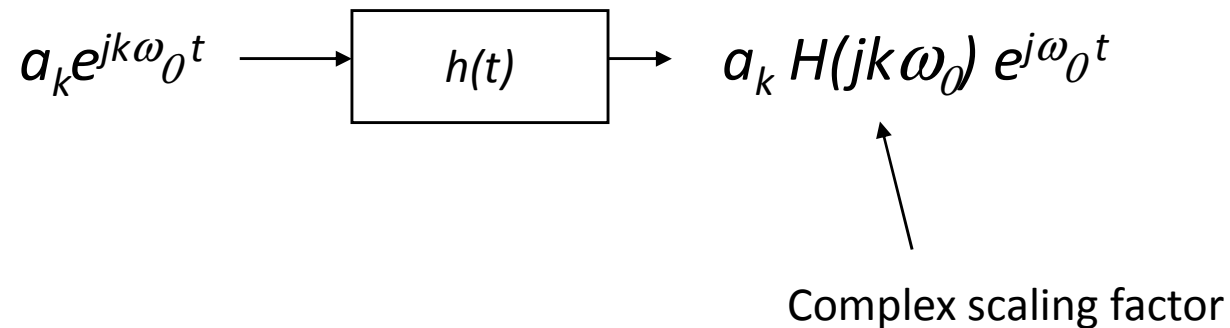


Basic signals

- Why use complex exponentials?
 - Because they are useful building blocks which can be used to represent large and useful classes of signals
 - Response of LTI systems to these basic signals is particularly simple and useful.



Fourier Series Representation of CT Periodic Signals

$$x(t) = x(t+T) \text{ for all } t$$

- Smallest such T is the *fundamental period*
- $\omega_0 = 2\pi/T$ is the *fundamental frequency*

$$e^{j\omega t} \text{ periodic with period } T \Leftrightarrow \omega = k\omega_0$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \stackrel{\Downarrow}{=} \sum_{k=-\infty}^{+\infty} a_k e^{j2\pi kt/T}$$

- Periodic with period T
- $\{a_k\}$ are the *Fourier (series) coefficients*
- $k = 0$ DC
- $k = \pm 1$ first harmonic
- $k = \pm 2$ second harmonic

CT Fourier Series Pair

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \quad (\text{Synthesis Equation})$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \quad (\text{Analysis Equation})$$

Examples

- $x(t) = \cos(4\pi t) + 2\sin(8\pi t)$
- Periodic square wave (from lecture)
- Periodic impulse train

$$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

Even and Odd Functions

- Even functions are defined by the property:

$$f(x) = f(-x)$$

- Odd functions are defined by the property:

$$f(x) = -f(-x)$$

Some notes on CT Fourier series

- For $k = 0$, $a_0 = \frac{1}{T} \int_0^T x(t) dt \leftarrow$ mean value over the period (DC term)
- If $x(t)$ is real, $a_k = a_{-k}^*$
- If $x(t)$ is even, $a_k = a_{-k}$
- If $x(t)$ is odd, $a_k = -a_{-k}$
- If $x(t)$ is real and even, $a_k = a_{-k}^* = a_{-k} \rightarrow a_k$'s will only have real terms
- If $x(t)$ is real and odd, $a_k = a_{-k}^* = -a_{-k} \rightarrow a_k$'s will only have odd terms

The CT Fourier Transform Pair

$$x(t) \leftrightarrow X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t} d\omega$$