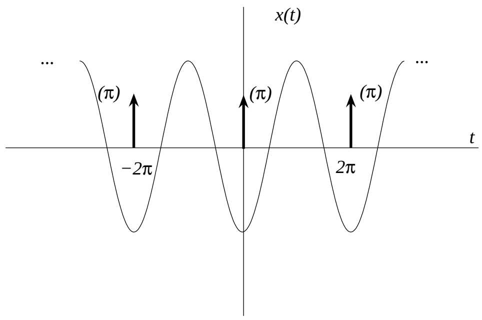
Recitation problem Set 7

1. Compute the FS for $x(t)$



$T_o = 2\pi \Rightarrow \omega_o = 1$; $x(t) = -\cos(t) + \pi\delta(t)$, $-\pi \leq t \leq \pi$

$$\begin{aligned} x_k &= \frac{1}{T_o}\int_{-\pi}^{\pi} x(t)e^{-jk\omega_o t}dt \\ &= -\frac{1}{2\pi}\int_{-\pi}^{\pi} [-\cos(t)e^{-jkt} + \pi\delta(t)e^{-jkt}]dt \\ &= -\frac{1}{4\pi}\int_{-\pi}^{\pi} [-(e^{jt} + (e^{-jt})e^{-jkt} + \pi\delta(t)e^{-jkt}]dt \\ &= \begin{cases} 0, & k = \pm 1 \\ \frac{1}{2}, & else \end{cases} \end{aligned}$$

Note that the cosine removed the fundamental component of impulse train. This signal is used in auditory tests. An impulse train sounds like a sequence of "". The signal with fundamental removed has been used in auditory studies-You hear the missing fundamental!

4. If $x(t)$ has fundamental period T_o and is an even signal, show that

$$x_k = \frac{2}{T_o}\int_{0}^{\frac{T_o}{2}} x(t)\cos(k\omega_o t)dt \qquad (real!)$$

$$x_k = \frac{1}{T_o}\int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} x(t)\cos(k\omega_o t)dt$$

$$= \frac{1}{T_o}\int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} \underbrace{x(t)}_{even}[\underbrace{\cos(k\omega_o t)}_{even} + j\underbrace{\sin(k\omega_o t)}_{odd}]dt$$

$$= \frac{1}{T_o}\int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} \cos(k\omega_o t)dt, \quad \textit{since even} \times \textit{odd} = \textit{odd}$$

$$= \frac{1}{T_o}\int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} \cos(k\omega_o t)dt, \quad \textit{since even} \times \textit{even} = \textit{even}$$

5. What are the complex-form Fourier series for $x(t) =$

   (a) $3\sin(2t) + 2e^{-j4t} + 2e^{j4t}$
   $\omega_o = 2$
   $x_1 = \frac{3}{2j} = x^*_{-1}$
   $x_2 = 2 = x_{-2}$
   Al other x_k's are zero.

   (b) $3\sin(2t) + 2e^{-j3t} + 2e^{j3t}$
   $\omega_o = 1$
   $x_2 = \frac{3}{2j} = x^*_{-2}$
   $x_3 = 2 = x_{-3}$
   Al other x_k's are zero.

   (c) $3\sin(2t) + 2\cos(\pi t)$
   Not periodic.

   Plot the magnitude and phase sketch.