(1) For an LTI system with system function

$$H(\omega) = \frac{1}{2+j\omega}$$

and input signal

$$x(t) = 1 + \cos(2t)$$

compute the response $y(t)$. (using Fourier Transf.)

Table $\Rightarrow$ $X(\omega) = 2\pi\delta(\omega) + \pi\delta(\omega-2) + \pi\delta(\omega+2)$

$$Y(\omega) = H(\omega)X(\omega) = \frac{1}{2+j\omega} X(\omega)$$

use mult. prop.

$$= \pi\delta(\omega) + \frac{\pi}{2+j2}\delta(\omega-2) + \frac{\pi}{2-j2}\delta(\omega+2)$$

$$= \pi\delta(\omega) + \frac{\pi}{\sqrt{8}} e^{-j\pi/4}\delta(\omega-2) + \frac{\pi}{\sqrt{8}} e^{j\pi/4}\delta(\omega+2)$$

$\Rightarrow$ from Table

$$y(t) = \frac{1}{2} + \frac{\pi}{2\pi\sqrt{8}} e^{-j\pi/4} e^{j2t} + \frac{\pi}{2\pi\sqrt{8}} e^{j\frac{\pi}{4}} e^{-j2t}$$

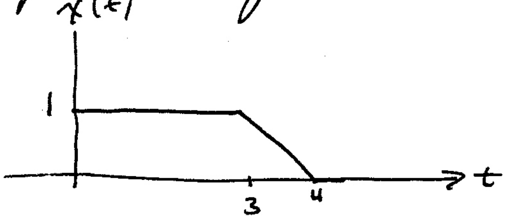
$$= \frac{1}{2} + \frac{1}{4\sqrt{2}}\left[e^{j(2t-\pi/4)} + e^{-j(2t-\pi/4)}\right]$$

$$= \frac{1}{2} + \frac{1}{2\sqrt{2}}\cos(2t-\pi/4)$$

(Different method of simplification gives $\frac{1}{2} + \frac{1}{4}\cos 2t - \frac{1}{4}\sin 2t$

520.214 Week 12 Recitation

① If $x(t)$ is given by



then compute the Laplace transform of

$$z(t) = \int_0^t x(\sigma)\, d\sigma \, u(t).$$

$$x(t) = u(t) - r(t-3) + r(t-4)$$

$$\Rightarrow \quad X(s) = \frac{1}{s} - \frac{e^{-3s}}{s^2} + \frac{e^{-4s}}{s^2}$$

$$= \frac{s - e^{-3s} + e^{-4s}}{s^2}$$

$$Z(s) = \frac{1}{s} X(s) = \frac{s - e^{-3s} + e^{-4s}}{s^3}$$

2) If $\mathcal{L}[x(t)u(t)] = \frac{s+1}{s^2+2s+3}$, what is $\mathcal{L}[\hat{x}(t)]$ when $\hat{x}(t) = x(3t-4)u(3t-4)$?

Note $\hat{x}(t)$ is one-sided. $3t-4>0 \Rightarrow 3t>4 \Rightarrow t>4/3$

$$\mathcal{L}[\hat{x}(t)] = \int_0^\infty \hat{x}(t)e^{-st}\,dt = \int_0^\infty x(3t-4)u(3t-4)e^{-st}\,dt$$

Let $\sigma = 3t-4$

$$= \int_{-4}^\infty x(\sigma)u(\sigma)e^{-s\left(\frac{\sigma+4}{3}\right)}\,\tfrac{1}{3}d\sigma$$

Note lower limit can be raised to zero.

$$= \tfrac{1}{3}e^{-s\frac{4}{3}}\int_0^\infty x(\sigma)u(\sigma)e^{-\frac{s}{3}\sigma}\,d\sigma$$

$$= \tfrac{1}{3}e^{-s\frac{4}{3}}\,\frac{s/3+1}{\left(\frac{s}{3}\right)^2+\frac{2s}{3}+3}$$

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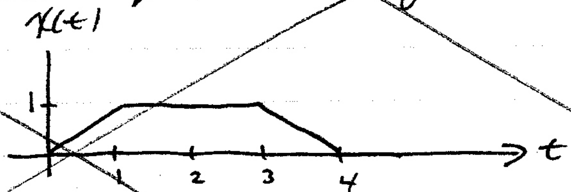
3) ~~Compute the Laplace transform of~~



$x(t) = r(t) - r(t-1) - r(t-3) + r(t-4)$

$X(s) = \frac{1}{s^2} - e^{-s}\frac{1}{s^2} - e^{-3s}\frac{1}{s^2} + e^{-4s}\frac{1}{s^2}$

③ Given

$$\dot{y}(t) + 2y(t) = x(t) + \dot{x}(t)$$

with $y(0^-) = 0$ and $x(t) = e^{-t}u(t)$,
compute $y(t)$

$$sY(s) + 2Y(s) = X(s) + sX(s)$$

(note $x(0^-)=0$)

$$\Rightarrow \quad Y(s) = \frac{s+1}{s+2}X(s) = \frac{s+1}{s+2}\cdot\frac{1}{s+1} = \frac{1}{s+2}$$

$$\Rightarrow \quad y(t) = e^{-2t}u(t)$$

④ If a right-sided signal $x(t)$ has the following Laplace transform $X(s)$, what is the Fourier transform, $X(\omega)$, of the signal?

(a) $X(s) = \dfrac{10}{s^2+2s-3}$ ($X(\omega)$ does not exist)

(b) $X(s) = \dfrac{e^{-s}}{s}$ $\left(X(\omega) = \dfrac{e^{-j\omega}}{j\omega} + e^{-j\omega}\pi\delta(\omega) = \dfrac{e^{-j\omega}}{j\omega} + \pi\delta(\omega)\right)$

(c) $X(s) = \dfrac{1}{s^2+3s+2}$ $\left(X(\omega) = \dfrac{1}{2-\omega^2+j3\omega}\right)$