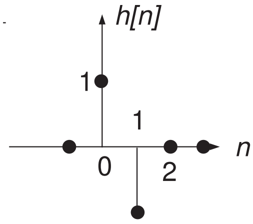
520:214: Recitation Problem Set 3

1. Answer student questions on Homework 2.

2. For the LTI system with



compute the responses (convolution) to the inputs:

   (a) $x[n] = u[n] \quad A: \; y[n] = \delta[n]$

   (b) $x[n] = r[n] \quad A: \; y[n] = u[n-1]$

   This system is called a *first difference.*

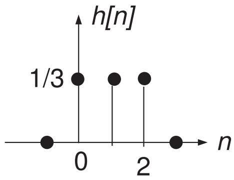
3. Is the LTI system with $h[n] = (0.5)^n u[n]$

   (a) causal? *(yes)*

   (b) stable? *(no)*

   (c) memoryless? *(yes)*

4. For the LTI system with $h[n]$ given by



derive a simple formula for the output signal in terms of the input signal.

A:

$$y[n] = \frac{x[n] + x[n+1] + x[n+2]}{3}$$

5. If $h[n] = u[n]$ and $x[n] = (1/2)^{|n|}$ compute $y[n] = h[n] * x[n]$.

A:

$$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|} u[n-k] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|}$$

For $n < 0$,

$$y[n] = \sum_{k=-\infty}^{-|n|} \left(\frac{1}{2}\right)^{-k} = \sum_{j=\infty}^{|n|} \left(\frac{1}{2}\right)^{j} = \sum_{j=|n|}^{\infty} \left(\frac{1}{2}\right)^{j}$$

For $n \geq 0$,

$$y[n] = \sum_{k=-\infty}^{-1} \left(\frac{1}{2}\right)^{-k} = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} = \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j} + \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k}$$

These expressions can be made neater using some formulas that you may see in a homework problem:

For $\alpha \neq 1$:

$$\sum_{k=0}^{N-1} \alpha^k = \frac{1-\alpha^N}{1-\alpha}$$

For $|\alpha| < 1$:

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}$$

Therefore, for $n \geq 0$ we have

$$\begin{aligned} y[n] &= -1 + \sum_{j=0}^{\infty} \left(\frac{1}{2}\right)^{j} + \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \\ &= -1 + \frac{1}{1-\frac{1}{2}} + \frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} \\ &= -1 + 2 + 2 - 2\left(\frac{1}{2}\right)^{n+1} \\ &= 3 - \left(\frac{1}{2}\right)^{n} \end{aligned}$$

Similar manipulations can be written for $n < 0$.