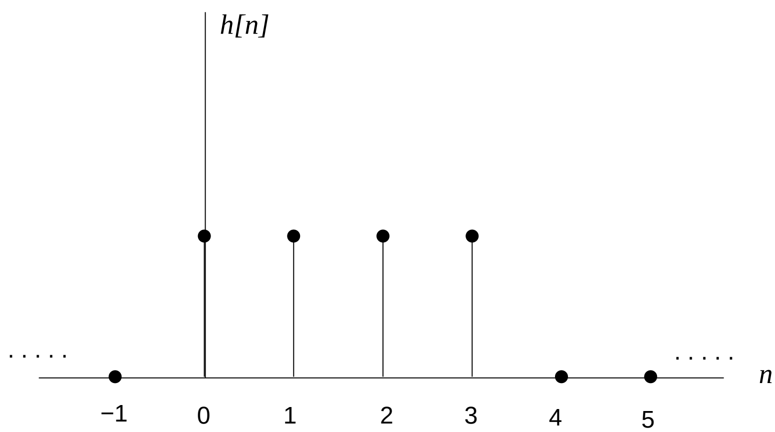
Recitation problem Set 4

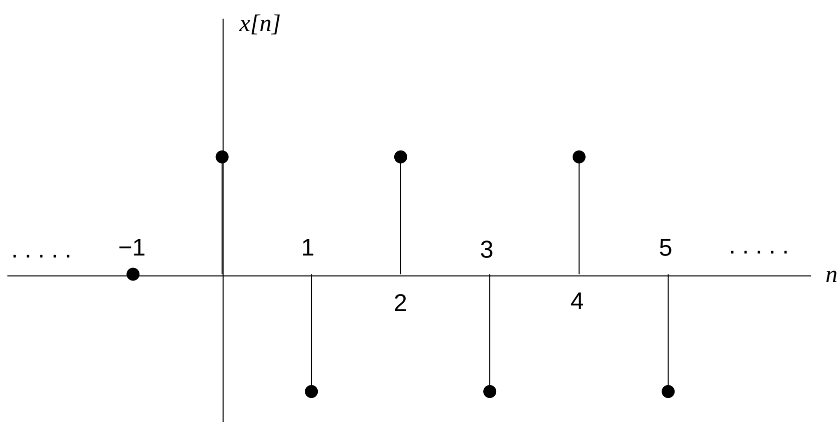
1. Compute

$$\begin{aligned} y[n] &= (h * x)[n] \\ h[n] &= u[n] - u[n-4] \\ x[n] &= (-1)^n u[n] \end{aligned}$$

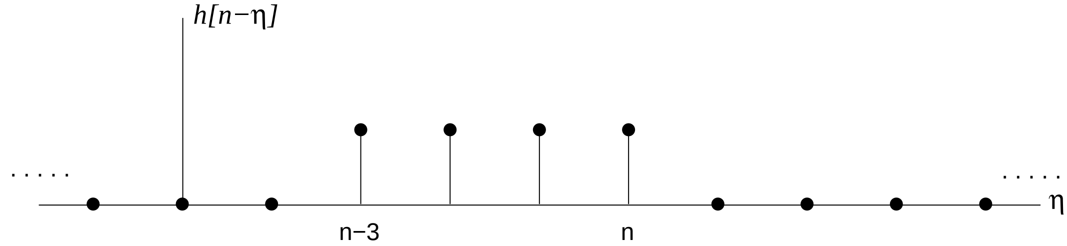
Let

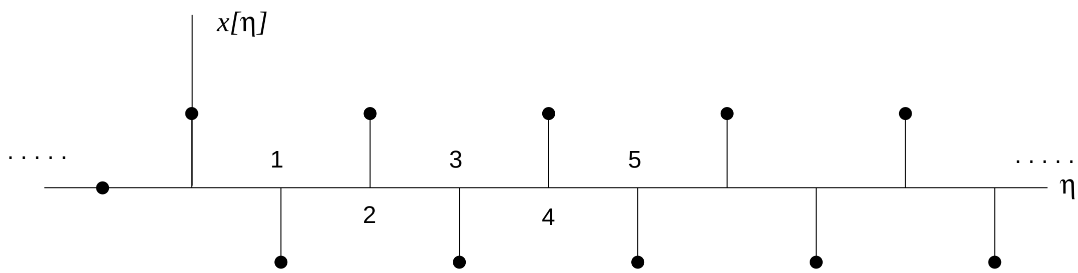
$$h[n] = u[n] - u[n-4]$$
$$x[n] = (-1)^n u[n]$$





$$y[n] = \sum_{\eta=-\infty}^{\eta=\infty} x[n]h[n-\eta]$$





From Plot

$$\begin{aligned} y[n] &= 0 \qquad for\ n\ <\ 0\ , \text{ Since no overlap.} \\ y[0] &= 1 \\ y[1] &= 0 \\ y[2] &= 0 \\ y[3] &= 0 \end{aligned}$$

Can see that for $n \geq 3$ $h[n-\eta]x[\eta]$ has four non-zero values, two +1 and two -1. Then $y[n] = 0, n \geq 3$.
Summing,

$$y[n] = \delta[n] + \delta[n-2]$$

2. Is $x[n] = e^{-j\frac{\pi}{2}n}$ periodic? If so, What is the fundamental period?
$\omega = -\frac{\pi}{2} = \frac{m}{N}2\pi$

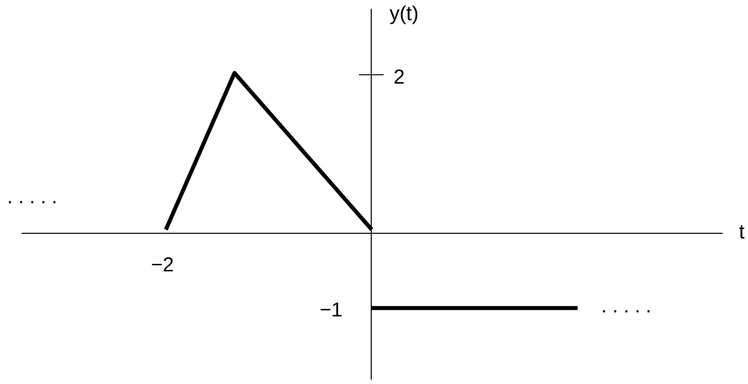
$$\begin{aligned} \Rightarrow m &= -1\ ,\ N = 4 \\ m &= -2\ ,\ N = 8 \\ &\vdots \quad etc \end{aligned}$$

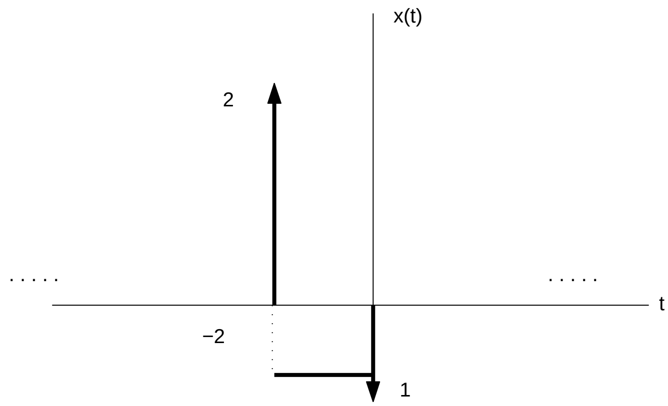
$\Rightarrow$ Periodic with $N_o = 4$

3.Shown below is a sketch of

$$y(t) = \int_{-\infty}^{t} x(\tau) d\tau$$

Sketch $x(t)$





Check this by doing a graphical running integral.

4. The system described by

$$y(t) = 5x(t) - \int_{t-2}^{t} 3x(\sigma) d\sigma$$

is
Linear?
Causal?
Stable?
Memoryless?
LTI, causal, stable, not memoryless

5. Is the signal

$$x(t) = \sin 5t + \cos(7.5t)$$

periodic? If so, what is the fundamental period?
Note:
$5 = 2 \times 2.5$, $7.5 = 3 \times 2.5$
So the frequencies are harmonically related.
$\Rightarrow$ Period $\omega_o = 2.5$ is the largest such fundamental frequency, so

$$T_o = \frac{2\pi}{2.5}$$