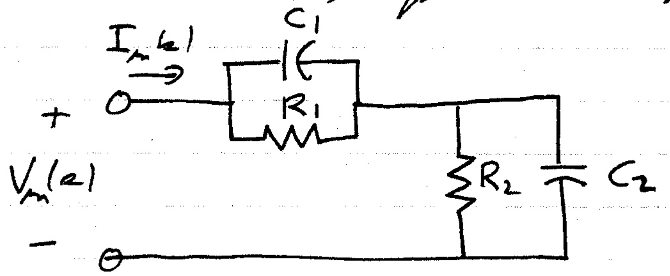
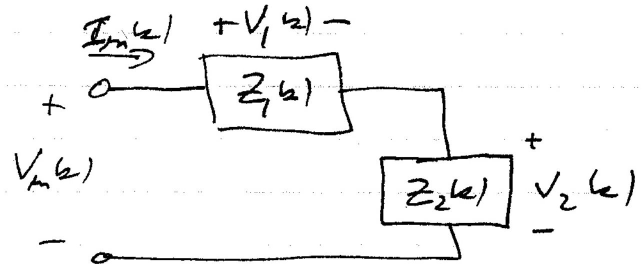
$Z_L = Ls$, $Z_C = \frac{1}{Cs}$

Recitation on Week 13

① Compute the input impedance $Z_{in}(s)$ of



Write as



$$V_{in}(s) = V_1(s) + V_2(s) = \big(Z_1(s) + Z_2(s)\big) I_{in}(s)$$

$\Rightarrow$

$$Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = Z_1(s) + Z_2(s)$$

Since parallel admittances add:

$$Z_1(s) = \frac{1}{Y_1(s)} = \frac{1}{C_1 s + \frac{1}{R_1}} = \frac{1/C_1}{s + 1/R_1C_1}$$

Similarly,

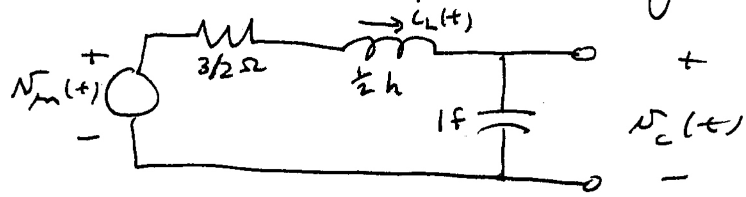
$$Z_2(s) = \frac{1/C_2}{s + 1/R_2C_2}$$

Thus

$$Z_{in}(s) = \frac{1/C_1}{s + 1/R_1C_1} + \frac{1/C_2}{s + 1/R_2C_2}$$

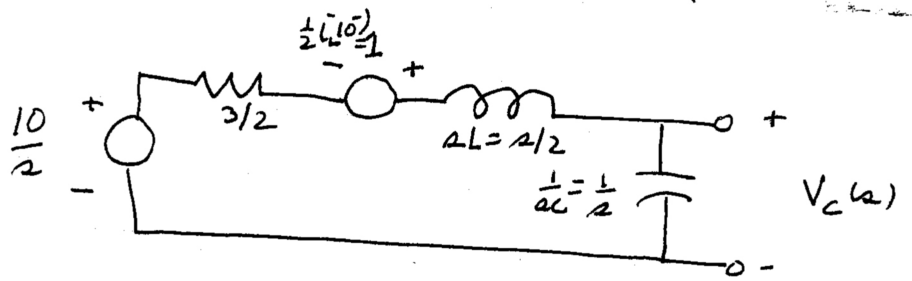
---

② Compute $v_c(t)$, $t \geq 0$ for

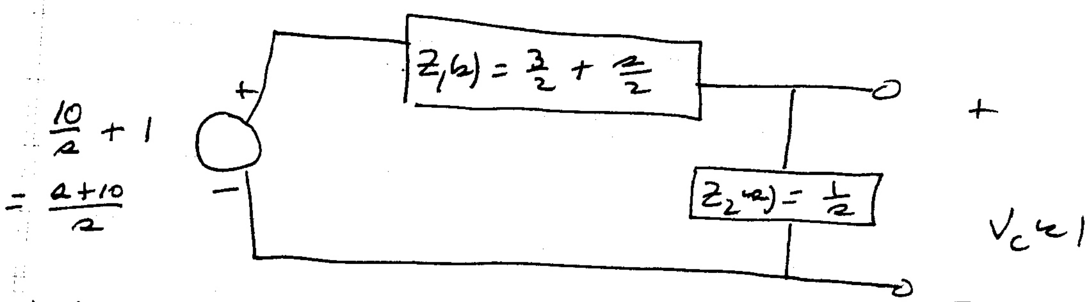


where $v_{in}(t) = 10u(t)$, $i_L(0^-) = 2$, $v_c(0^-) =$

Redraw in Laplace domain:



Combining voltage sources:



Voltage divider:

$$V_c(s) = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} V_{in}(s) = \frac{1/s}{\frac{s+3}{2} + \frac{1}{s}} \cdot \frac{s+10}{s}$$

$$= \frac{s+10}{s(s^2+3s+2)}$$

--- use PFE and table look up...

$v_c(t) = K_1 u(t) + K_2 e^{-t} u(t) + K_3 e^{-2t} u(t)$

(3) For a system described by

$$\ddot{y}(t) + \dot{y}(t) = 3x(t) + \dot{x}(t)$$

with $y(0^-) = 0$, $\dot{y}(0^-) = -1$, $x(t) = e^{-3t}u(t)$ compute $y(t)$.

$$s^2 Y(s) - s y(0^-) - \dot{y}(0^-) + sY(s) - y(0^-)$$
$$= 3X(s) + sX(s)$$

$$\Rightarrow (s^2 + s)Y(s) = (s+3)X(s) - 1$$

$$X(s) = \frac{1}{s+3} \Rightarrow (s^2+s)Y(s) = 0$$
$$\Rightarrow Y(s) = 0$$
$$\Rightarrow y(t) = 0, \ t \geq 0.$$

Note: This leads to an interpretation of the zeros of a transfer function!

ie

$$H(s) = \frac{s+3}{s(s+1)}$$

for the system above.