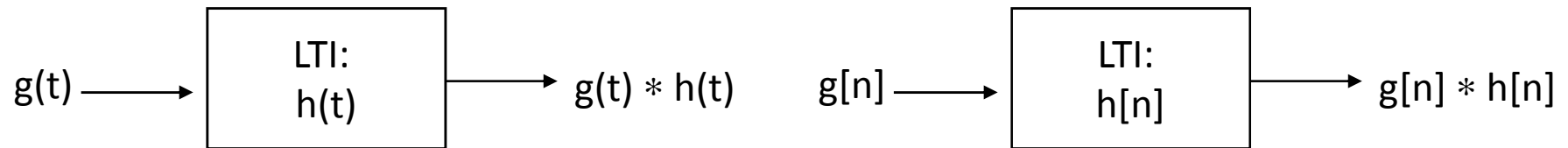


Convolution

$$g(t) * h(t) = \int_{-\infty}^{\infty} g(\tau) h(t - \tau) d\tau$$

$$g[n] * h[n] = \sum_{k=-\infty}^{\infty} g[k] h[n - k]$$



Example: $g[n] = u[n] - u[3-n]$
 $h[n] = \delta[n] + \delta[n-1]$

Convolution methods:

- Method 1: “running sum”
 1. Plot x and h vs. m
 2. Flip h over vertical axis to get $h[-m]$
 3. Shift $h[-m]$ to obtain $h[n-m]$
 4. Multiply to obtain $x[m]h[n-m]$
 5. Sum on m , $\sum_m x[m] h[n-m]$
 6. Increment n and repeat steps 3~6

Convolution methods:

- **Method 2: Superposition method**

- We know $\delta[n] \rightarrow h[n]$ and the system is LTI.
- Therefore, $a_0 \delta[n-n_0] \rightarrow a_0 h[n-n_0]$
- $a_0 \delta[n-n_0] + a_1 \delta[n-n_1] + \dots \rightarrow a_0 h[n-n_0] + a_1 h[n-n_1] \dots$
- Since the input can be expressed as a sum of shifted unit samples:

$$x[n] = \dots x[-2] \delta[n+2] + x[-1] \delta[n+1] + x[0] \delta[n] + x[1] \delta[n-1] + \dots$$

- The out is then a sum of shifted unit sample responses:

$$x[n_0] \delta[n-n_0] + x[n_1] \delta[n-n_1] + \dots \rightarrow x[n_0] h[n-n_0] + x[n_1] h[n-n_1] \dots$$

$$\rightarrow \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

(The Convolution Sum!)