Recitation problem Set 11

Put table on Blackboard:

$x(t)$	$X(\omega)$
$\delta(t)$	1
$u(t)$	$\pi\delta(\omega) + 1/j\omega$
$e^{-at}ut, a > 0$	$\frac{1}{a+j\omega}$
	$\frac{2\sin(\omega T_1)}{\omega}$
1	$2\pi\delta(\omega)$
$\cos(\omega_o t)$	$\pi\delta(\omega-\omega_o) + \pi\delta(\omega+\omega_o)$

Also put properties and definitions on board:

$$
\begin{aligned}
&X(\omega) = \int_{-\infty}^{\infty} x(t)e^{j\omega t}dt \\
&f[x(t-t_o)] = e^{j\omega t_o}X(\omega) \\
&f[x(at)] = \frac{1}{|a|}X(\frac{\omega}{a}) \\
&f[\dot{x}(t)] = j\omega X(\omega) \\
&f[\int_{-\infty}^{t} x(\sigma)d\sigma] = \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)
\end{aligned}
$$

1. Sketch the magnitude spectrum of

$$x(t) = u(-t) + e^{-t}u(t)$$

$$
\begin{aligned}
X(\omega) &= \frac{1}{j(-\omega)} + \pi\delta(\omega) + \frac{1}{1+j\omega} \\
&= \frac{-1}{j\omega} + \frac{1}{1+j\omega} + \pi\delta(\omega) \\
&= \frac{-1}{j\omega(1+j\omega)} + \pi\delta(\omega) \\
\Rightarrow \quad |X(\omega)| &= \frac{1}{|\omega|\sqrt{1+\omega^2}} + \pi\delta(\omega)
\end{aligned}
$$

(Treat impulse separately) **(Figure needed here...see handwritten notes.)**

2. Sketch the magnitude spectrum of **(Figure needed!)**
A: Same as 1m since $\hat{X}(\omega) = e^{-j\omega 2}X(\omega)$.
3. Compute the Fourier transform $X(\omega)$ for

$$x(t) = sgn(t) = \begin{cases} 1 & ,t > 0 \\ -1 & ,t < 0 \end{cases}$$

Note direct computation fails, must be more clever! So write

$$x(t) = u(t) - u(-t)$$

From tables and scaling property

$$\begin{aligned} f[u(t)] &= \frac{1}{j\omega} + \pi\delta(\omega) \\ f[u(-t)] &= \frac{-1}{j\omega} + \pi\delta(-\omega) = \frac{-1}{j\omega} + \pi\delta(\omega) \text{Treat impulse as even!} \end{aligned}$$

Then

$$\begin{aligned} X(\omega) &= \frac{1}{j\omega} + \pi\delta(\omega) - [\frac{-1}{j\omega} + \pi\delta(\omega)] \\ &= \frac{2}{j\omega} \end{aligned}$$

(Note there are no generalized function!)

4. Compute the Fourier transform of **(Figure needed here!)**
Again, direct computation fails, need to be clever.

$$\begin{aligned} x(t) &= u(t-1) + u(-t-1) \\ \Rightarrow \quad X(j\omega) &= e^{-j\omega} f[u(t)] + \int_{-\infty}^{\infty} u(-t-1)e^{-j\omega t}dt \quad \text{Let } \sigma = -t-1 \\ &= \ldots \\ &= e^{-j\omega}[\frac{1}{j\omega} + \pi\delta(\omega)] + e^{j\omega}\int_{-\infty}^{\infty} u(\sigma)e^{-j(-\omega)\sigma}d\sigma \\ &= e^{-j\omega}[\frac{1}{j\omega} + \pi\delta(\omega)] + e^{j\omega}[\frac{-1}{j\omega} + \pi\delta(\omega)], \quad (\delta(-\omega) = \delta(\omega)) \\ &= -2\frac{e^{j\omega} - e^{-j\omega}}{2j\omega} + (e^{-j\omega} + e^{j\omega})\pi\delta(\omega) \\ &= -2\frac{\sin(\omega)}{\omega} + 2\pi\delta(\omega) \end{aligned}$$

More clever! write

$$\begin{aligned} x(t) &= 1 - [u(t+1)) - u(t-1)] \\ X(j\omega) &= 2\pi\delta(\omega) - 2\frac{\sin(\omega)}{\omega} \end{aligned}$$

5. If

$$X(j\omega) = \frac{e^{-j\omega}}{2+j\omega}, \text{What is } x(t)?$$

$x(t)$ is $e^{-2t}u(t)$ delayed by 1 , i.e. $x(t) = e^{-2(t-1)u(t-1)}$