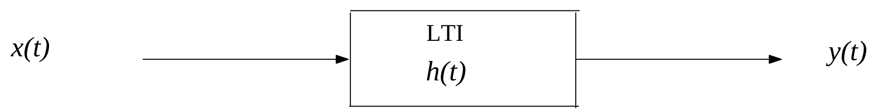
Recitation problem Set 6

1.



For $h(t) = e^{-t}u(t)$ and $x(t) = \cos(t)$, use the eigenfunction property to compute $y(t)$

Stable System $x(t) = e^{j\omega_o t} \quad \Rightarrow$

$$\begin{aligned} y(t) &= H(\omega_o)e^{j\omega_o t}, \\ H(\omega_o) &= \int_{-} \infty^{\infty} h(\tau)e^{-j\omega_o \tau} d\tau \end{aligned}$$

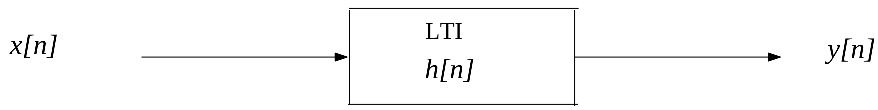
Note the system is stable.

$$\begin{aligned} H(\omega_o) &= \int_0^{\infty} h(\tau)e^{-(1+j\omega_o)\tau} d\tau \\ &= \frac{1}{1+j\omega_o} \\ x(t) &= \cos(t) = \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt} \\ \Rightarrow & \\ y(t) &= \cos(t) = \frac{1}{2}\frac{1}{(1+j)}e^{jt} + \frac{1}{2}\frac{1}{(1-j)}e^{-jt} \\ &= \frac{1}{2}[\frac{1}{\sqrt{2}}e^{j(t-\frac{\pi}{4})} + \frac{1}{\sqrt{2}}e^{-j(t-\frac{\pi}{4})} \\ &= \frac{1}{\sqrt{2}}\cos(t - \frac{\pi}{4}) \end{aligned}$$

Or, write $x(t) = \mathcal{R}e\{e^{jt}\}$ (Better!)

$$\begin{aligned} y(t) &= \mathcal{R}e\{H(1)e^{jt}\} \\ &= x(t) = \mathcal{R}e\{\frac{1}{(1+j)}e^{jt}\} \\ &= x(t) = \mathcal{R}e\{\frac{1}{\sqrt{2}}e^{j(t-\frac{\pi}{4})}\} \\ &= \frac{1}{\sqrt{2}}\cos(t - \frac{\pi}{4}) \end{aligned}$$

2.



For $h[n] = \delta[n] + \delta[n-1]$ and $x[n] = \cos(\frac{\pi}{4}n)$, use the eigenfunction property to compute $y[n]$

Stable System $x([n] = e^{j\omega_o n}$ $\Rightarrow$

$$\begin{aligned} y[n] &= H(\omega_o)e^{j\omega_o n}, \\ H(\omega_o) &= \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega_o k} \end{aligned}$$

Note the system is stable.

$$\begin{aligned} x[n] &= \cos(\frac{\pi}{4}n) = \mathcal{R}e\{e^{j\frac{\pi}{4}n}\} \\ H(\frac{\pi}{4}) &= \sum_{k=-\infty}^{\infty} (\delta[k] + \delta[k-1])e^{-j\frac{\pi}{4}k} \\ &= 1 + e^{-j\frac{\pi}{4}} \\ \Rightarrow & \\ y[n] &= \mathcal{R}e\{(1 + e^{-j\frac{\pi}{4}})e^{j\frac{\pi}{4}n}\} \\ &= \mathcal{R}e\{e^{j\frac{\pi}{4}n} + e^{j\frac{\pi}{4}(n-1)}\} \\ &= \mathcal{R}e\{e^{j\frac{\pi}{4}n}\} + \mathcal{R}e\{e^{j\frac{\pi}{4}(n-1)}\} \\ &= \cos(\frac{\pi}{4}n) + \cos(\frac{\pi}{4}(n-1)) \end{aligned}$$

3. Is the CT LTI system with

$$h(t) = te^{-t}u(t+1)$$

Memoryless? Causal? Stable?

(No,No,Yes)

4. Is the CT LTI system with

$$h(t) = \sum_{k=0}^{\infty} \delta(t - 2k)$$

Memoryless? Causal? Stable?

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} (\delta(\tau - 2k)) x(t - \tau) d\tau \\
&= \sum_{k=0}^{\infty} \int_{-\infty}^{\infty} (\delta(\tau - 2k)) x(t - \tau) d\tau \\
&= \sum_{k=0}^{\infty} x(t - 2k)
\end{aligned}
$$

Not memoryless, causal, not stable.