

Performance analysis of spectral-phase-encoded optical code-division multiple-access system regarding the incorrectly decoded signal as a nonstationary random process

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Abstract. The performance of a spectral-phase-encoded (SPE) optical code-division multiple-access (OCDMA) system is analyzed. Regarding the incorrectly decoded signal (IDS) as a nonstationary random process, we derive a novel probability distribution for it. The probability distribution of the IDS is considered a chi-squared distribution with degrees of freedom $r=1$, which is more reasonable and accurate than in previous work. The bit error rate (BER) of an SPE OCDMA system under multiple-access interference is evaluated. Numerical results show that the system can sustain very low BER even when there are multiple simultaneous users, and as the code length becomes longer or the initial pulse becomes shorter, the system performs better. © 2005 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.2128127]

Subject terms: spectral-phase encoding (SPE); OCDMA; nonstationary random process.

Paper 040786R received Oct. 20, 2004; revised manuscript received Mar. 31, 2005; accepted for publication Apr. 27, 2005; published online Nov. 11, 2005.

1 Introduction

The optical code-division multiple-access (OCDMA) communication system has recently attracted much attention. The essence of OCDMA is that multiple users can share the whole bandwidth simultaneously. In such a system, each user is assigned a unique code. The transmitter imprints the code onto the bit sequence to be sent. When the datum is 1, an encoded signal burst is sent; when the datum is 0, the transmitter keeps silent. At the receiver, a matched filter is used to distinguish the desired transmitter from the others, and the data are recovered. Since the OCDMA system has many advantages over traditional WDM or OTDM systems, such as higher spectrum utilization efficiency, simpler network control, and improved security, it is considered a promising technology for optical LANs.

There have been many different schemes to build OCDMA networks.¹⁻⁴ Direct-sequence spread spectrum (DSSS) implements encoding in the time domain, spectral amplitude or phase encoding (SAE or SPE) implements encoding in the frequency domain, and fast frequency hopping (FFH) encodes the pulse in both time and frequency domain. Figure 1 illustrates a typical setup for a spectral-phase encoder using a liquid-crystal spatial light modulator (SLM).⁵ First, the input pulse is reflected by a blazed grating. Different frequency components are separated on the focal plane of the lens. Then the SLM, acting as a phase mask, introduces a phase shift into each of them according to the code. Finally, the second grating recombines different frequency components to form the output beam.

2 Characteristics of Incorrectly Decoded Signal

In an SPE OCDMA system, an ultrashort optical pulse is used as the light source. At the transmitter, the spectrum of the initial pulse is sliced into many narrow bands, and each band is subjected to an additional phase shift (0 or π) by the encoder. The encoded signal is a long-duration, low-level, noiselike burst. The decoder is the same as the encoder except for the code it uses. When its code is conjugate to the encoder's, the additional phase shift introduced by the encoder is removed and the pulse is recovered (correctly decoded signal). Otherwise, the decoded signal remains a noiselike signal (incorrectly decoded signal). The incorrectly decoded signal has been studied in Refs. 6-8, in which it was viewed for simplicity as a sample function of a stationary, ergodic random process, and the root-mean-square (rms) width was used in bit error rate (BER) evaluation. Actually, the incorrectly decoded signal has a deterministic envelope (sinc^2) and is nonstationary. In this section, we take this envelope into account and derive a more accurate probability distribution of the IDS.

Assume the initial optical field $E(t)$ and intensity $I(t)$ are

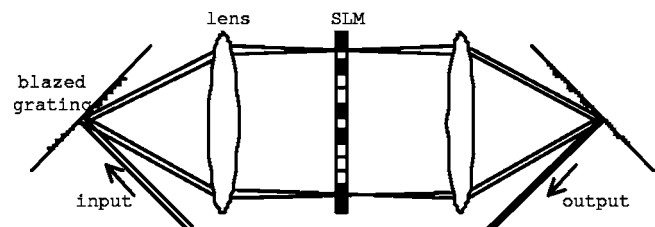


Fig. 1 Encoder using liquid-crystal spatial light modulator.

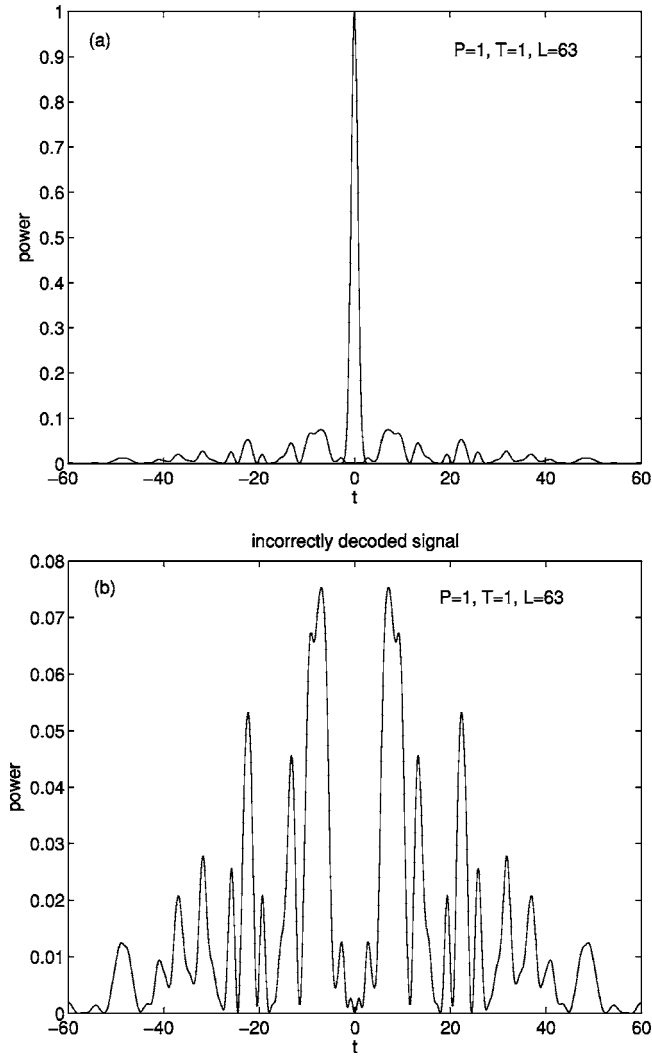


Fig. 2 (a) Initial ultrashort pulse and the encoded signal. (b) The encoded signal.

$$E(t) = \sqrt{P} \exp\left(-\frac{t^2}{2T^2}\right), \quad (1)$$

$$I(t) = P \exp\left(-\frac{t^2}{T^2}\right), \quad (2)$$

where P is the peak power and T is the pulse width. The pulse's total energy is $\sqrt{\pi} \cdot PT$. The spectrum of $E(t)$ is

$$E(\omega) = (2\pi P)^{1/2} T \exp\left(-\frac{\omega^2 T^2}{2}\right). \quad (3)$$

To fully utilize the pulse's energy, we encode the pulse in a wide spectrum range W so that the loss of energy can be neglected:

$$-W/2 < \omega < W/2 \quad (W = 2\pi/T).$$

The code C consists of L bits, and each bit is 1 or -1 with equal probability $1/2$. To ensure that the incorrectly decoded signal remains noise-like, we can assign each user a

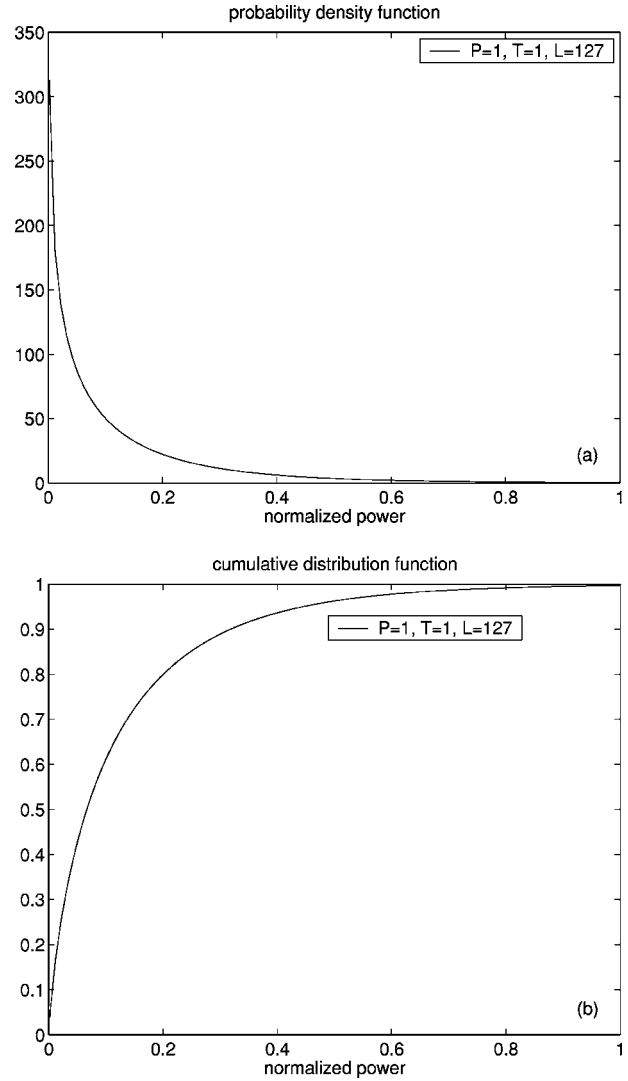


Fig. 3 Theoretical PDF and CDF of $I(t)$.

different phase shifted version of the same m -sequence. According to the m -sequence property, the incorrectly decoded signal has the same statistical characteristic as the encoded signal.

The encoder acts as a phase mask:

$$M(\omega) = \begin{cases} C_i & \text{for } -W/2 + i \delta\omega < \omega < -W/2 + (i+1)\delta\omega, \\ 0 & \text{elsewhere,} \end{cases}$$

where

$$0 \leq i \leq L-1, \quad \delta\omega = W/L, \quad C_i = \pm 1.$$

The impulse response of the encoder is

$$M(t) = \frac{\delta\omega}{2\pi} \text{Sa}\left(\frac{\delta\omega}{2}t\right) \sum_{i=0}^{L-1} C_i \exp[jz(i)t],$$

where

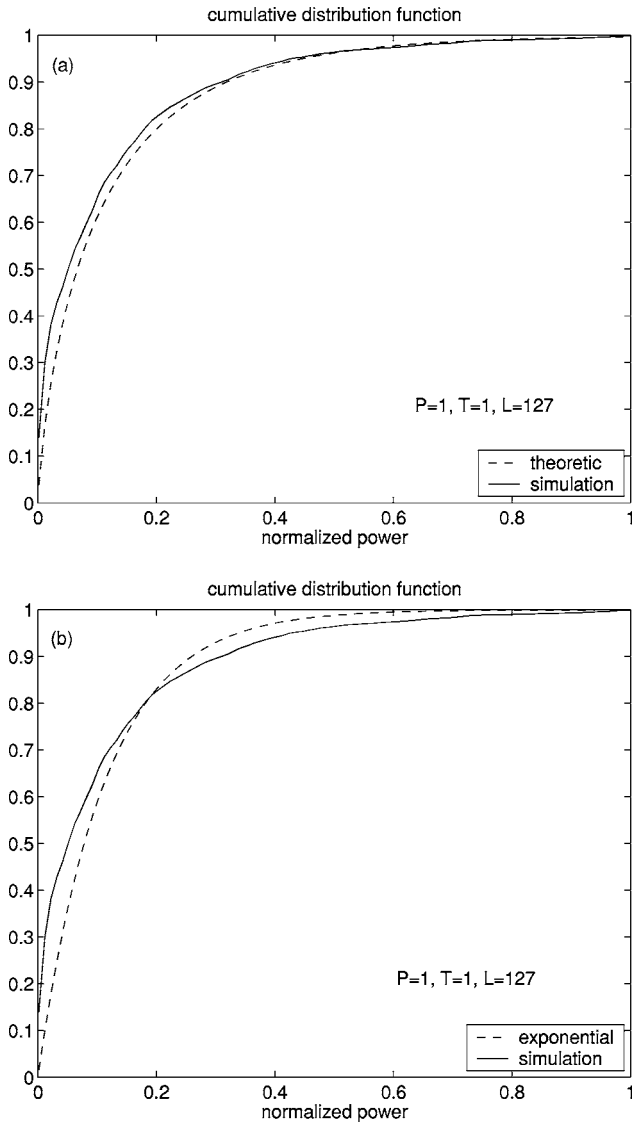


Fig. 4 (a) Theoretical CDF of $I(t)$ and the simulation curve. (b) CDF of $I(t)$ given in previous work and the simulation curve.

$$z(i) = -\frac{W}{2} + \left(i + \frac{1}{2}\right) \delta\omega.$$

So the encoded signal (shown in Fig. 2) is

$$\begin{aligned} E(t) &= F^{-1}[E(\omega) \cdot M(\omega)] \\ &= \int_{-\infty}^{+\infty} \sqrt{P} \exp\left[-\frac{(\tau-t)^2}{2T^2}\right] \cdot \frac{\delta\omega}{2\pi} \\ &\quad \times \text{Sa}\left(\frac{\delta\omega}{2} \tau\right) \cdot \sum_{i=0}^{L-1} C_i \exp[z(i)\tau] d\tau \\ &\approx \sqrt{\frac{P}{2\pi}} \cdot \delta\omega \cdot T \text{Sa}\left(\frac{\delta\omega}{2} t\right) \sum_{i=0}^{L-1} C_i \exp[jz(i)t]. \end{aligned} \quad (4)$$

Denote

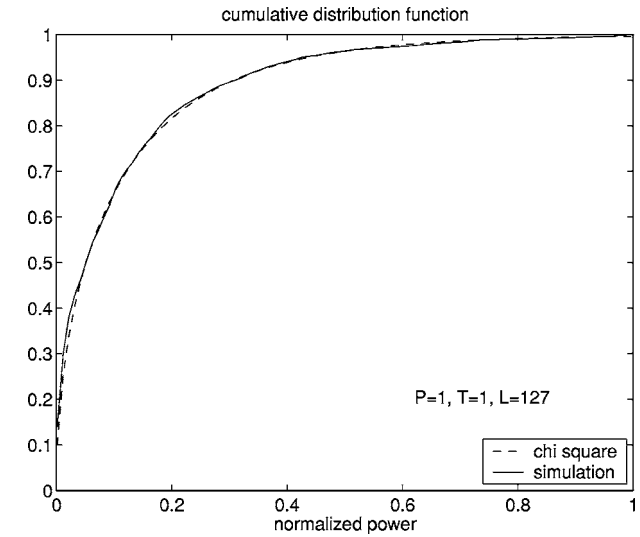


Fig. 5 The chi-squared distribution and the simulation curve.

$$U(t) = \sqrt{\frac{P}{2\pi}} \cdot \delta\omega \cdot T \sum_{i=0}^{L-1} C_i \exp[jz(i)t],$$

$$V(t) = |U(t)|^2, \quad S(t) = \text{Sa}^2\left(\frac{\delta\omega}{2} t\right). \quad (5)$$

We have

$$I(t) = S(t)V(t). \quad (6)$$

Here $U(t)$ is a complex signal whose real and imaginary parts are

$$U_R(t) = \sqrt{\frac{P}{2\pi}} \delta\omega T \sum_{i=0}^{L-1} C_i \cos[z(i) \cdot t],$$

$$U_I(t) = \sqrt{\frac{P}{2\pi}} \delta\omega T \sum_{i=0}^{L-1} C_i \sin[z(i) \cdot t].$$

The functions $U_R(t)$ and $U_I(t)$ can be considered sample functions of a stationary, ergodic random process, for which the ensemble and time averages are interchangeable.⁶ Assume $U_R(t)$ and $U_I(t)$ are independent random variables and each follows a Gaussian distribution; then V follows a negative-exponential distribution:

$$f_V(x) = \frac{1}{\sigma_V} \exp\left(-\frac{x}{\sigma_V}\right). \quad (7)$$

In Eq. (6), $S(t)$ represents the slowly varying envelope $\text{Sa}^2((\delta\omega/2)t)$. Most of the energy of the encoded signal (>90%) concentrates in the time range $[-T_0, T_0]$, where T_0 is the first zero point of $S(t)$:

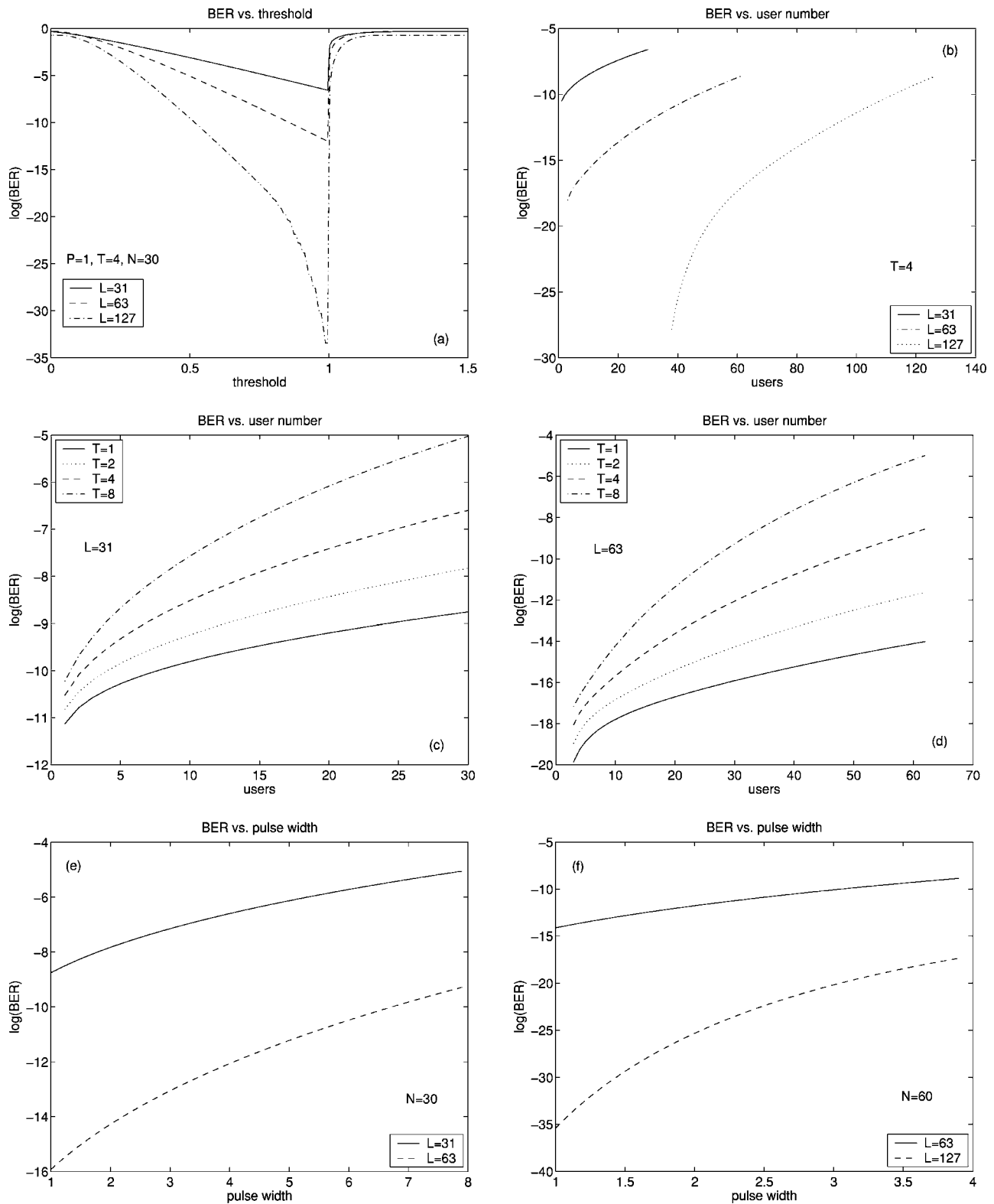


Fig. 6 (a) BER versus threshold with different code lengths. (b) BER versus user number with different code lengths. (c) BER versus user number ($L=31$). (d) BER versus user number ($L=63$). (e) BER versus initial pulse width ($N=30$). (f) BER versus initial pulse width ($N=60$).

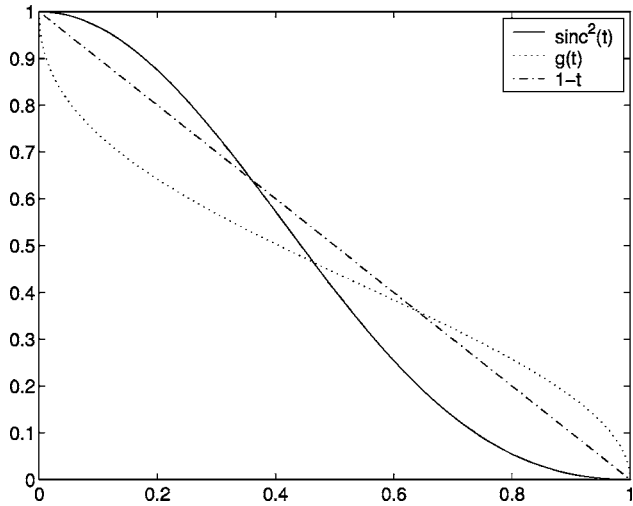


Fig. 7 $\text{sinc}^2(t)$ and its inverse function $g(t)$. The dot-dash line is a straight line $1-t$.

$$T_0 = \frac{2\pi}{\delta\omega} = LT.$$

Thus, the signal outside this range can be neglected. Assuming t is uniformly distributed in $[-T_0, T_0]$, after some calculation (see Appendix), we can derive the cumulative distribution function (CDF) of $I(t)$:

$$\begin{aligned} F_I(x) &\approx 1 - \exp\left(-\frac{x}{\sigma_V}\right) + \int_0^1 \frac{x/t}{\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt \\ &= 1 - \exp\left(-\frac{x}{\sigma_V}\right) + \int_1^\infty \frac{x/t}{\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt, \end{aligned} \quad (8)$$

and the probability density function (PDF) of $I(t)$:

$$f_I(x) \approx \int_0^1 \frac{1}{t\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt = \int_1^\infty \frac{1}{t\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt. \quad (9)$$

plots of $F_I(x)$ and $f_I(x)$ are shown in Fig. 3. The optical power is normalized, meaning that the maximum power of IDS is seen as 1.

To validate our derivation, we compare the theoretical CDF of $I(t)$ with the simulation curve in Fig. 4(a). Moreover, we show in Fig. 4(b) the result in previous work in which $I(t)$ was considered to follow a negative-exponential distribution. It is clear that our model gives a more accurate CDF of $I(t)$ than the previous work and matches the simulation better.⁷

As Fig. 4(a) shows, the theoretical curve departs from the simulation curve slightly, because we have replaced the probability distribution of S with a uniform distribution and used the approximation (15) in our derivation.

3 BER Evaluation

In Eqs. (8) and (9), the CDF and PDF of $I(t)$ are in complicated integral forms and have no clear, explicit expressions. Based on them, it is hard to derive BER directly. Note that, in Fig. 3(a), the PDF of $I(t)$ is infinite near zero,

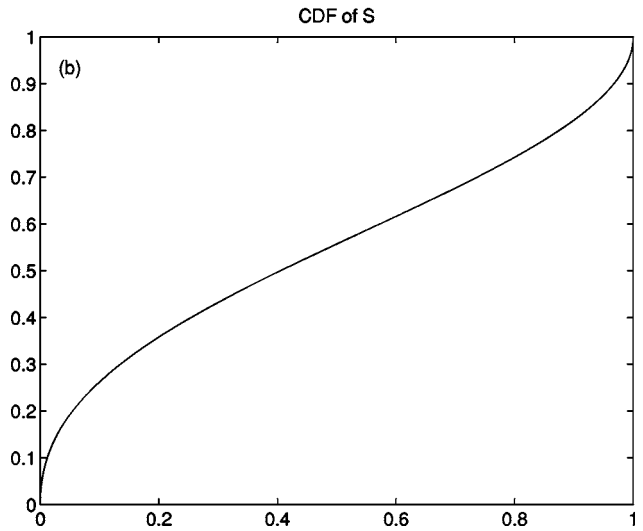
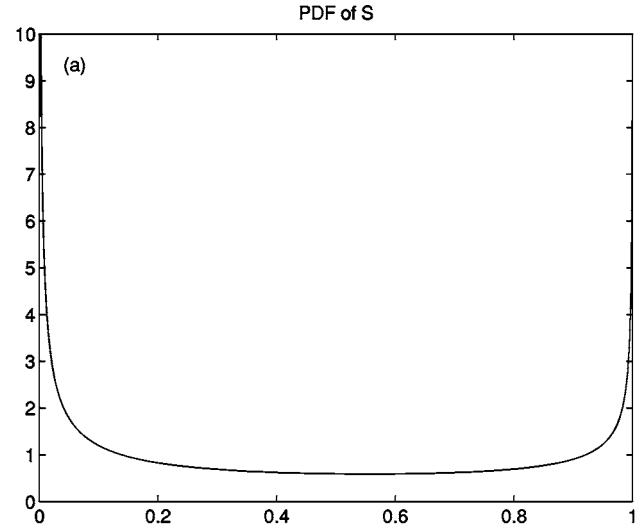


Fig. 8 PDF and CDF of $S(t)$.

and decreases quickly as the optical power grows. It is very close to a chi-squared distribution with degrees of freedom $r=1$. Assume the optical power $I(t)$ follows a chi-squared distribution with $r=1$:

$$f_I(x) = \frac{(x/\sigma_I)^{-1/2} \exp(-x/2\sigma_I)}{\Gamma(1/2) \cdot 2^{1/2} \sigma_I}, \quad (10)$$

where σ_I is the time average of $I(t)$ in $[-T_0, T_0]$. Denote

$$\alpha = \int_{-1}^1 \text{sinc}^2(x) dx \approx 0.903. \quad (11)$$

Then α is the ratio of the energy of the encoded signal in $[-T_0, T_0]$ to the total energy. Because the energy in $[-T_0, T_0]$ is $\alpha\sqrt{\pi PT}$, then

$$\sigma_I = \frac{\alpha\sqrt{\pi PT}}{2T_0}.$$

We compare the chi-squared distribution (10) with the simulation curve in Fig. 5. It is seen that the CDF of the chi-squared distribution matches the simulation result perfectly. The perfect match proves that our assumption is valid.

Based on Eq. (10), the BER of the SPE OCDMA system can be evaluated. For simplicity, we neglect the defects of the fiber channel, such as dispersion and nonlinearity. Like a wireless CDMA system, the OCDMA system is limited by multiple-access interference (MAI). In other words, for a given user, the main noise comes from the interfering users; other noises are less important. So we neglect other noises in the BER evaluation.

Assume that the duration of encoded signal is no longer than the bit period and that the duty cycle is

$$r = \frac{2T_0}{T_b}. \quad (12)$$

In an asynchronous system, each user transmits its own data stream randomly. Assume that synchronization is maintained between the receiver and the desired transmitter so that the decision time is always at the peak of the correctly decoded signal. Then, at the decision time, the probability that k users are transmitting a 1 is

$$P(n, k) = C_n^k p^k (1-p)^{n-k},$$

where $p = r/2 = T_0/T_b$ and n is the number of users. The received optical power is the summation of k independent random variables, and each of them follows a chi-squared distribution with $r=1$:

$$f_I(x) = \frac{(x/\sigma_I)^{-1/2} \exp(-x/2\sigma_I)}{\Gamma(1/2) \cdot 2^{1/2} \sigma_I}.$$

According to probability theory, the received power follows a chi-squared distribution with $r=k$:

$$f_k(x) = \frac{(x/\sigma_I)^{k/2-1} \exp(-x/2\sigma_I)}{\Gamma(k/2) \cdot 2^{k/2} \sigma_I}.$$

Denote by θ the decision threshold. The probability that an error occurs is

$$P_{\text{err}} = \frac{1}{2} P(r < \theta | b = 1) + \frac{1}{2} P(r > \theta | b = 0).$$

When 0 is transmitted,

$$P(r > \theta | b = 0) = \sum_{k=1}^n C_n^k p^k (1-p)^{n-k} \int_{\theta/2\sigma_I}^{\infty} \frac{x^{k/2-1} \exp(-x)}{\Gamma(k/2)} dx.$$

When 1 is transmitted, the optical power received is at least P , so

$$\text{for } \theta < P, \quad P(r < \theta | b = 1) = 0;$$

$$\text{for } \theta > P, \quad P(r < \theta | b = 1)$$

$$= \sum_{k=1}^n C_n^k p^k (1-p)^{n-k} \int_0^{(\theta-P)/2\sigma_I} \frac{x^{k/2-1} \exp(-x)}{\Gamma(k/2)} dx.$$

It is obvious that the optimum decision threshold is $\theta_0 = P$.

Using the incomplete gamma function

$$\Gamma(x, n) = \int_0^x \frac{t^{n-1} \exp(-t)}{\Gamma(n)} dt, \quad (13)$$

we get

$$P(r > \theta | b = 0) = \sum_{k=1}^n C_n^k p^k (1-p)^{n-k} \left[1 - \Gamma\left(\frac{\theta}{2\sigma_I}, \frac{k}{2}\right) \right],$$

$$P(r < \theta | b = 1) = \sum_{k=1}^n C_n^k p^k (1-p)^{n-k} \cdot \Gamma\left(\frac{\theta-P}{2\sigma_I}, \frac{k}{2}\right)$$

(for $\theta > P$).

The BER under different conditions is shown in Fig. 6. Because we have neglected the defects of the fiber channel and other noises, our analysis gives a lower limit on the BER the SPE OCDMA system could achieve. In a practical system, the dispersion of the fiber channel will degrade the performance, especially when the initial pulse is very short or the transmission distance is long. Dispersion compensation is needed in these scenarios. Using a dispersion-shifted fiber link instead of a single-mode fiber link will also enable longer transmission distance.⁸ The shot noise and thermal noise of the receiver may degrade system performance more or less, depending on the detector and environment.

Further experimental study of SPE OCDMA systems will be reported in our future work.

4 Conclusion

In an SPE-OCDMA system, the incorrectly decoded signal should be viewed as a nonstationary random process. It follows a chi-squared distribution with degrees of freedom $r=1$ rather than the negative-exponential distribution described in previous work. Under the condition that MAI is the dominant noise and the dispersion of the fiber channel is negligible, the SPE OCDMA system can sustain very low BER even there are multiple simultaneous users. Using a longer code or a shorter initial pulse can improve the system performance significantly.

5 Appendix

To find the probability distribution of $I(t)$, we should first find the probability distribution of $S(t)$. Assume t is uniformly distributed in $[-T_0, T_0]$; then S falls in $[0, 1]$ and follows the same distribution as $\text{sinc}^2(t)$ when t is uniformly distributed in $[0, 1]$. Let $g(t)$ be the inverse function of $\text{sinc}^2(t)$ in $[0, 1]$ (shown in Fig. 7), the cumulative distribution function (CDF) of S is

$$F_S(x) = 1 - g(x),$$

and the probability density function (PDF) of S is

$$f_S(x) = -g'(x).$$

$F_S(x)$ and $f_S(x)$ are shown in Fig. 8.

Since $I(t)$ is the product of two independent random variables, $S(t)$ and $V(t)$, according to probability theory, the CDF of $I(t)$ is

$$\begin{aligned} F_I(x) &= \int_0^1 -g'(t) \cdot \left[1 - \exp\left(-\frac{x/t}{\sigma_V}\right) \right] dt \\ &= 1 + \int_0^t g'(t) \exp\left(-\frac{x/t}{\sigma_V}\right) dt. \end{aligned} \quad (14)$$

Using

$$g(t) \approx 1 - t, \quad g'(t) \approx -1, \quad (15)$$

we get

$$\begin{aligned} F_I(x) &\approx 1 - \exp\left(-\frac{x}{\sigma_V}\right) + \int_0^1 \frac{x/t}{\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt \\ &= 1 - \exp\left(-\frac{x}{\sigma_V}\right) + \int_1^\infty \frac{x/t}{\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt, \end{aligned} \quad (16)$$

and the PDF of $I(t)$ is

$$f_I(x) \approx \int_0^1 \frac{1}{t\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt = \int_1^\infty \frac{1}{t\sigma_V} \exp\left(-\frac{x/t}{\sigma_V}\right) dt. \quad (17)$$

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