Analysis of Time-Spreading OCDMA Systems With Partially Coherent Sources

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***Abstract*—The effect of partial coherence of optical sources on the performance of time-spreading optical code-division multiple access (TS-OCDMA) systems is investigated. We develop a simple analytic model for the evaluation of bit-error-rate performance, and verify the validity of the model using Monte Carlo simulations. We present performance comparisons with fully coherent sources as well as statistically multiplexed sources, to indicate a desirable operation strategy of TS-OCDMA systems.**

***Index Terms*—Optical code-division multiple access (OCDMA), optical thresholding, partial coherence, spectral efficiency, statistical multiplexing, time-spreading optical code-division multiple access (TS-OCDMA).**

I. INTRODUCTION

IN AN optical code-division multiple access (OCDMA) system, the communication media (optical fiber) is shared among multiple users by using a code uniquely assigned to each user. Among various OCDMA schemes, time-spreading optical code-division multiple access (TS-OCDMA) refers to the scheme for which OCDMA coding is performed over the time domain. The performance of OCDMA systems is usually limited by the multiple access interference (MAI), and the performance of asynchronous TS-OCDMA systems limited by MAI was originally analyzed by Wang *et al.* [1]. The analysis involved integrate-and-dump receivers and was based on the assumption that signal–interference and interference–interference beatings are mutually independent.

It was suggested in [1] that the use of partially coherent sources improves performance of TS-OCDMA systems with integrate-and-dump receivers, since the integration process benefits from averaging the multiple phases resulting from partial coherence. However, OCDMA receivers using optical thresholders before optical-to-electrical conversion are known to perform more efficiently than simple integrate-and-dump receivers [2], [3], and there is a concern that the performance improvement described in [1] may not be achievable for systems with optical thresholders. Because the receivers with an optical thresholder perform "OR" operations with interference signals with fluctuating phase as input, the use of an incoherent source can actually increase the error probability when a "0" bit is sent. More recently, an analysis of asynchronous TS-OCDMA systems with fully coherent optical sources was given in [4], which took into account the correlation between the beating terms.

The purpose of this letter is three-fold: First, we extend the analytical bit-error-rate (BER) model developed in [4] to the case of partially coherent optical sources and demonstrate its accuracy. Second, by using the proposed model, we show that the utilization of incoherent sources can indeed improve system performance even when optical thresholders are used at the receivers. Finally, we compare the performance resulting from partial coherence sources and the statistical multiplexing described in [3] and [4] to show that their relative performance strongly depends on the number of interfering users.

II. THEORY

The system analyzed here is identical to that in [4], except that the optical source is not fully coherent. A transmitter has a short pulse laser with a pulsewidth of t_p and repetition rate of $1/T_B$, where T_B is the bit period of the system. The coherence time of the laser is $t_{\mathrm{coh}} = t_p/N_{\mathrm{coh}}$. It is assumed that the phase of the laser is constant within each coherence time period and at the end of a coherence time period, it makes a random jump to a new value. After binary amplitude modulation of the transmitted information, the pulses enter an OCDMA encoder. For a single input pulse, we have a stream of N_0 pulses (or chips) separated by a chip period of $T_C = t_p$ at the output of the encoder and N_0 pulses are binary phase modulated according to the OCDMA code assigned to each transmitter.

The signals from multiple transmitters are then combined by a coupler and distributed to multiple receivers. The received optical signal first goes into a correlator (OCDMA decoder), which is similar to the encoder. The phase and delay of each arm of the correlator are matched to those of the desired transmitter so that the correlation output from the desired signal has a strong autocorrelation (AC) peak.

The correlator output enters an ideal ultrafast optical thresholder, which produces a "HIGH" or "LOW" signal depending on the optical intensity of the correlator output. The output of the optical thresholder is then fed into an electronic peak detector which determines the received symbol by examining the presence of "HIGH" at the output of the optical thresholder over the gating period $T_G = \beta T_C$. The gating period represents the response time of electrical detectors [4]. For details of the system configuration, see [4].

In [4], it was shown that BER of TS-OCDMA systems with fully coherent sources can be represented by

$$\mathrm{BER} = \sum_{m=0}^{M} P(m)\,\mathrm{BER}(m) \qquad (1)$$

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where M is the total number of active interference users, $\mathrm{BER}(m)$ is the BER conditioned on m effective interfering correlation signals at sampling instants, and

$$P(m) = \binom{M}{m}\left(\frac{1}{2K}\right)^{m}\left(1-\frac{1}{2K}\right)^{M-m} \tag{2}$$

is the probability that m effective interference signals exist at the instants. Here $K \equiv T_B/(N_0 t_p)$ is the statistical multiplexing factor [3], [4]. The $\mathrm{BER}(m)$ can be expressed by

$$\mathrm{BER}(m) = \frac{1}{2}\left[1-\gamma(m)+\rho(m)\gamma^{\beta-1}(m)\right] \tag{3}$$

where

$$\rho(m) = \frac{1}{\frac{m}{N_0}}\int_0^{I_{\mathrm{th}}} \exp\left(-\frac{I+1}{\frac{m}{N_0}}\right) I_0\left(\frac{\sqrt{I}}{\frac{m}{2N_0}}\right) dI \tag{4}$$

is the probability of having the LOW output at the thresholder despite the presence of the AC peak from the desired signal, and

$$\gamma(m) = \frac{1}{\frac{m}{N_0}}\int_0^{I_{\mathrm{th}}} \exp\left(-\frac{I}{\frac{m}{N_0}}\right) dI \tag{5}$$

is the probability of having the LOW output when the AC peak is not present. Equation (3) can be recognized from the fact that an error occurs if a pulse is detected when bit "0" is sent, whereas an error occurs if a pulse is *not* detected when bit "1" is sent.

For the case of incoherent optical sources, (3) needs to be modified. If $t_{\mathrm{coh}} = T_C/N_{\mathrm{coh}}$, there are $N_{\mathrm{coh}}\beta$ coherence intervals within a gating period. When a "0" bit is sent, an error occurs if a pulse is detected in any of the $N_{\mathrm{coh}}\beta$ intervals. If we assume that the interference field in each coherence interval is independent from each other, then the probability of detecting a pulse can be calculated as

$$P_E(e|0,m) = 1-\gamma^{N_{\mathrm{coh}}\beta}(m). \tag{6}$$

Similarly, when a "1" bit is sent, an error occurs if no pulse is detected within the $N_{\mathrm{coh}}\beta$ coherence intervals. With the same approximation of statistical independence, the probability of not detecting a pulse can be calculated as

$$P_E(e|1,m) = \rho^{N_{\mathrm{coh}}\beta}(m)\gamma^{N_{\mathrm{coh}}(\beta-1)}(m). \tag{7}$$

Using (4) and (5), the modified $\mathrm{BER}(m)$ can be expressed as

$$\mathrm{BER}(m) = \frac{1}{2}\left[1-\gamma^{N_{\mathrm{coh}}\beta}(m)+\rho^{N_{\mathrm{coh}}}(m)\gamma^{N_{\mathrm{coh}}(\beta-1)}(m)\right]. \tag{8}$$

It should be noted that $\rho(m)$ and $\gamma(m)$ in (8) are identical to their fully coherent counterparts in (3) and can be calculated by (4) and (5), respectively.

III. PARTIAL COHERENCE AND PERFORMANCE OF TS-OCDMA

The validity of the assumption of statistical independence among the interference signals that belong to different coherence intervals but are all in a same chip period needs to be verified for the accuracy of the derived expressions. Since they are sums of the same interference signal components and only the phases between the components are different in each coherence interval, a certain degree of correlation may exist.

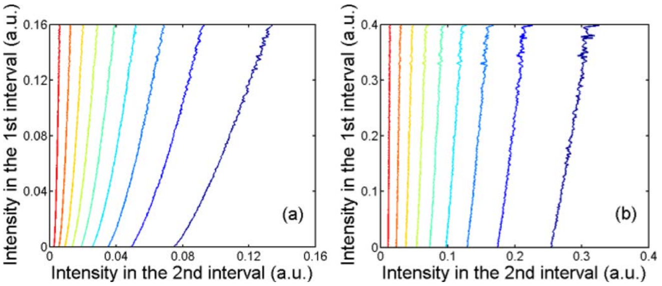

Fig. 1. The pdf of the intensity of interference in a coherence interval conditioned on the intensity of the signal in the previous interval. (a) $N_{\mathrm{user}} = 10$; (b) $N_{\mathrm{user}} = 30$.

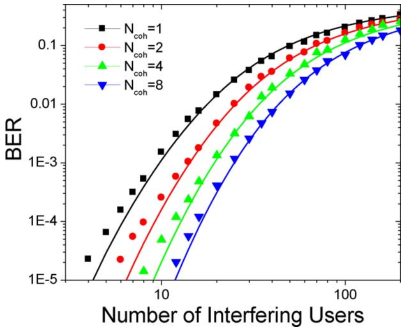

Fig. 2. Comparison of M-C simulations (symbols) with analytic model (lines) with various N_{coh}. For all cases, the pulsewidth was identical and $N_0 = 128$ and $\beta = 1$.

We tested the validity of the assumption using Monte Carlo (M-C) simulations. To assess the amount of correlation, we performed M-C simulations with $N_{\mathrm{coh}} = 2$ and $N_0 = 128$. In Fig. 1, x- and y-axes represent the intensity in the second and the first coherence intervals, respectively, and each curve represents a set of points with equal probability densities for the intensity in the second coherence interval conditioned on the intensity in the first interval, i.e., $P_I(I(t=t_{\mathrm{coh}})|I(t=0))$. Note that a set of vertical contour lines would mean statistical independence between the intensities in the first and the second coherence intervals. In Fig. 1(a), we can observe that when $N_{\mathrm{user}} = 10$, the conditional probability density function (pdf) exhibits some dependency on the intensity in the first interval. However, as N_{user} increases to 30 [Fig. 1(b)], the dependency is greatly reduced and, when $N_{\mathrm{user}} = 100$ (not shown), the pdf is almost independent of the intensity in the first interval. Therefore, the assumption of statistical independency can be justified for a sufficient number of users as demonstrated by M-C simulations below.

Next, M-C simulations were performed with an increasing number of coherence intervals per chip period (N_{coh}) while fixing the OCDMA code-length (N_0) at 128. The electrical bandwidth parameter β was set to 1 for all cases, which means that the electronic peak detector was ON for one OCDMA chip period ($T_C = t_{\mathrm{coh}}N_{\mathrm{coh}}$). Fig. 2 compares the results of M-C simulations with the analytic model. An excellent agreement between the simulations and the analytic model is observed.

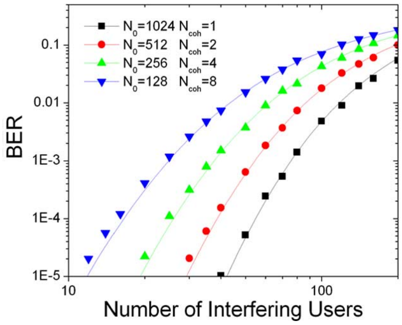

Fig. 3. Comparison of M-C simulation results (symbols) with analytic model (lines) with identical optical and electrical bandwidths but with different $N_{\rm coh}$. $t_{\rm coh} = T_B/(N_0 N_{\rm coh}) = T_B/1024$ and $T_G = 8t_{\rm coh}$.

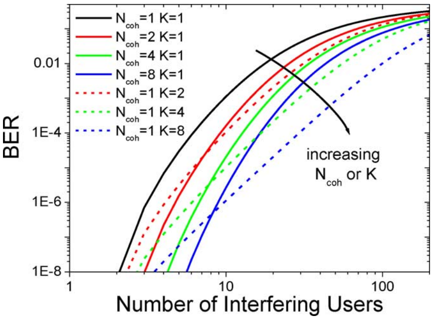

Fig. 4. Comparison of partial coherence and statistical multiplexing. $N_{\rm coh}$ or K is increased at the cost of optical bandwidth. $N_0 = 128, \beta = 1$ for all cases. (Solid lines: partial coherence; dotted lines: statistical multiplexing; black line: fully coherent source without statistical multiplexing.)

Furthermore, it is observed that the system performance improves as $N_{\rm coh}$ increases, even when optical thresholders are used in the receivers.

The above performance improvement was obtained at the cost of increased optical bandwidth because the optical bandwidth is inversely proportional to the coherence time of the carrier. In many systems, it may be desired that the use of partially coherent sources does not incur the bandwidth increase; thus we also compare systems with identical optical bandwidths. We repeated simulations while keeping $N_{\rm coh}N_0 = 1024$ so that the coherence interval $t_{\rm coh}$ is fixed at $T_B/1024$. The electronic peak detector was "ON" for the duration of $8t_{\rm coh}$ in all cases to keep the electronic bandwidth equal. Fig. 3 shows the results of this comparison, which again confirms a very close agreement between the simulations and the analytic model. Furthermore, it is indicated that the use of partially coherent sources in systems limited by optical bandwidth actually degrades the system performance. Therefore, when the optical bandwidth is at a premium, a desirable strategy would be to use fully coherent pulses.

The use of partially coherent sources can be considered as a trade-off between the performance and optical bandwidth. A similar trade-off exists with statistical multiplexing [3], [4]. Here, we compare the partial coherence with statistical multiplexing. Fig. 4 compares the performance of the systems when the code length and the electrical bandwidth are fixed. Note that the curves for the partially coherent sources are identical to those in Fig. 2. We observe that as $N_{\rm user}$ increases, the statistical multiplexing becomes more efficient than partial coherence in improving the system performance. However, for a fair comparison, it is noted that for a given optical bandwidth a partially coherent source generates longer pulses ($t_p = N_{\rm coh}t_{\rm coh}$) and the cost of a system using longer partially coherent pulses may be lower than that of a system using very short fully coherent pulses.

Actually, an advantage of using partial coherence is in the ease of implementation because the use of partial coherence simply lessens the requirement on the coherence time (i.e., optical bandwidth) of the laser.

In this work, we used random codes for both the coherent and partially coherent systems so that our results do not depend on special properties of a particular code set. However, since optimal codes for partially coherent systems may differ from those for fully coherent systems, a more thorough investigation of the impacts of different code sets seems to be required.

IV. CONCLUSION

In this work, we examined the effect of the partial coherence of optical sources on the performance of TS-OCDMA systems. An analytic expression for the BER performance was presented, which was verified using M-C simulation. Both the analytic and simulation results showed that, even when optical thresholders are used, partially coherent sources can improve the BER at the expense of spectral efficiency. When the spectral efficiency is fixed, the use of partial coherence actually degrades the performance. It was also found that the use of statistical multiplexing becomes more effective in improving the system performance than that of partially coherent sources as the number of interfering users increases.

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