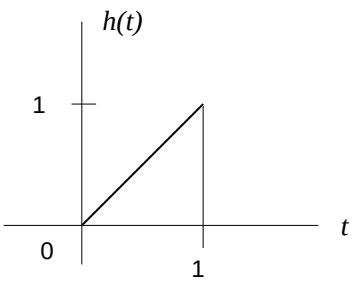
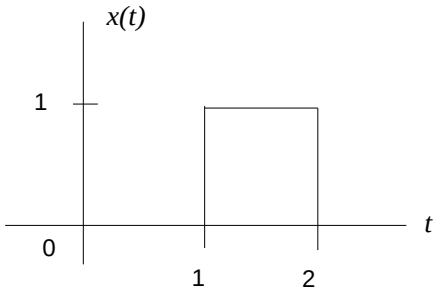
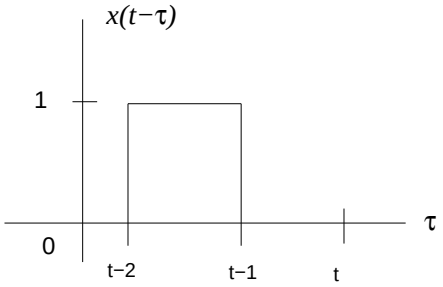
Recitation problem Set 5

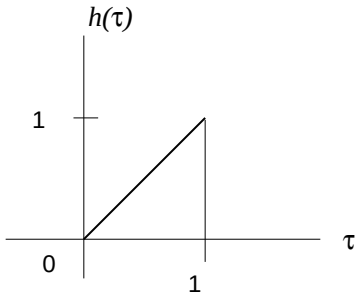
1. Compute $y(t) = (h * x)(t)$ where





Graphically $y(t) = \int_{-\infty}^{\infty} h(\tau)x(\tau)d\tau$





For $t - 1 < 0$ i.e. $t < 1$
$y(t) = 0$ "no overlap"

For $t-2>1$ i.e. $t>3$
$y(t)=0$ "no overlap"

For $0 \le t-1 \le 1$ i.e. $1 \le t \le 2$

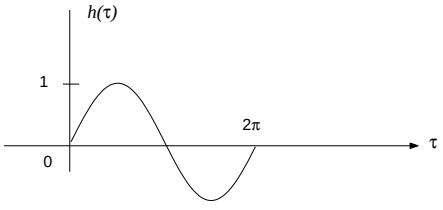
$$\begin{aligned} y(t) &= \int_0^{t-1} 1.\tau d\tau \\ &= \frac{1}{2}\tau^2 \mid_t -2^1 \\ &= \frac{1}{2} - \frac{1}{2}(t-2)^2 \end{aligned}$$

2. What is the unit step response of the LTI system with

$$h(t) = \begin{cases} \sin(t) & 0 \le t \le 2\pi \\ 0 & else \end{cases}$$

What is the response of the system to x(t)=1 (for all t)

$$\begin{aligned} x(t) &= u(t) \\ \Rightarrow y(t) &= \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau \\ &= \int_{-\infty}^{t} h(\tau)d\tau \end{aligned}$$



Clearly,$y(t)=0$ for $t \le 0$ or $t \ge 2\pi$.

For $t \in (0, 2\pi)$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} \sin\tau d\tau \\ &= -\cos\tau \mid_0^t \\ &= 1-\cos(t) \end{aligned}$$

So summarizing,

$$y(t) = \begin{cases} 1 - \cos(t) & t \in (0, 2\pi) \\ 0 & else \end{cases}$$

$x(t) = 1 \qquad \Rightarrow \qquad y(t) = \int_{-\infty}^{\infty} h(\tau) d\tau = 0$, for all t.

3. Is the DT LTI system with

$$h[n] = (-1.01)^n u[n]$$

Causal? Stable? Memoryless?

(Yes, No, No)

4. Is the CT LTI system with

$$h(t) = 2u(t) - 2u(t-1) + \delta(t+2)$$

Causal? Stable? Memoryless?

(No, Yes, No)