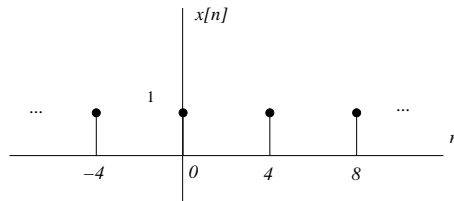


Recitation problem Set 9

1. Compute the DTFS coefficients for

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$$



$$\begin{aligned} \Rightarrow N_o &= 4, \quad \omega_o = \text{frac}2\pi4 = \frac{\pi}{2} \\ x_k &= \frac{1}{N_o} \sum_{n=0}^{N_o-1} x[n]e^{-jk\omega_o n} = \frac{1}{4} \sum_{n=0}^3 x[n]e^{-jk\omega_o n} \\ &= \frac{1}{4}(1 + 0 + 0 + 0) = \frac{1}{4} \\ \Rightarrow x[n] &= \sum_{k=0}^3 \frac{1}{4} e^{jk\frac{\pi}{2}n} \end{aligned}$$

Check:

$$\begin{aligned} x[1] &= \frac{1}{4}(e^{j0} + e^{j\frac{\pi}{2}} + e^{j\pi} + e^{j\frac{3\pi}{2}}) \\ &= \frac{1}{4}(1 + j + (-1) + (-j)) = 0 \end{aligned}$$

2. Compute the DTFS coefficients for

$$x[n] = 1 - \cos\left(\frac{\pi}{3}n + \pi\right)$$

Write

$$\begin{aligned} x[n] &= 1 - \frac{1}{2}e^{j(\frac{\pi}{3}n+\pi)} - \frac{1}{2}e^{-j(\frac{\pi}{3}n+\pi)} \\ &= 1 + \frac{1}{2}e^{j\frac{\pi}{3}n} + \frac{1}{2}e^{-j\frac{\pi}{3}n} \end{aligned}$$

Clearly, $N_o = 6$, $\omega_o = \frac{2\pi}{6} = \frac{\pi}{3}$.

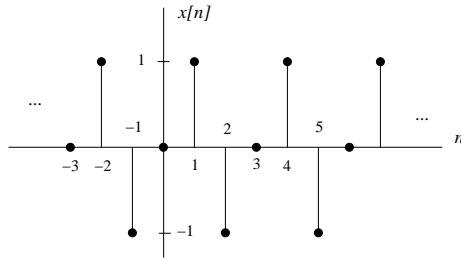
Choose

$$\langle N_o \rangle = -2, -1, 0, 1, 2, 3$$

and match the $x[n]$ expression with

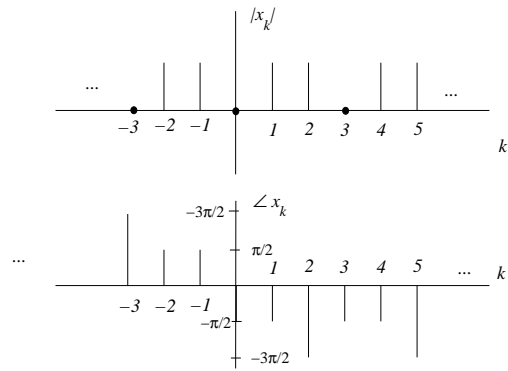
$$\begin{aligned} x[n] &= \sum_{k=-2}^3 x_k e^{jk\frac{\pi}{3}n} \\ \Rightarrow \quad x_o &= 1, \quad x_1 = x_{-1} = \frac{1}{2}, \quad x_{-2} = x_2 = x_3 = 0, \quad x_{k+6} = x_k \end{aligned}$$

3. Plot the magnitude and phase spectra for



Clearly $N_o = 3$. Need to plot $|x_k|$. And $\angle x_k$ is k (or is $k\frac{2\pi}{3}$).

$$\begin{aligned} x_k &= \frac{1}{3} \sum_{n=-1}^1 x[n] e^{-jk\frac{2\pi}{3}n} = \frac{1}{3} [-e^{jk\frac{2\pi}{3}} + 0 + e^{-jk\frac{2\pi}{3}}] \\ &= \frac{-2j}{3} \left(\frac{e^{jk\frac{2\pi}{3}} - e^{-jk\frac{2\pi}{3}}}{2j} \right) = -\frac{2j}{3} \sin\left(\frac{2\pi}{3}k\right), \quad k = 0, 1, 2 \\ |x_k| &= \frac{2}{3} \sin\left(\frac{2\pi}{3}k\right) = \begin{cases} 0, & k = 0 \\ \frac{2}{3} \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}, & k = 1 \\ \frac{\sqrt{3}}{3}, & k = 2 \end{cases} \\ \angle x_k &= -\frac{\pi}{2} + \angle \sin\left(\frac{2\pi}{3}k\right) = \begin{cases} -\frac{\pi}{2}, & k = 0 \\ -\frac{\pi}{2}, & k = 1 \\ -\frac{3\pi}{2}, & k = 2 \end{cases} \end{aligned}$$



Careful here ... we are computing the angle if a real number that is the ... of an angle.