega)]}{\text{Re}[\tilde{\mathbf{E}}(\omega)]} \right\}$$

or

$$\varphi(\omega) = -\text{Im}\{\ln[\tilde{\mathbf{E}}(\omega)]\}$$

The Gaussian pulse

For almost all calculations, a good first approximation for any ultrashort pulse is the *Gaussian pulse* (with zero phase).

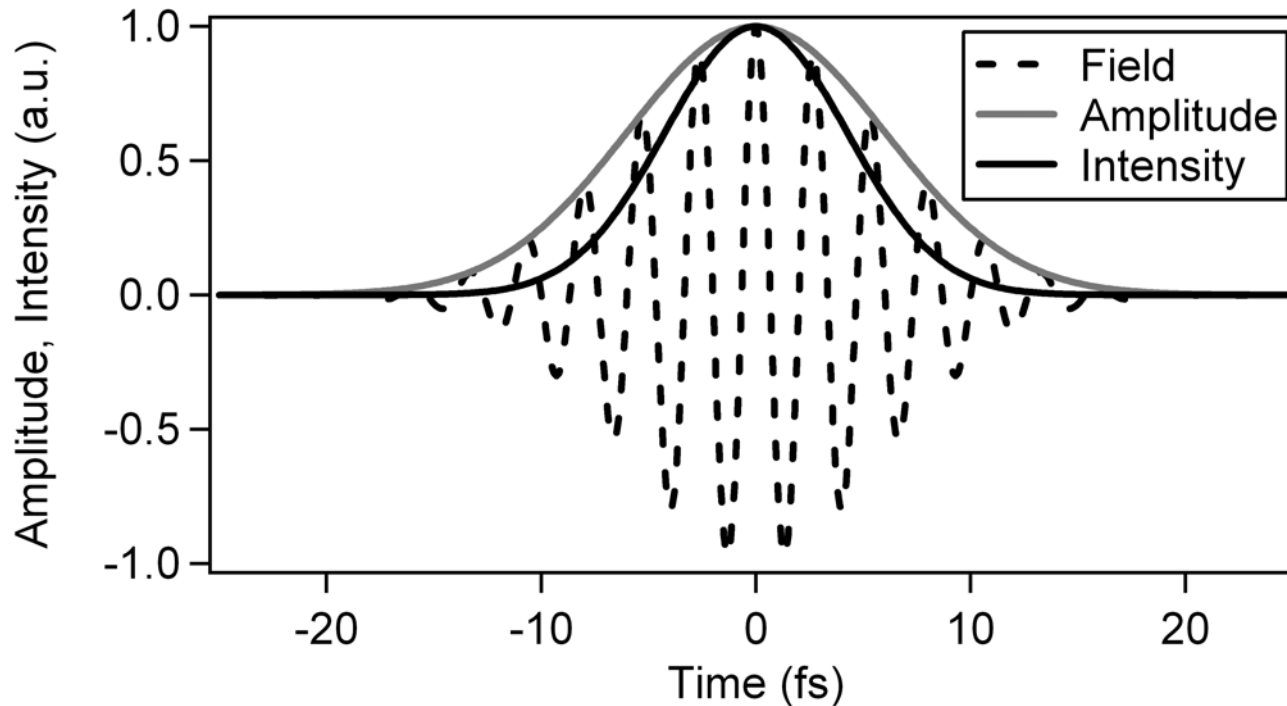
$$\begin{aligned} E(t) &= E_0 \exp\left[-(t / \tau_{HW1/e})^2\right] \\ &= E_0 \exp\left[-2 \ln 2 (t / \tau_{FWHM})^2\right] \\ &= E_0 \exp\left[-1.38 (t / \tau_{FWHM})^2\right] \end{aligned}$$

where τ_{FWHM} is the field half-width-half-maximum, and τ_{FWHM} is intensity the full-width-half-maximum.

The intensity is:

$$\begin{aligned} I(t) &= |E_0|^2 \exp\left[-4 \ln 2 (t / \tau_{FWHM})^2\right] \\ &= |E_0|^2 \exp\left[-2.76 (t / \tau_{FWHM})^2\right] \end{aligned}$$

Intensity vs. amplitude



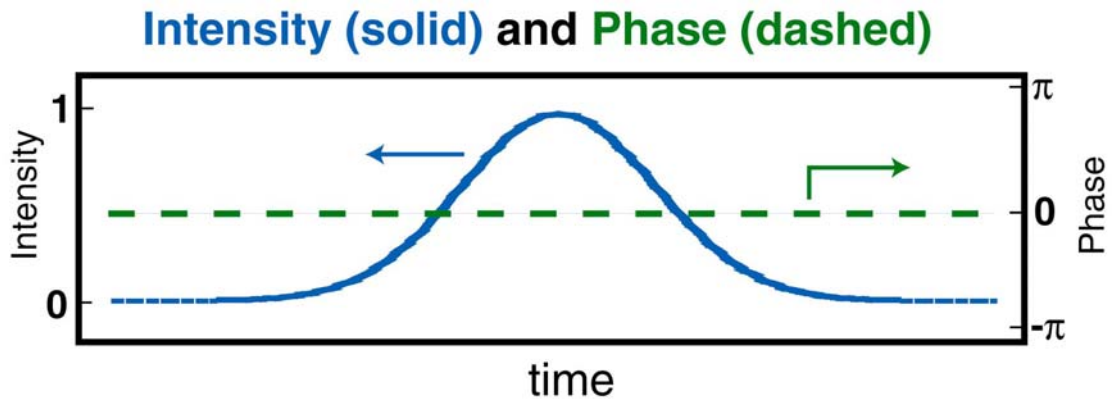
The phase of this pulse is constant, $\phi(t) = 0$, and is not plotted.

The intensity of a *Gaussian* pulse is $\sqrt{2}$ shorter than its real amplitude. This factor varies from pulse shape to pulse shape.

Intensity and phase of a Gaussian

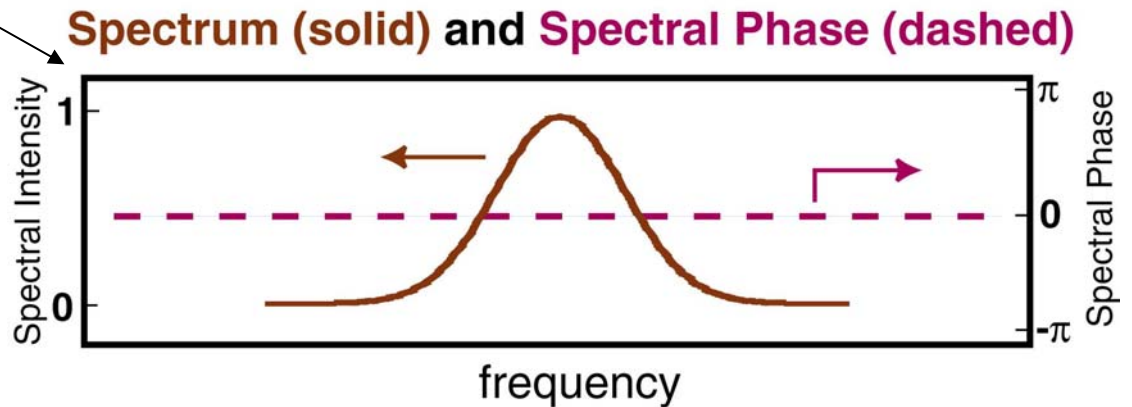
The Gaussian is real, so its phase is zero.

Time domain:



A Gaussian
transforms
to a Gaussian

Frequency domain:



So the spectral phase
is zero, too.

The instantaneous frequency

The temporal phase, $\phi(t)$, contains frequency-vs.-time information.

The pulse *instantaneous angular frequency*, $\omega_{inst}(t)$, is defined as:

$$\omega_{inst}(t) \equiv \omega_0 - \frac{d\phi}{dt}$$

This is easy to see. At some time, t , consider the total phase of the wave. Call this quantity ϕ_0 :

$$\phi_0 = \omega_0 t - \phi(t)$$

Exactly one period, T , later, the total phase will (by definition) increase to $\phi_0 + 2\pi$:

$$\phi_0 + 2\pi = \omega_0 [t + T] - \phi(t + T)$$

where $\phi(t+T)$ is the slowly varying phase at the time, $t+T$.

Subtracting these two equations:

$$2\pi = \omega_0 T - [\phi(t + T) - \phi(t)]$$

Instantaneous frequency (cont'd)

Dividing by T and recognizing that $2\pi/T$ is a frequency, call it $\omega_{inst}(t)$:

$$\omega_{inst}(t) = 2\pi/T = \omega_0 - [\phi(t+T) - \phi(t)] / T$$

But T is small, so $[\phi(t+T) - \phi(t)] / T$ is the derivative, $d\phi/dt$.

So we're done!

Usually, however, we'll think in terms of the *instantaneous frequency*, $\nu_{inst}(t)$, so we'll need to divide by 2π :

$$\nu_{inst}(t) = \nu_0 - [d\phi/dt] / 2\pi$$

While the instantaneous frequency isn't always a rigorous quantity, it's fine for ultrashort pulses, which have broad bandwidths.

Group delay

While the temporal phase contains frequency-vs.-time information, the spectral phase contains time-vs.-frequency information.

So we can define the *group delay vs. frequency*, $\tau_{gr}(\omega)$, given by:

$$\tau_{gr}(\omega) = d\phi / d\omega$$

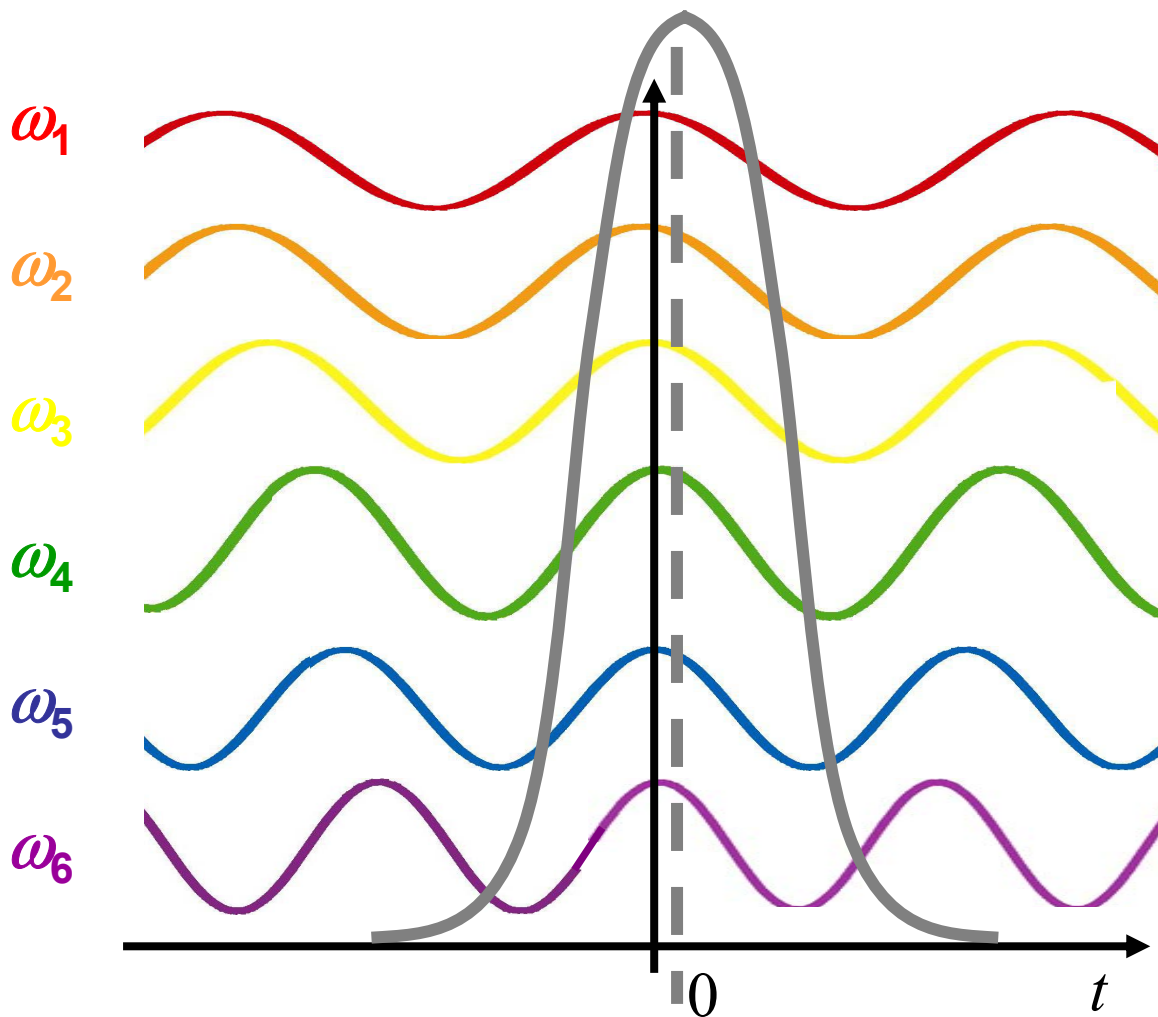
A similar derivation to that for the instantaneous frequency can show that this definition is reasonable.

Also, we'll typically use this result, which is a real time (the rad's cancel out), and never $d\phi/d\nu$, which isn't.

Always remember that $\tau_{gr}(\omega)$ is *not* the inverse of $\omega_{inst}(t)$.

What is the spectral phase?

The spectral phase is the phase of each frequency in the wave-form.



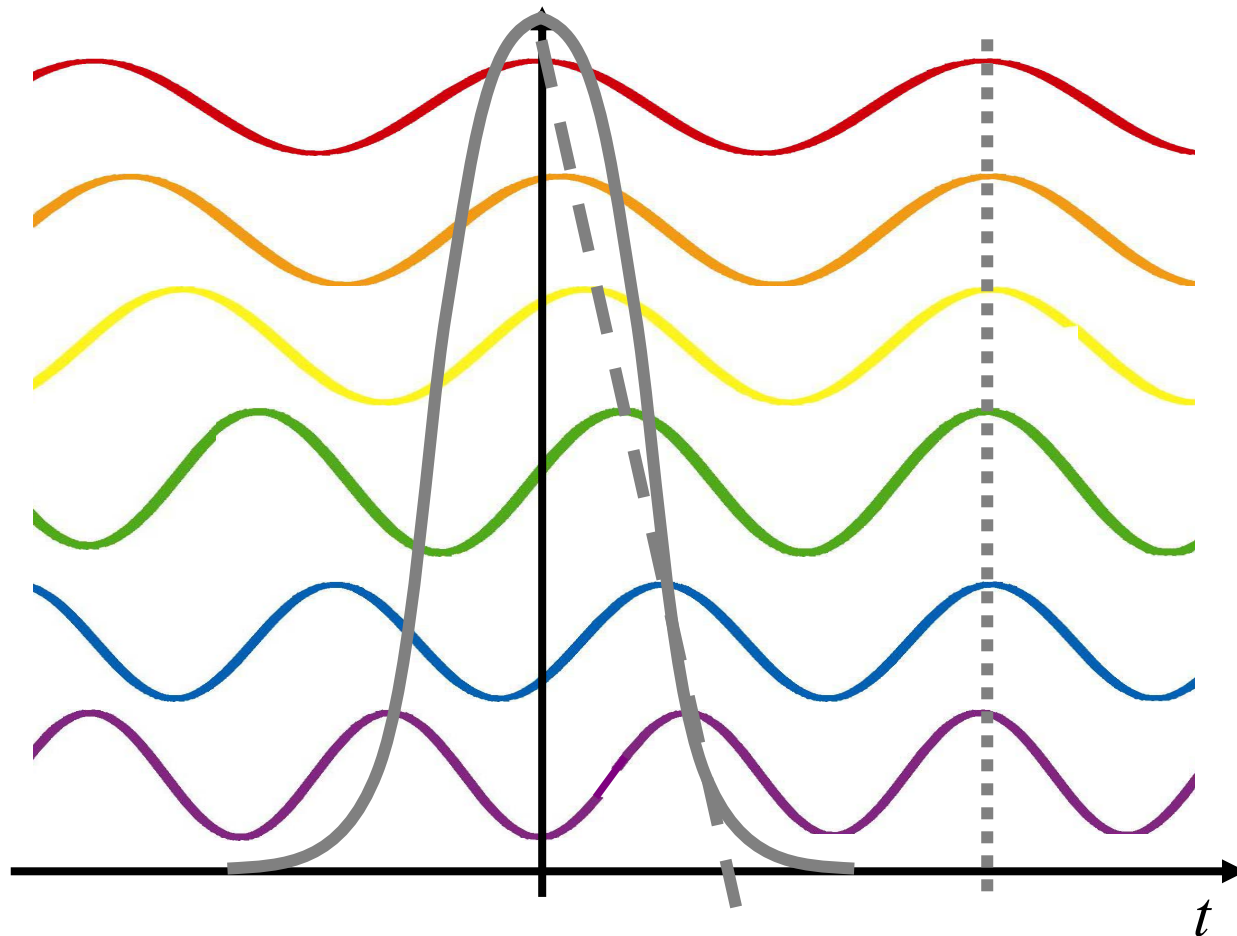
All of these frequencies have zero phase. So this pulse has:

$$\phi(\omega) = 0$$

Note that this wave-form sees constructive interference, and hence peaks, at $t = 0$.

And it has cancellation everywhere else.

Now try a linear spectral phase: $\varphi(\omega) = \alpha\omega$.



$$\varphi(\omega_1) = 0$$

$$\varphi(\omega_2) = 0.2 \pi$$

$$\varphi(\omega_3) = 0.4 \pi$$

$$\varphi(\omega_4) = 0.6 \pi$$

$$\varphi(\omega_5) = 0.8 \pi$$

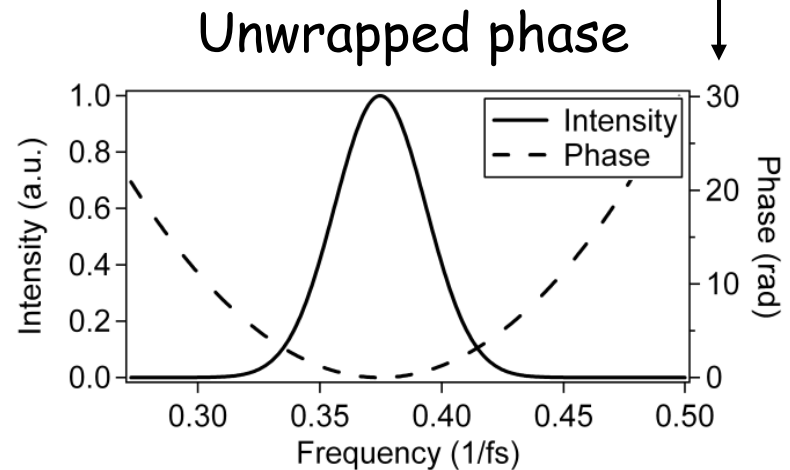
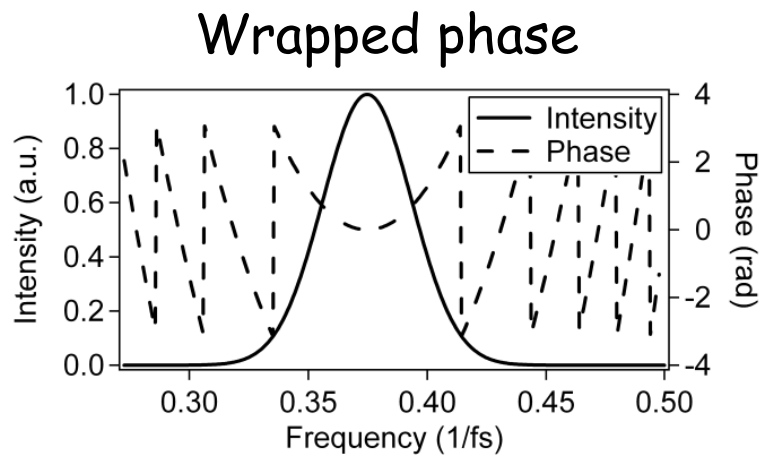
$$\varphi(\omega_6) = \pi$$

Phase wrapping and unwrapping

Technically, the phase ranges from $-\pi$ to π . But it often helps to “unwrap” it. This involves adding or subtracting 2π whenever there’s a 2π phase jump.

Example: a pulse with quadratic phase

Note the scale!

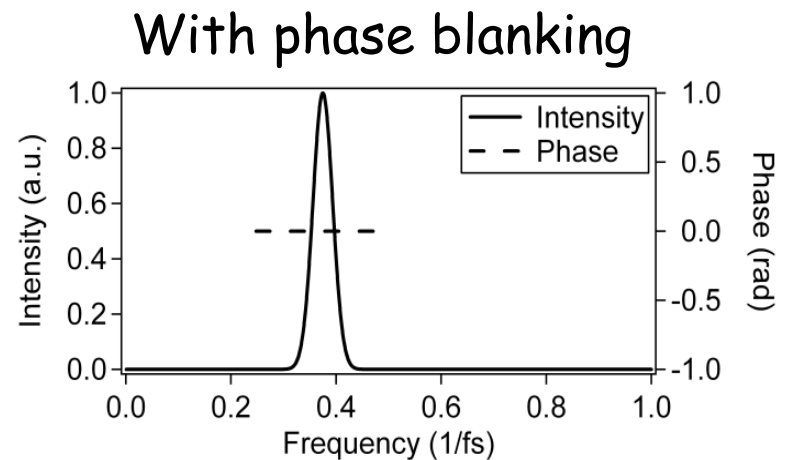
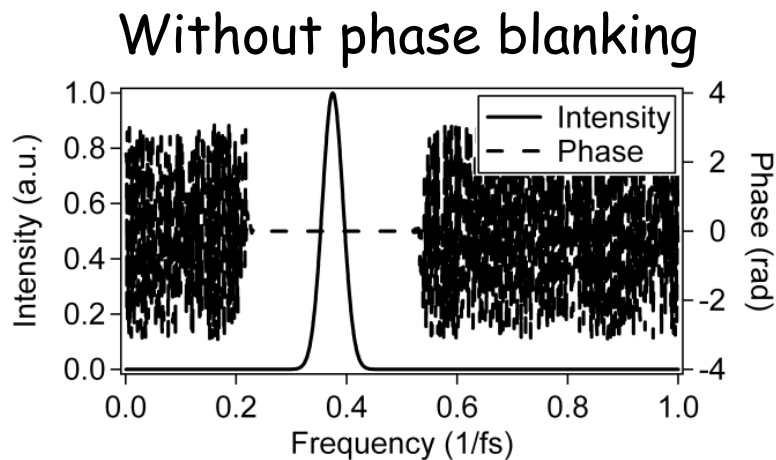


The main reason for unwrapping the phase is aesthetics.

Phase-blanking

When the intensity is zero, the phase is meaningless.
When the intensity is nearly zero, the phase is nearly meaningless.

Phase-blanking involves simply not plotting the phase when the intensity is close to zero.



The only problem with phase-blanking is that you have to decide the intensity level below which the phase is meaningless.

Phase Taylor Series expansions

We can write a Taylor series for the phase, $\phi(t)$, about the time $t = 0$:

$$\phi(t) = \phi_0 + \phi_1 \frac{t}{1!} + \phi_2 \frac{t^2}{2!} + \dots$$

where

$$\phi_1 = \left. \frac{d\phi}{dt} \right|_{t=0} \quad \text{is related to the instantaneous frequency.}$$

where only the first few terms are typically required to describe well-behaved pulses. Of course, we'll consider badly behaved pulses, which have higher-order terms in $\phi(t)$.

Expanding the phase in time is not common because it's hard to measure the intensity vs. time, so we'd have to expand it, too.

Frequency-domain phase expansion

It's more common to write a Taylor series for $\varphi(\omega)$:

$$\varphi(\omega) = \varphi_0 + \varphi_1 \frac{\omega - \omega_0}{1!} + \varphi_2 \frac{(\omega - \omega_0)^2}{2!} + \dots$$

where

$$\varphi_1 = \left. \frac{d\varphi}{d\omega} \right|_{\omega=\omega_0} \quad \text{is the group delay!}$$

$$\varphi_2 = \left. \frac{d^2\varphi}{d\omega^2} \right|_{\omega=\omega_0} \quad \text{is called the "group-delay dispersion."}$$

As in the time domain, only the first few terms are typically required to describe well-behaved pulses. Of course, we'll consider badly behaved pulses, which have higher-order terms in $\varphi(\omega)$.

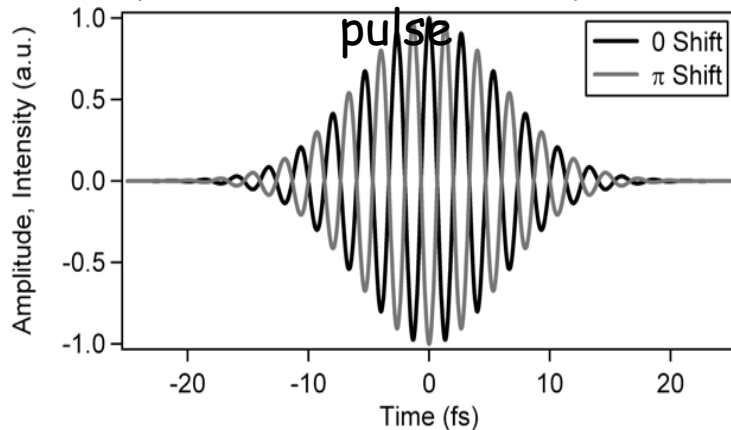
1. Zeroth-order phase: the absolute phase

The absolute phase is the same in both the time and frequency domains.

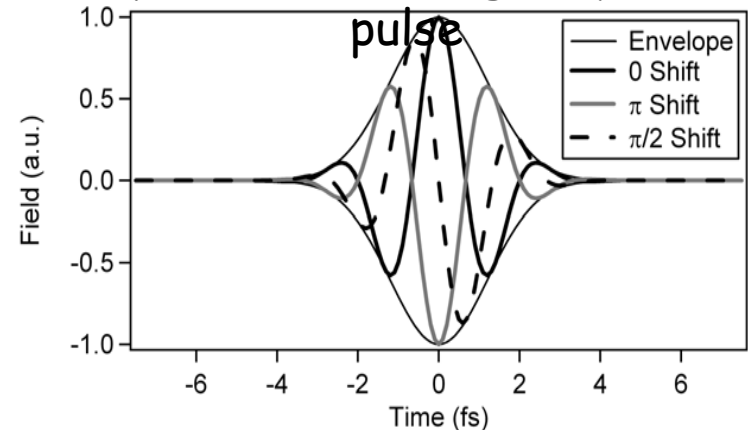
$$f(t)\exp(i\phi_0) \supset F(\omega)\exp(i\phi_0)$$

An absolute phase of $\pi/2$ will turn a cosine carrier wave into a sine. It's usually irrelevant, unless the pulse is only a cycle or so long.

Different absolute phases for a four-cycle pulse



Different absolute phases for a single-cycle pulse

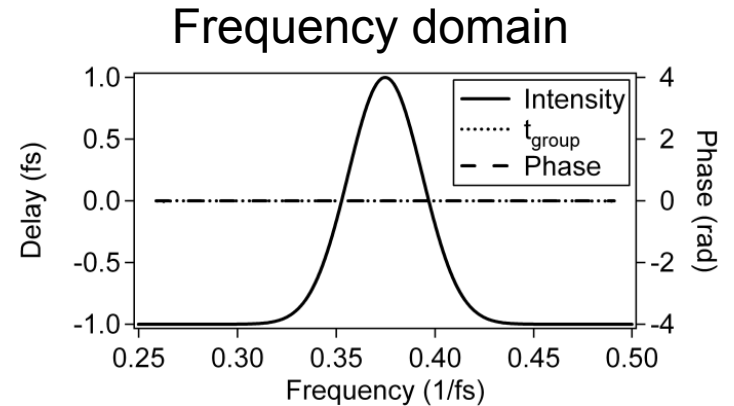
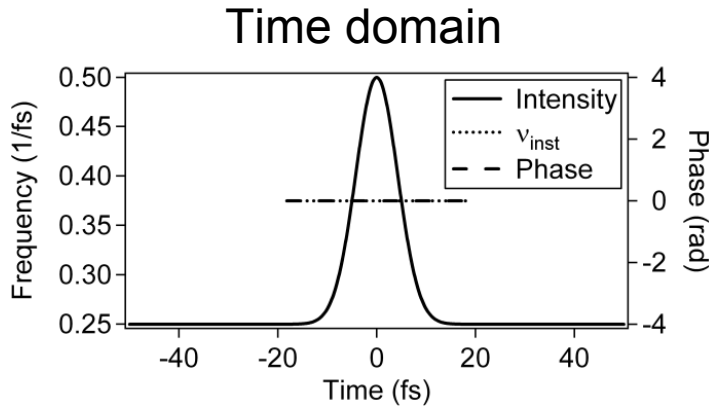


Notice that the two four-cycle pulses look alike, but the three single-cycle pulses are all quite different.

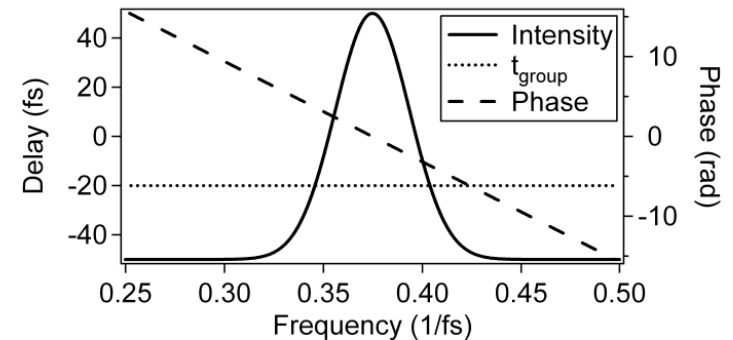
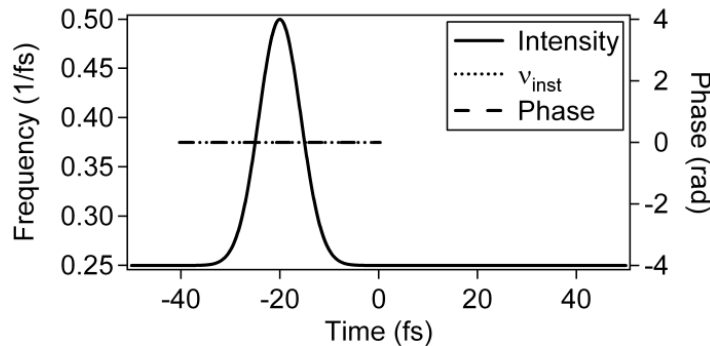
2. First-order phase in frequency: a shift in time

By the Fourier-Transform Shift Theorem, $f(t - \varphi_1) \supset F(\omega) \exp(i\omega\varphi_1)$

$$\varphi_1 = 0$$



$$\varphi_1 = -20 \text{ fs}$$



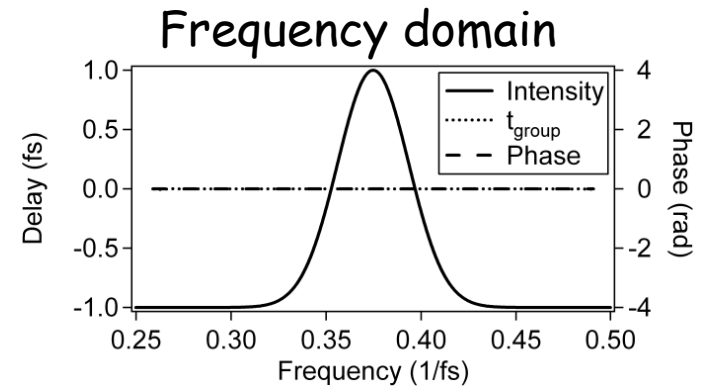
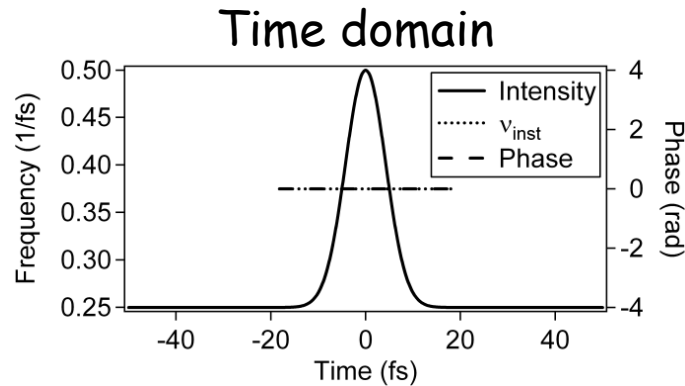
Note that φ_1 does not affect the instantaneous frequency, but the group delay = φ_1 .

First-order phase in time: a frequency shift

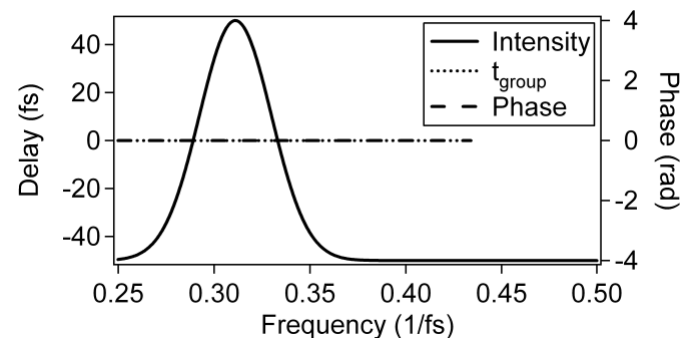
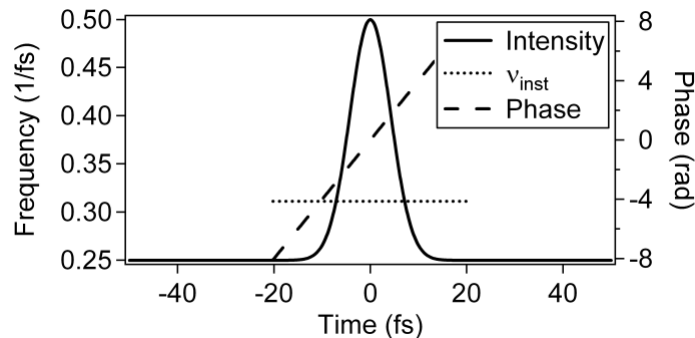
By the Inverse-Fourier-Transform Shift Theorem,

$$F(\omega - \phi_1) \subset f(t) \exp(-i \phi_1 t)$$

$$\phi_1 = 0 / \text{fs}$$



$$\phi_1 = -.07 / \text{fs}$$



Note that ϕ_1 does not affect the group delay, but it does affect the instantaneous frequency = $-\phi_1$.

Linear spectral phase: $\varphi(\omega) = \alpha\omega$.

„Konstruktive Interferenz zu einem späteren Zeitpunkt!“

