Phase Transition between Small and Large Field Models of Inflation

Ely D. Kovetz, Tel-Aviv University

Based on:

N. Itzhaki and E. D. Kovetz, “A Phase Transition between Small and Large Field Models of Inflation,” Submitted to Phys Rev D. [arXiv:hep-th/0810.4299]

N. Itzhaki and E. D. Kovetz, “Inflection Point Inflation and Time Dependent Potentials in String Theory,” JHEP 1007, 054 (2007) [arXiv:hep-th/0708.2798]

Outline

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  A period of accelerated expansion follows shortly after the big-bang. Consequently, the Universe is flattened, the Hubble Radius is shrinked, Relics are heavily diluted and quantum fluctuations are imprinted onto the CMB.

Scalar Field Inflation

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- We measure the amount of inflation in the slow roll region by counting the number of e-foldings in the growth of the scale factor:

$$N = \ln\frac{a(t_f)}{a(t_i)} = \int_{t_i}^{t_f} H dt$$

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- The spectral index is: n_s and the tensor-to-scalar ratio is: $r \equiv \dfrac{P_t}{P_s}$

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These two categorizations, though in some cases complementary, are not equivalent (for example, a recent model in string theory involving a $\phi^{\alpha},\ \alpha = \frac{2}{3}$ potential (L. McAllister and E. Silverstein, hep-th/0710.2951) will be considered a large field model according to the first categorization, but a small field model according to the second).

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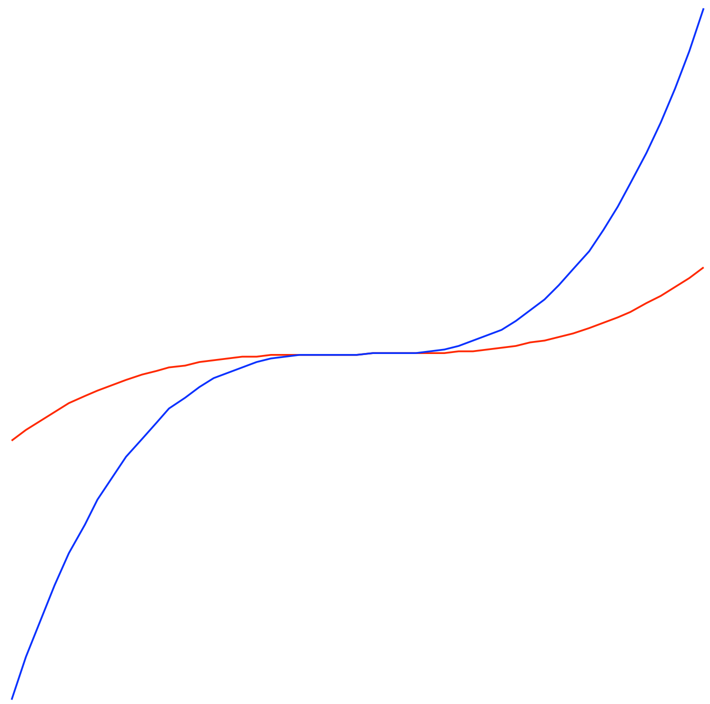
Differences between large field and small field models:



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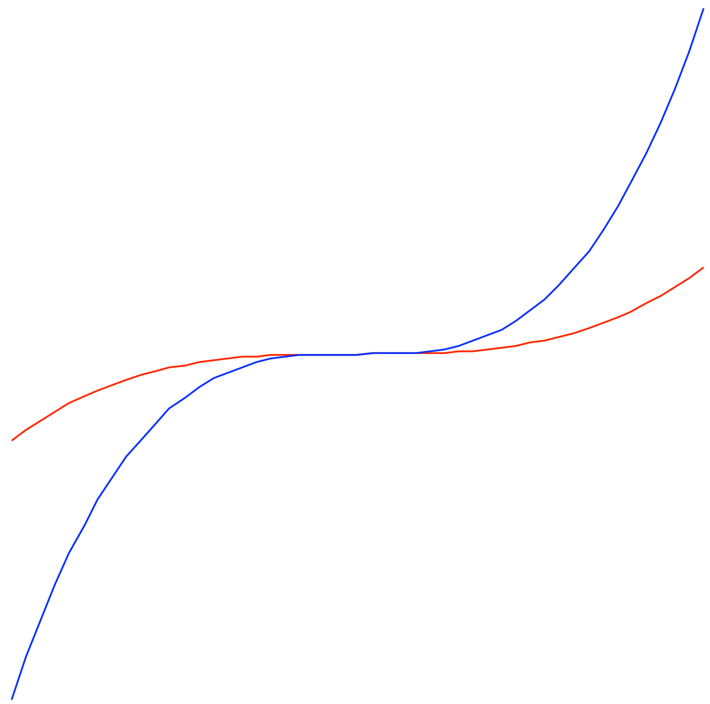

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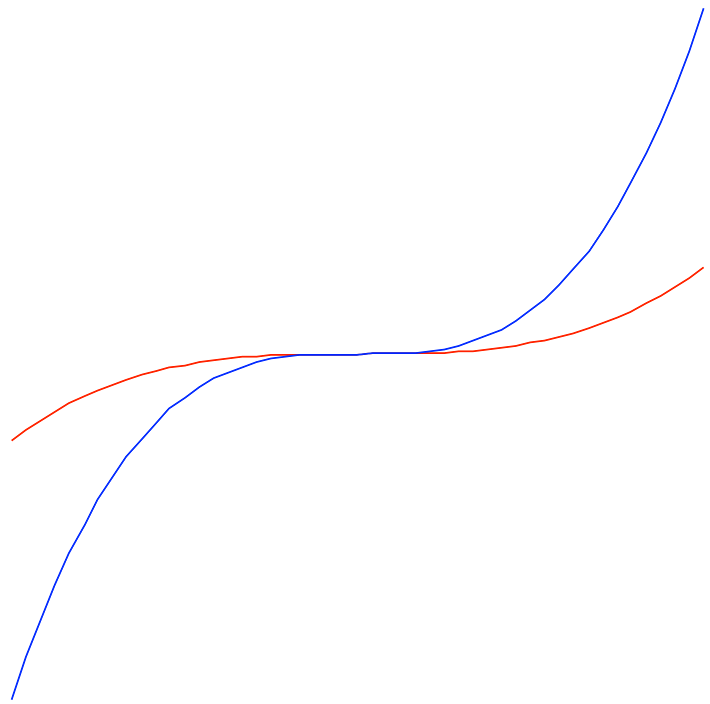

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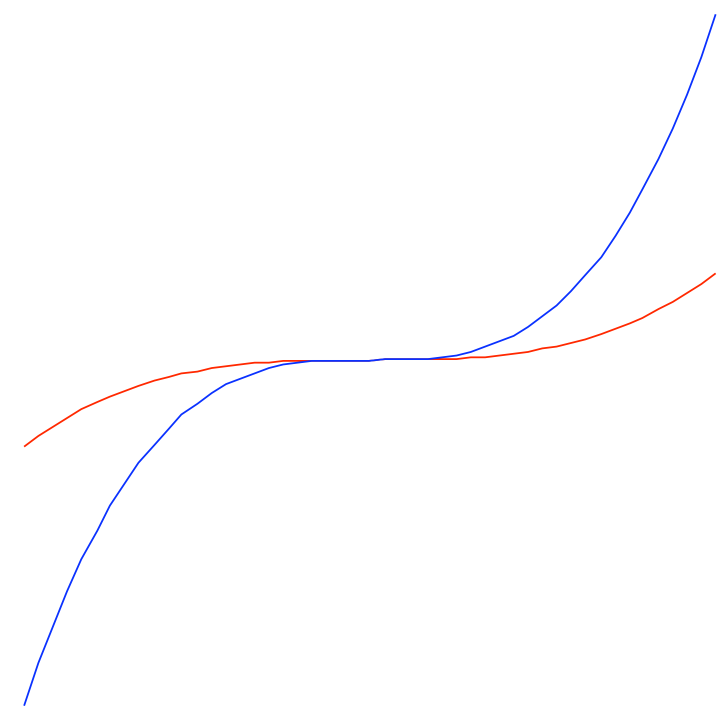

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- For single field models of inflation, the tensor-to-scalar ratio and the spectral index are related by:

$$r = \frac{8}{3}(1 - n_s) + \frac{16}{3}\frac{V''}{V}.$$

  This means that large field models will be easier to constrain in experiments.

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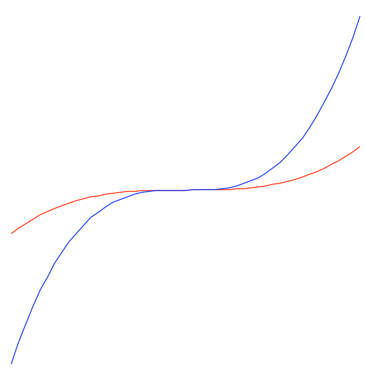

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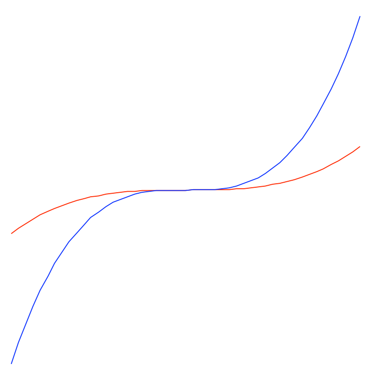

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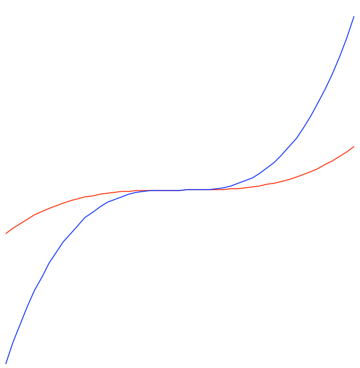

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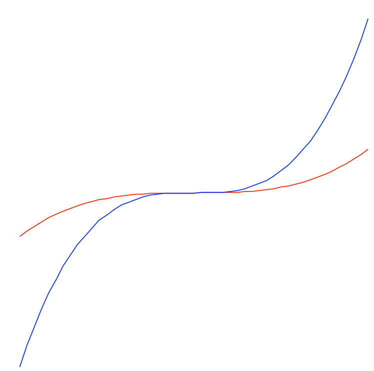

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Therefore, N is our candidate for the order parameter.

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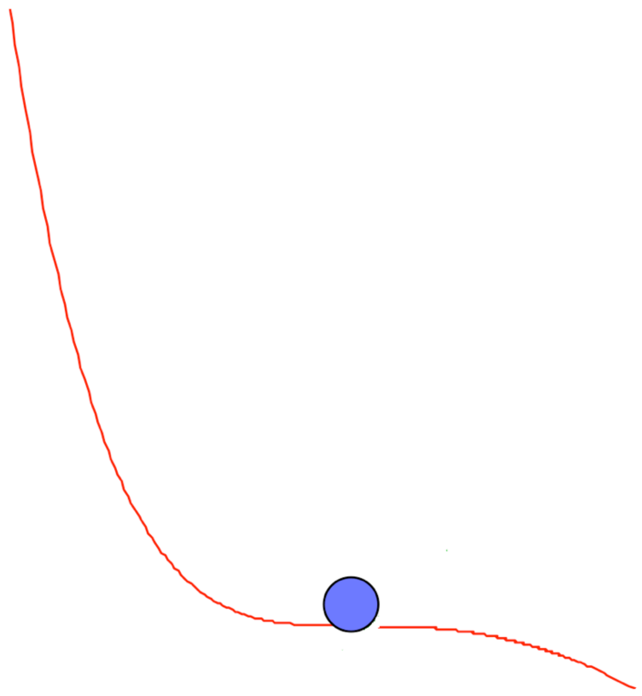

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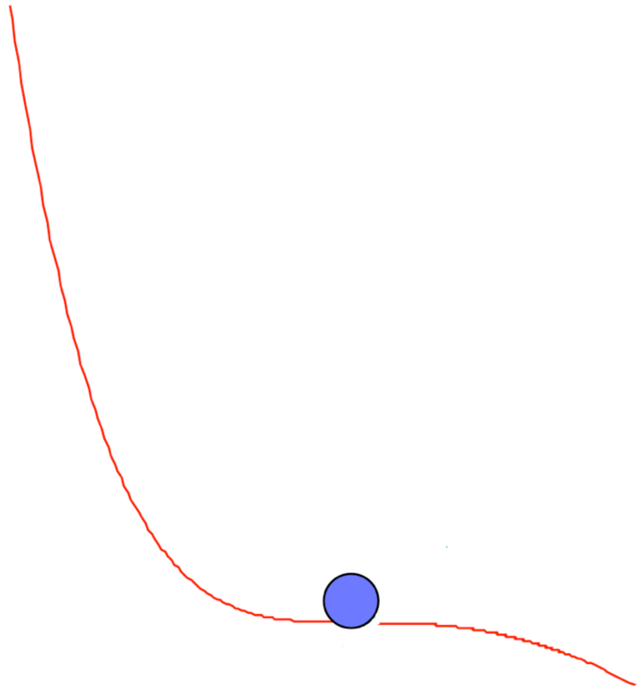

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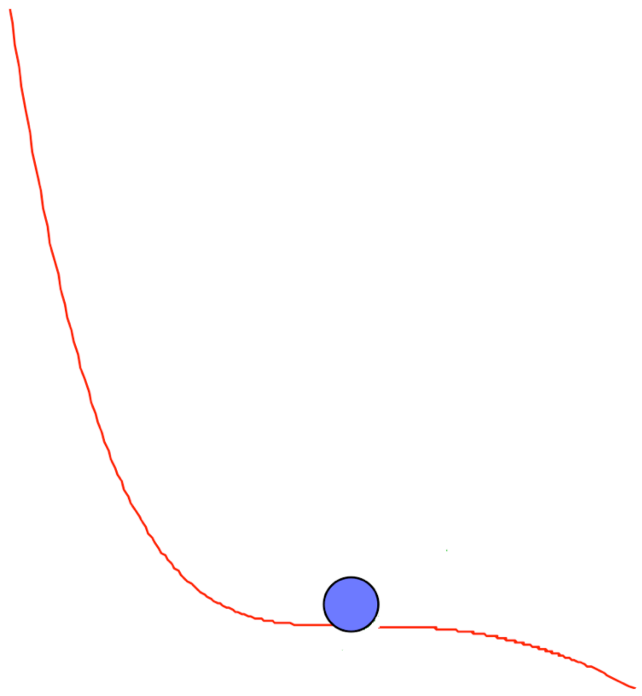

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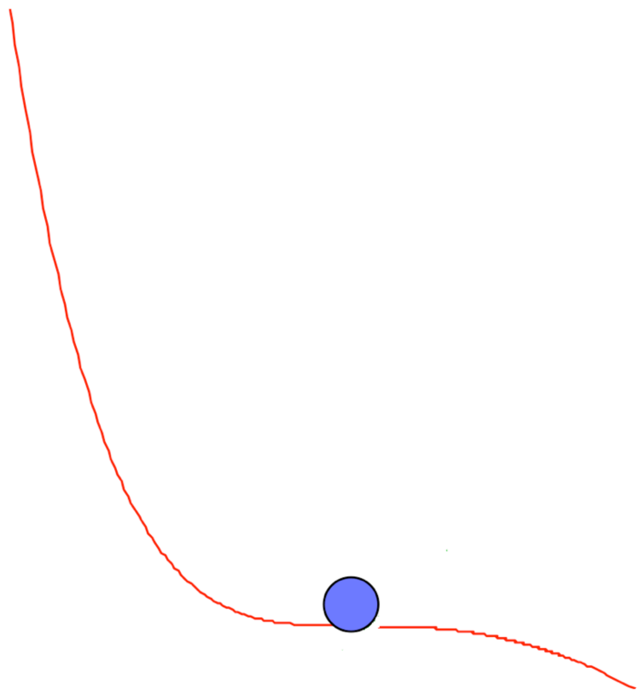

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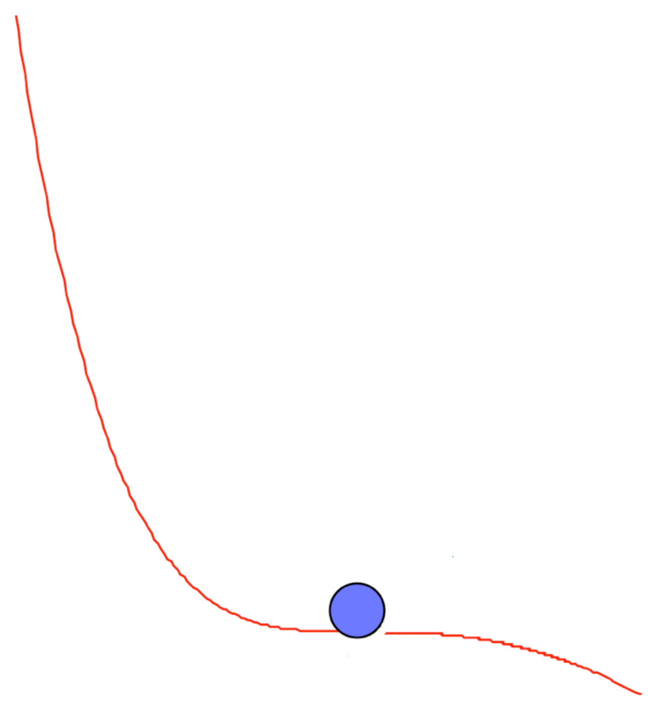

Solution:

- If the inflaton potential has an inflection point, then a large amount of inflation will be generated, provided that the inflaton spends enough time in this region.

Inflection Point Inflation

Properties of IPI:

- Can be approximated near the potential by: $V(\phi) = V_0\left(1 + \beta\phi^3\right)$
  where without loss of generality we assume that the inflection point is at zero.

- Recently became popular, especially in the context of String Theory.

- Fine-tuning of parameters is required to yield an inflection point.

- Highly sensitive to the initial condition (overshoot problem).

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- To demonstrate the PT property numerically, we notice that taking $\phi \rightarrow \lambda\phi$ and keeping the kinetic term fixed is equivalent to keeping ϕ fixed and transforming: $\beta \longrightarrow \lambda^3\beta$. Therefore, we can search for a critical β_c where a PT occurs between large and small field domains.

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$\beta \backslash \phi_{init}$	0.5	1	5	10	100
0.1	∞	∞	∞	∞	∞
0.7	∞	∞	∞	∞	∞
0.7744	∞	∞	∞	∞	∞
0.7745	∞	∞	38404.9097	38218.5366	38218.5365
0.8	∞	∞	48.3532	48.3529	48.3529
1	∞	∞	5.2579	5.2579	5.2579
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- Using the symmetry $V_0 \longrightarrow \alpha^2 V_0, \quad t \longrightarrow t/\alpha$, under which $N = \int H dt$, our order parameter, is invariant, we can fix: $V_0 = 1$.

- Solving for different initial conditions, we get:

β_c

$\beta \backslash \phi_{init}$	0.5	1	5	10	100
0.1	∞	∞	∞	∞	∞
0.7	∞	∞	∞	∞	∞
0.7744	∞	∞	∞	∞	∞
0.7745	∞	∞	38404.9097	38218.5366	38218.5365
0.8	∞	∞	48.3532	48.3529	48.3529
1	∞	∞	5.2579	5.2579	5.2579
5	0.96866	0.46226	0.35061	0.35061	0.35061

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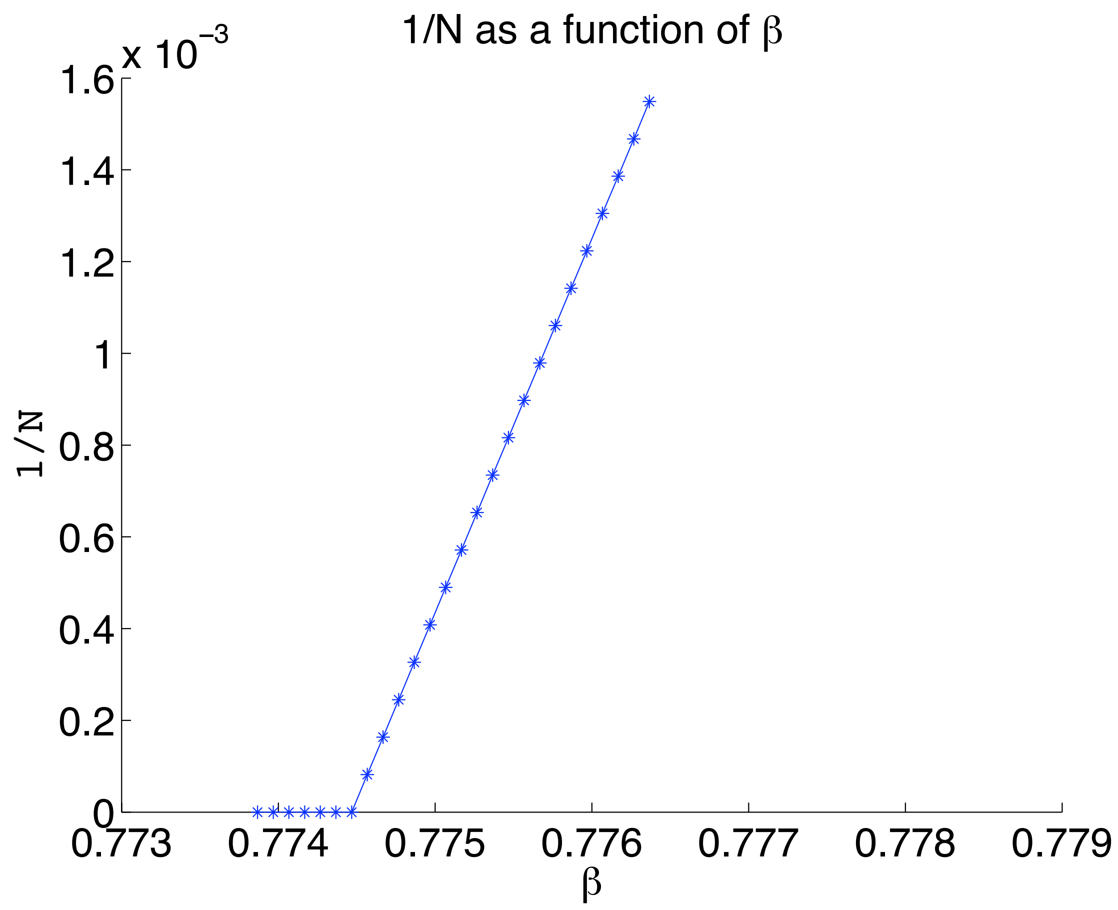

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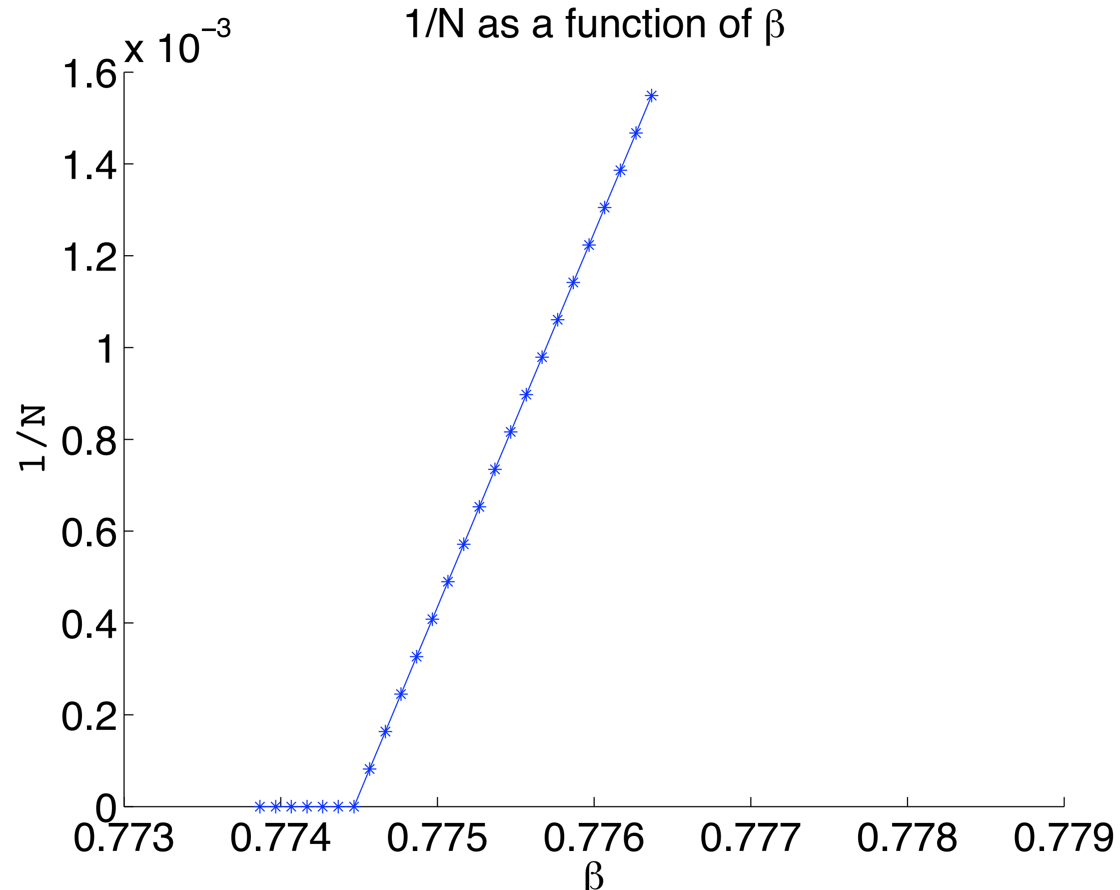

- $N_{e-folds}$ is time dependent. An alternative is to use: $\frac{1}{N} \sim \dot{\phi}(\phi = 0) \equiv \dot{\phi}_0$ and identify $\dot{\phi}_0$ as the order parameter.

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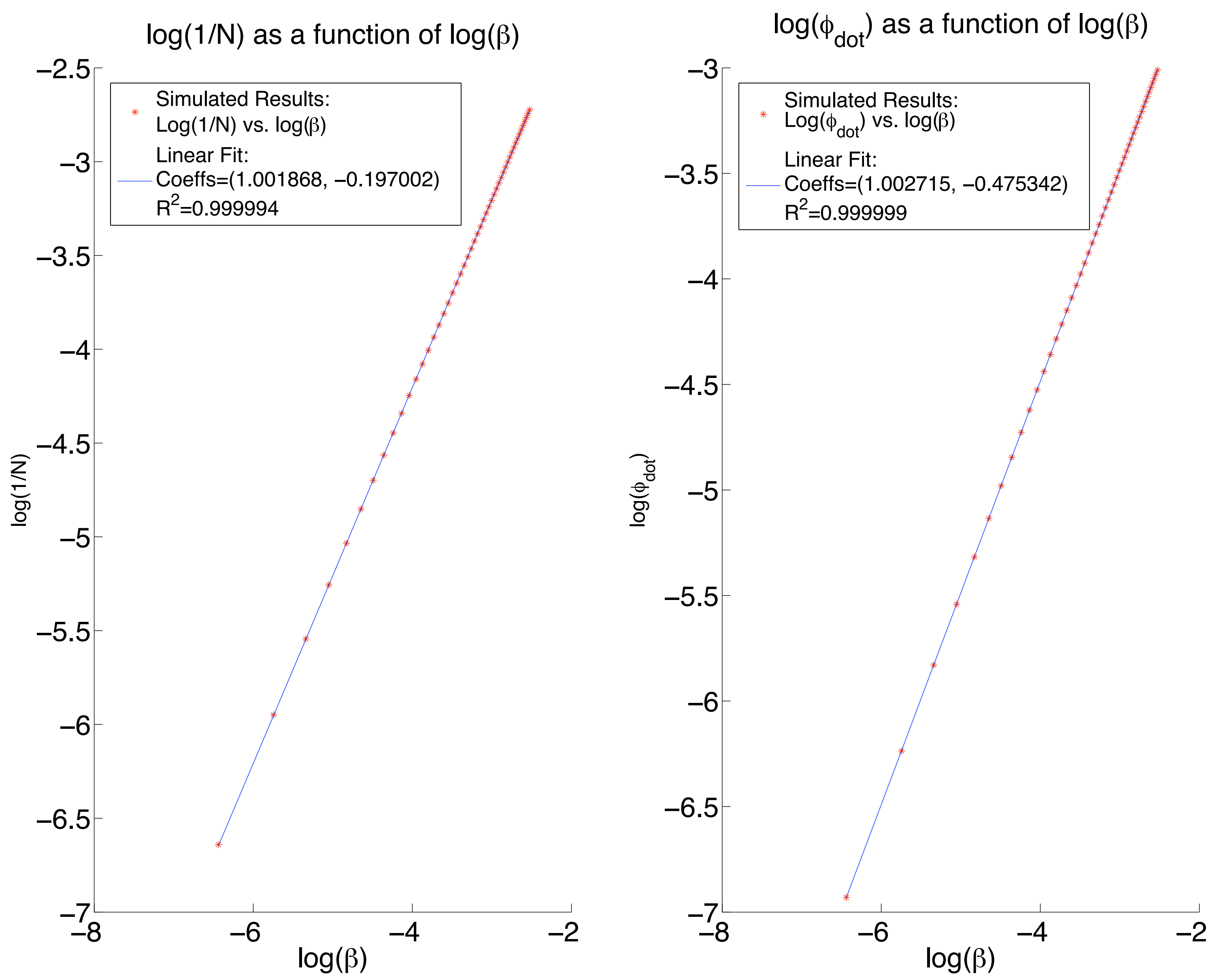

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- The critical exponent is integer: $\gamma = 1$
- The behavior is universal in the sense that it is independent of initial conditions. This is a direct result of the attractor property of slow roll inflation.

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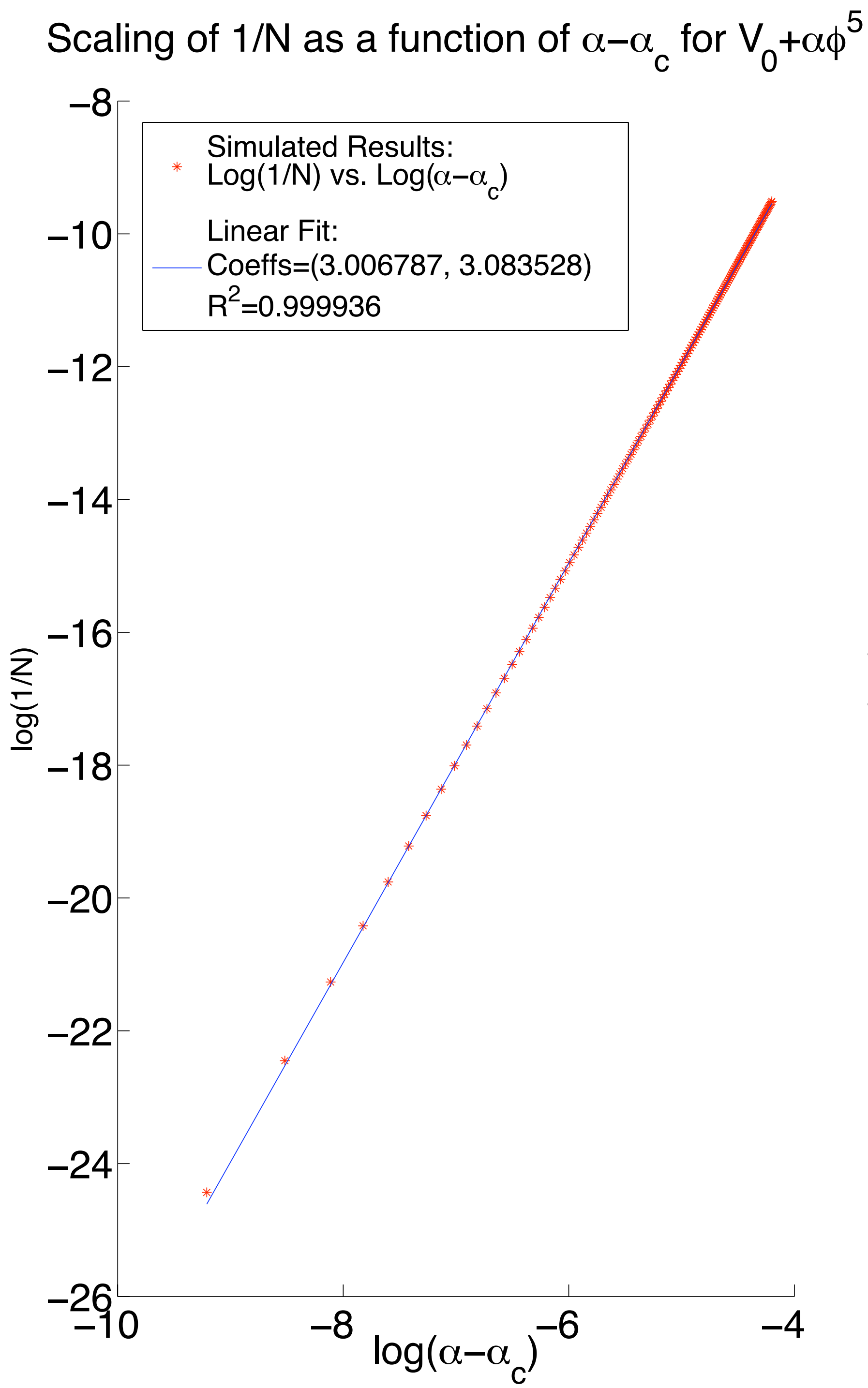
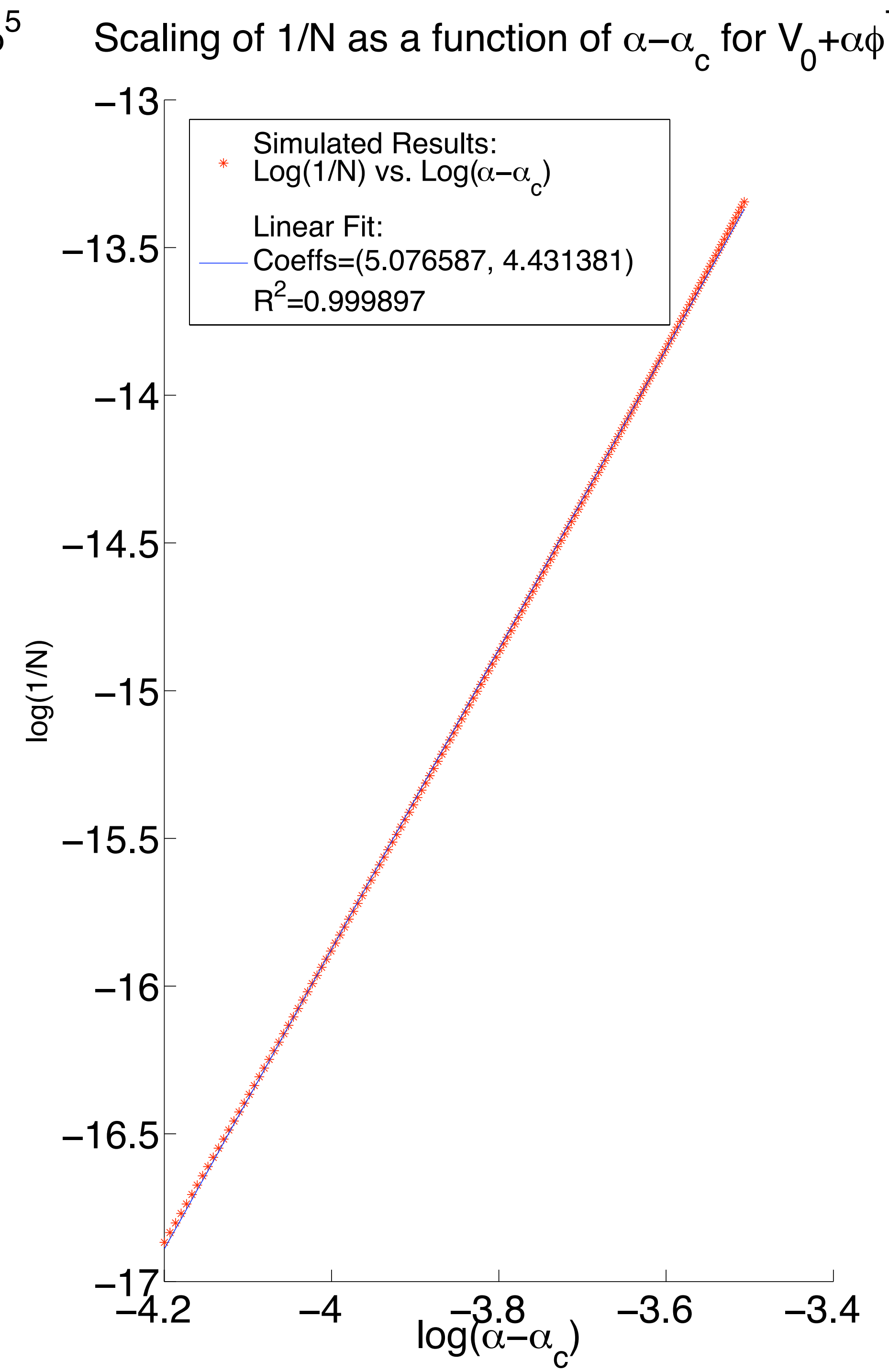
- Potentials in which the lowest non-zero derivative is even exhibit no critical behavior at all.

Different Critical Exponents





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- We find that for $\delta \neq 0$ there is a λ_c such that:

  - For $\lambda > \lambda_c$ the amount of inflation depends on the initial condition. Then, in order to maximize N we have to fine tune the initial condition.
  - For $\lambda < \lambda_c$, however, any non-singular initial condition gives $N = N_{max}$.

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- This is invariant under: $\phi \to C\phi, \;\; \beta \to \beta/C^2$

Using a Different Order Parameter

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- so if the solutions start with $\phi(0) = \phi_0$ and $\dot{\phi}(0) = 0$, then $\phi_0^2\beta$ can be used to parametrize them. Therefore, we can either fix β and vary the initial condition, or vary the initial condition with β fixed.

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  we can measure the time spent in the vicinity of the inflection point instead of the amount of e-foldings.

Damped Harmonic Oscillator

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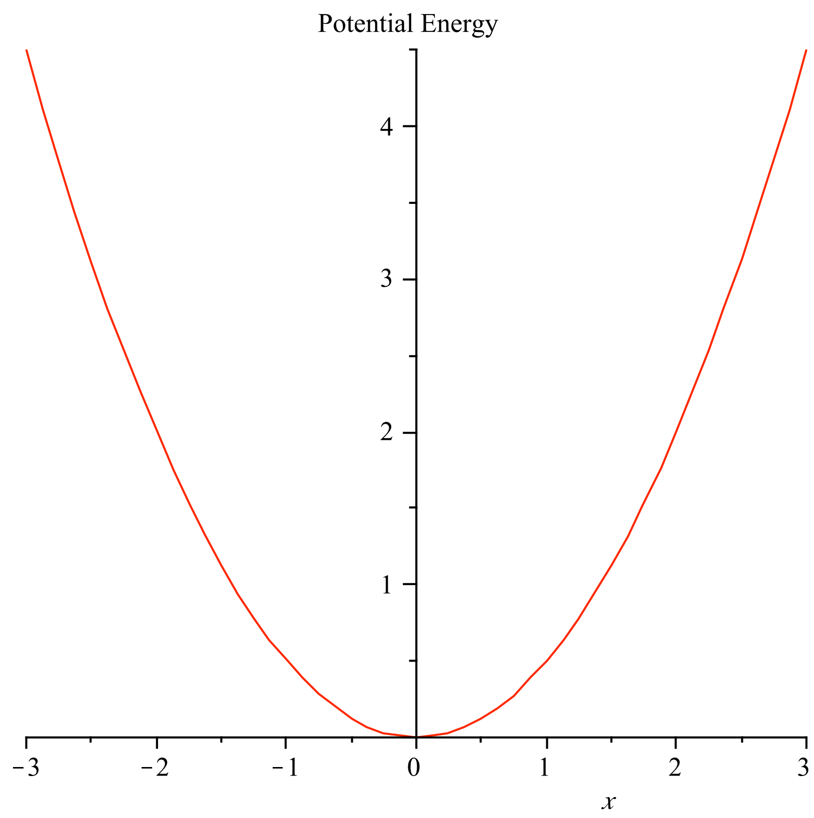
- A simpler example: a Harmonic Oscillator



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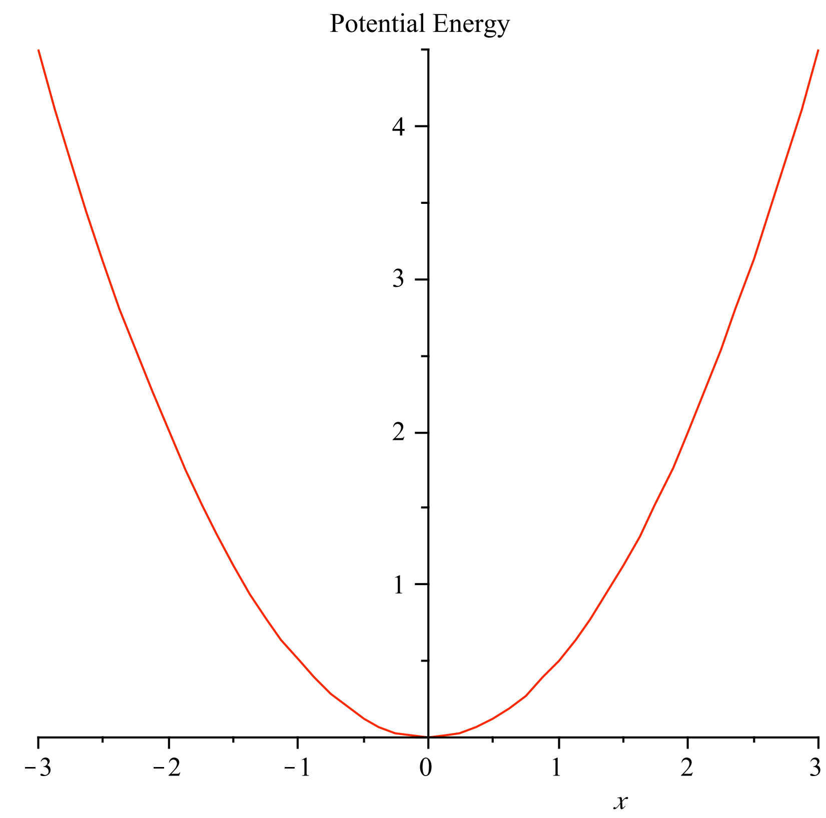

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$$\ddot{x} + \gamma\dot{x} + x = 0$$

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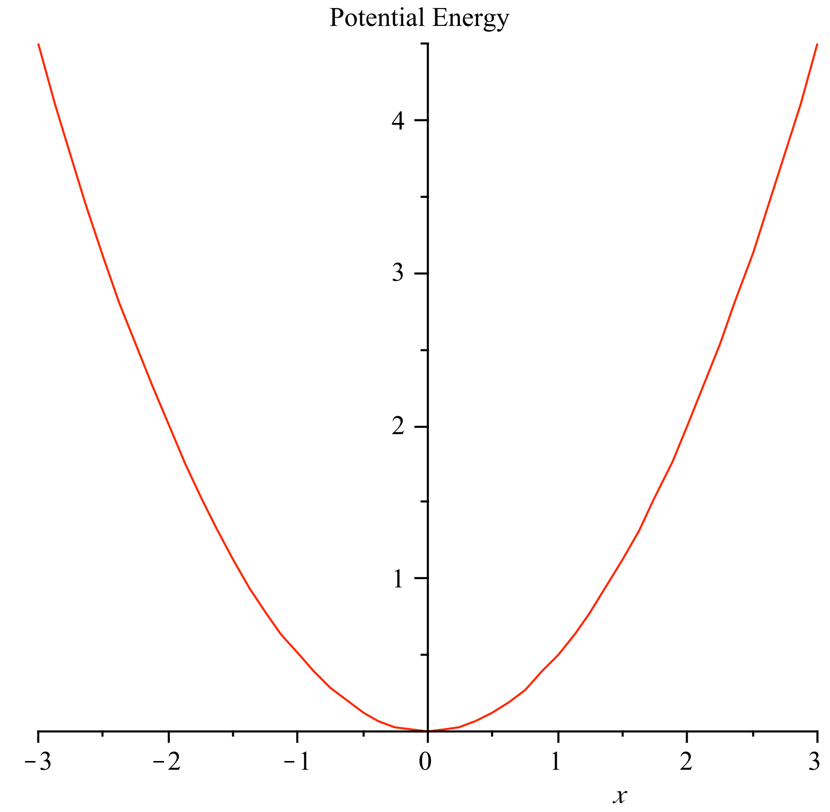

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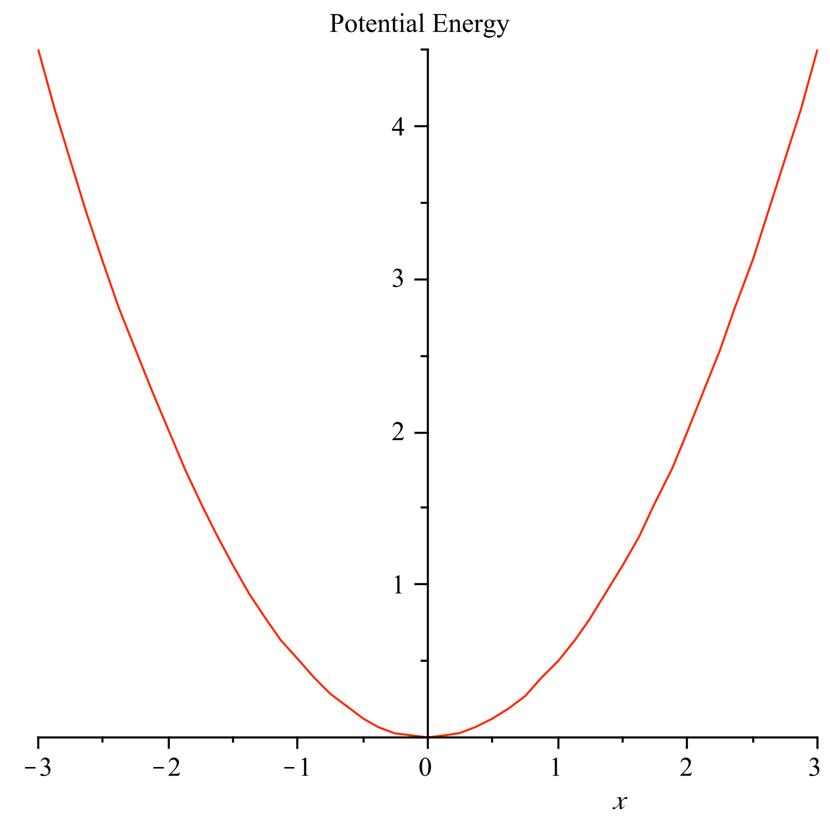

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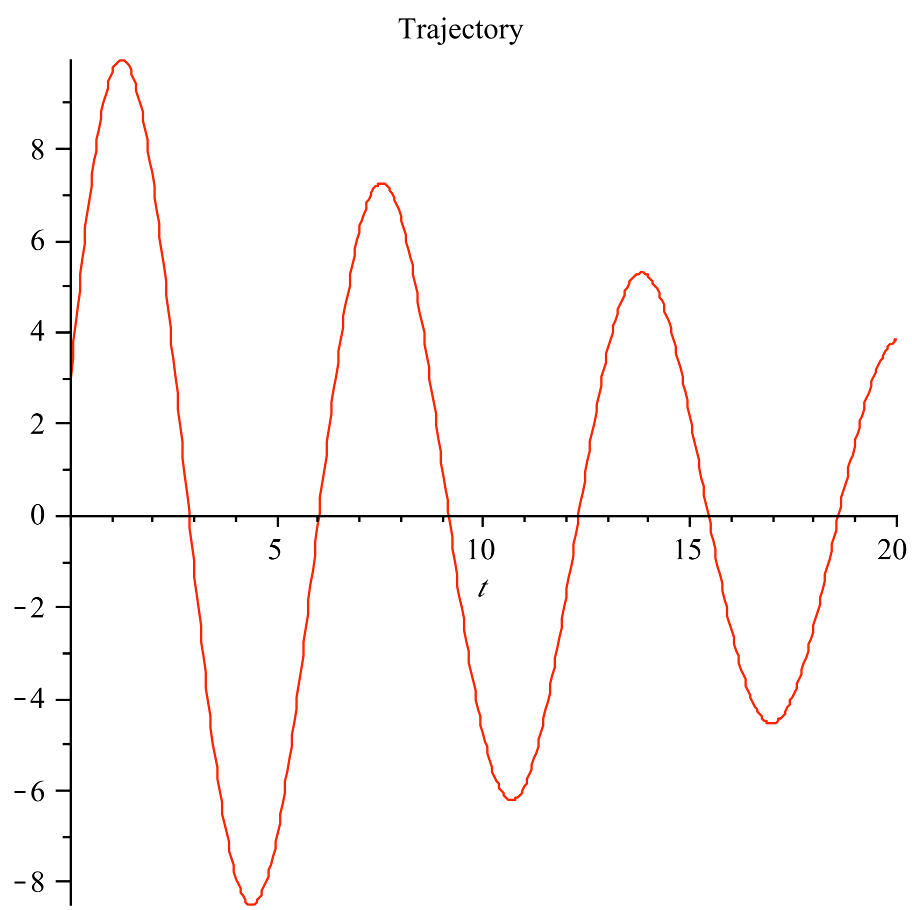
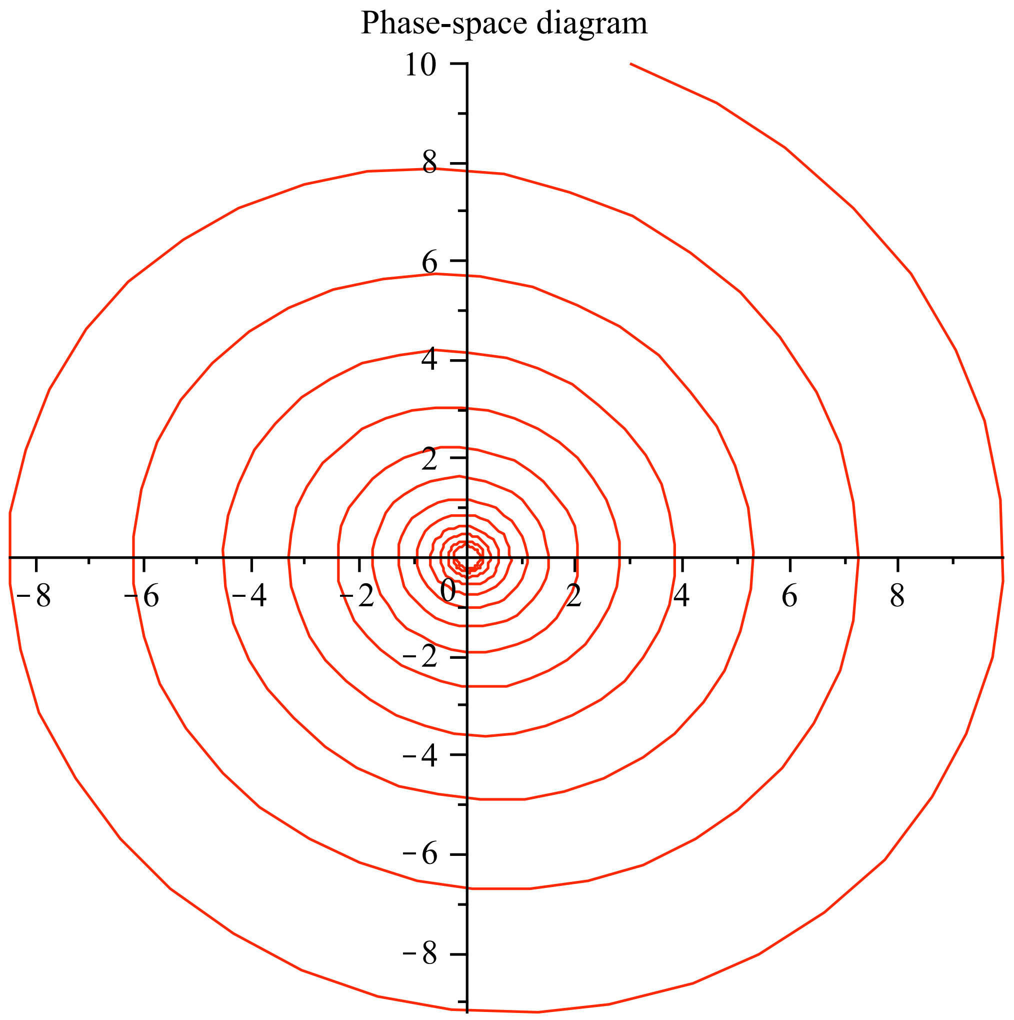
We can proceed to draw the solutions in phase space.

Harmonic Oscillator

- Left: trajectory for an arbitrary initial condition and $\gamma = 0.1$.
- Right: same trajectory in phase space.

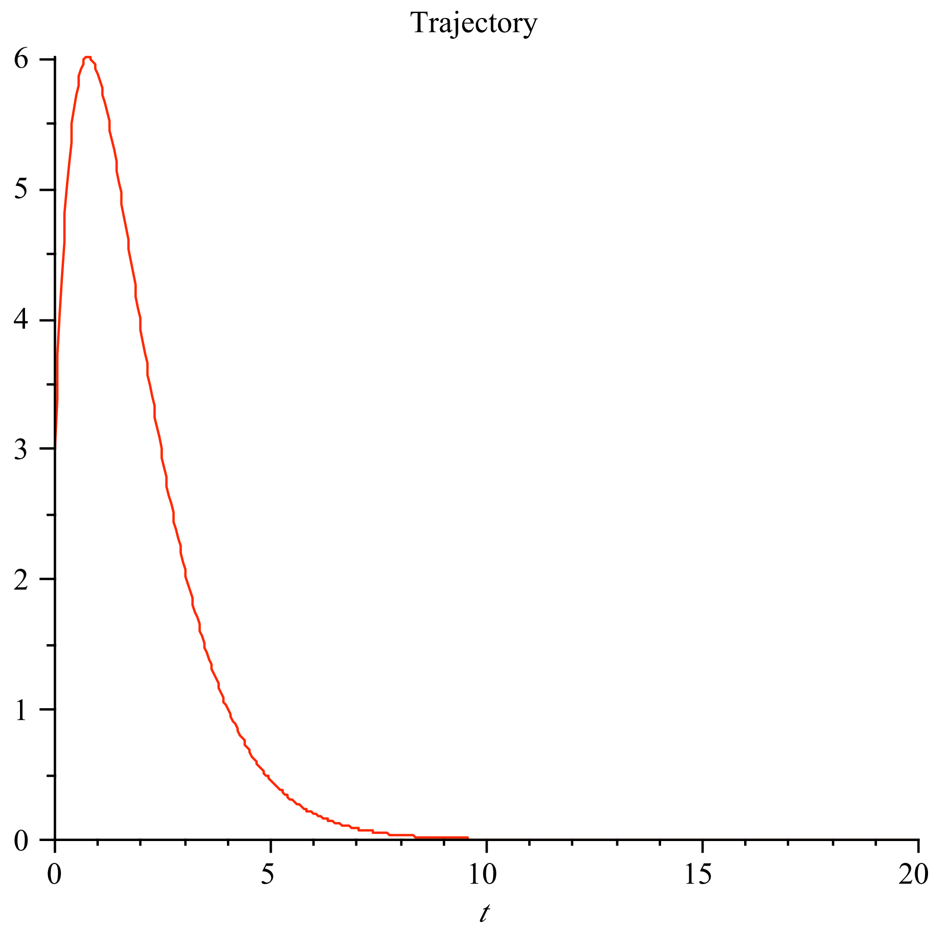


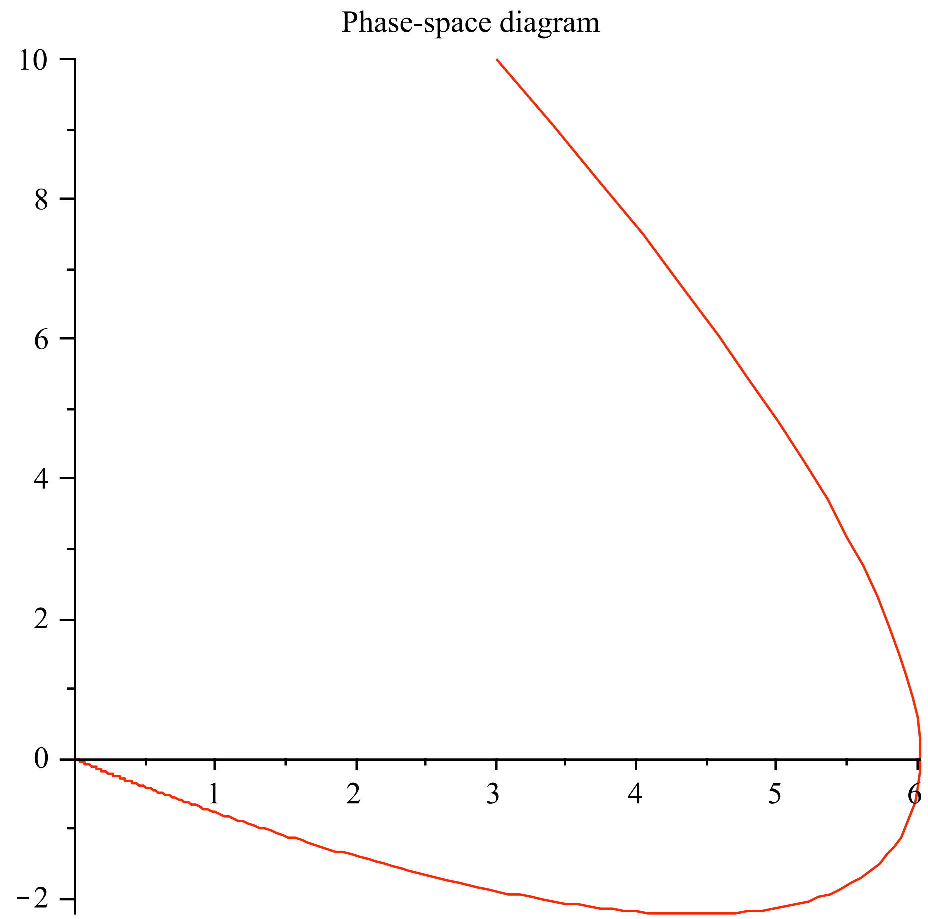


Harmonic Oscillator

- Left: trajectory for an arbitrary initial condition and $\gamma = 2$. The motion decays before any oscillations occur. The point x=0 is asymptotically approached.

- Right: same trajectory in phase space.





Duffing Oscillator

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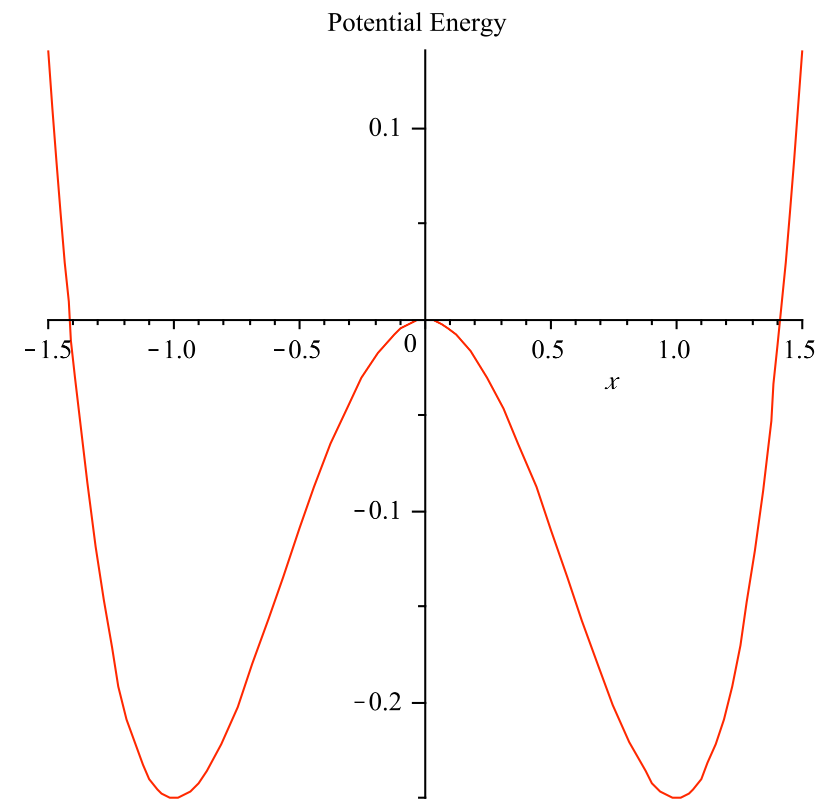

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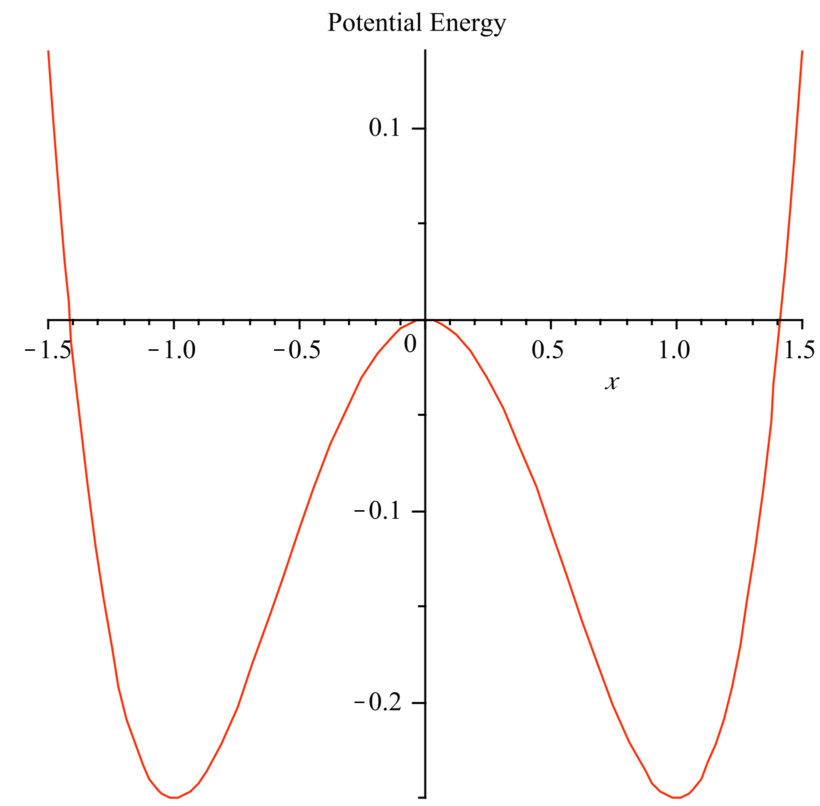

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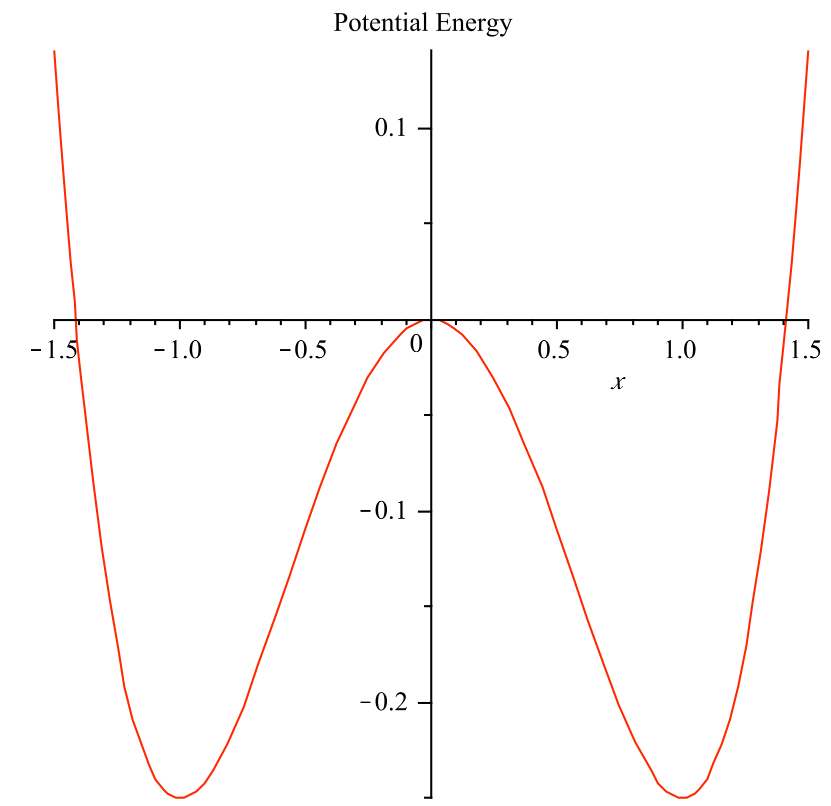

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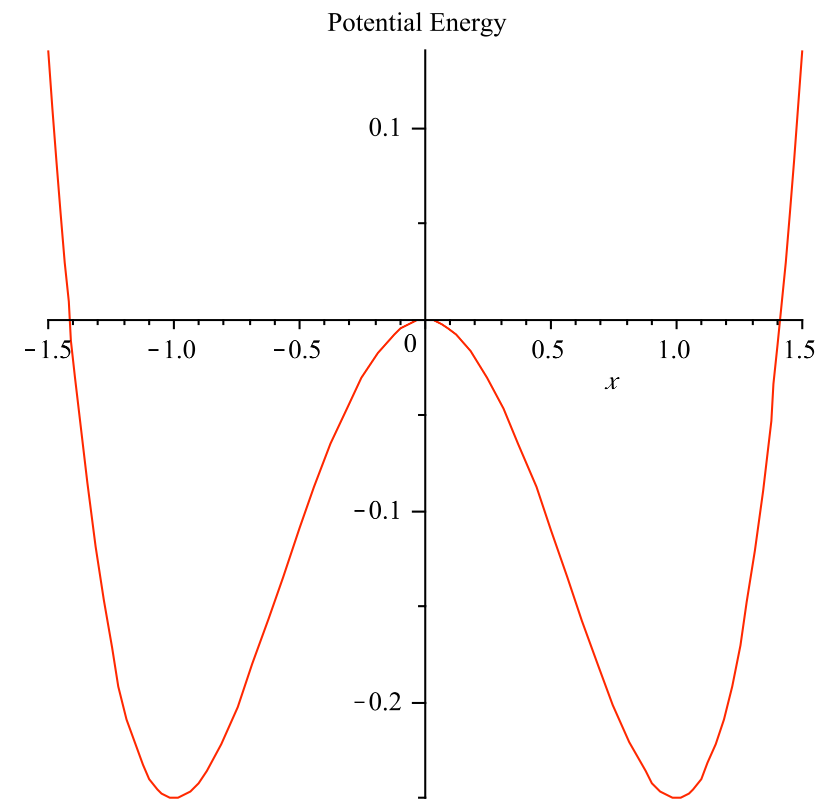
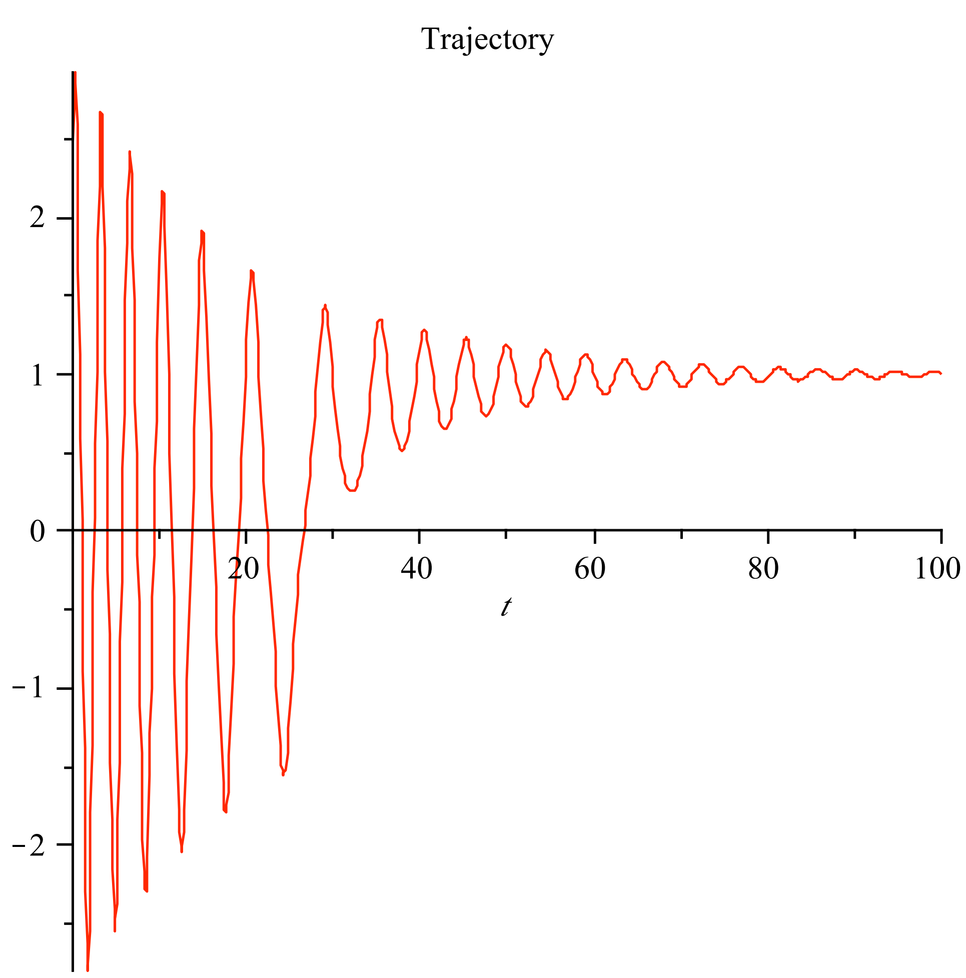
This system has the following properties:

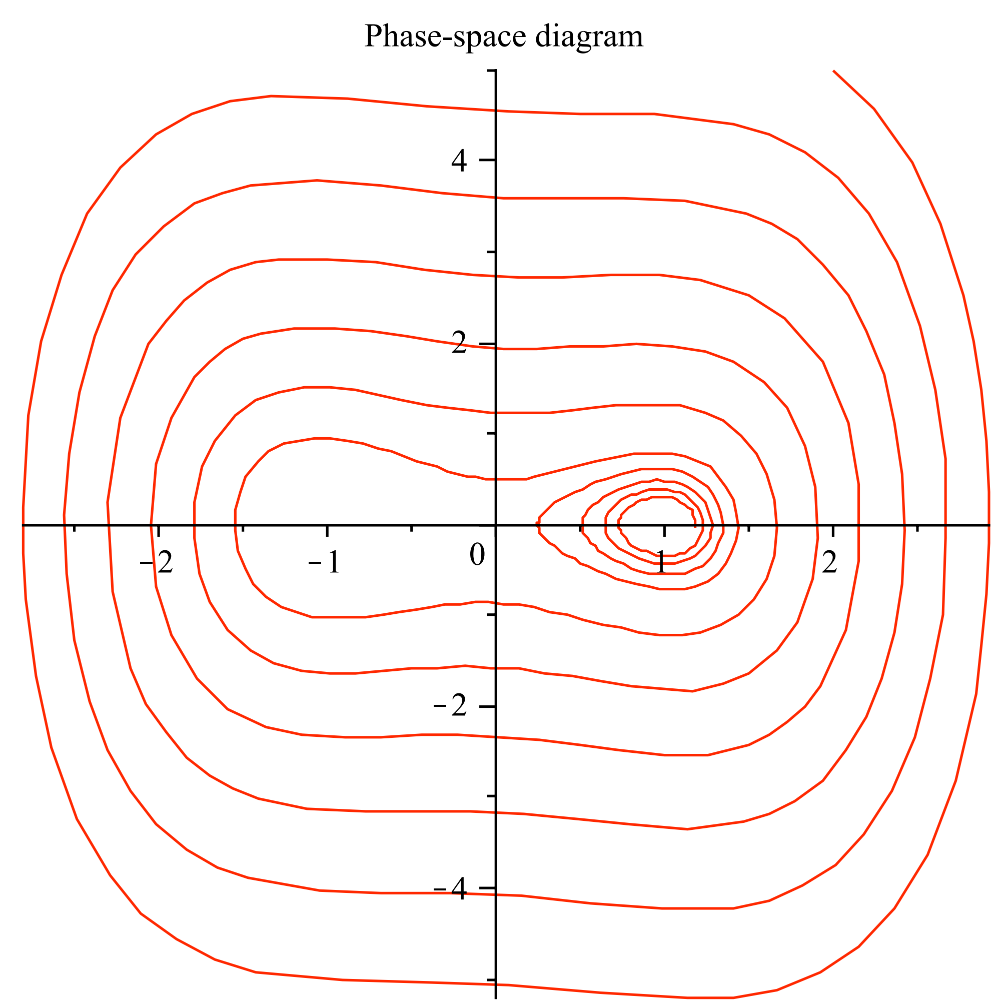
- Potential has a double-well structure.
- x =0 is an unstable equilibrium point.
- Particle motion ends in one of the wells.
- Unique phase-space trajectory per IC.
- Chaos appears as a result of the two wells connected by the unstable equilibrium point.



Duffing Oscillator

- Left: trajectory for arbitrary γ and initial condition. Particle falls into one well.
- Right: same trajectory in phase space. Trajectory converges to one side.





Phase Space Diagram

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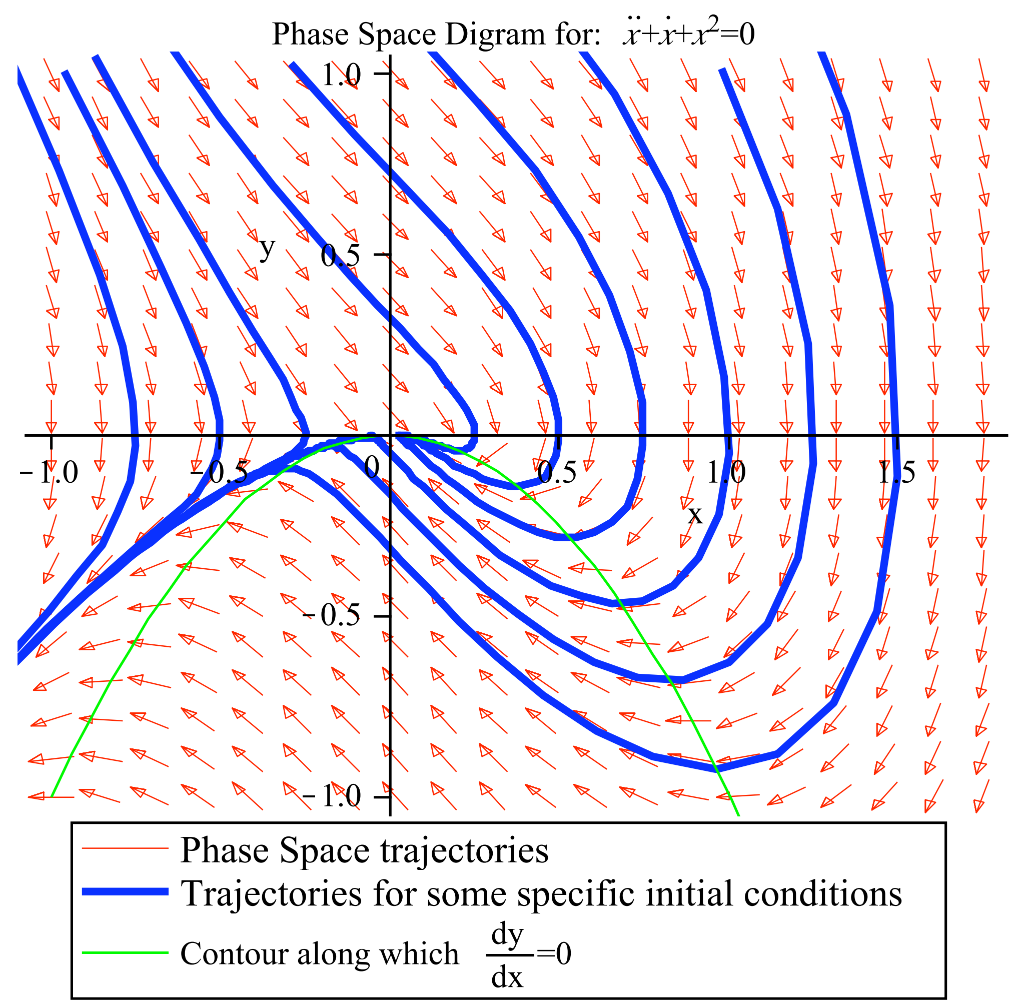
Reducing: $\ddot{x} + \dot{x} + x^2 = 0$ to:

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= -y - x^2\end{aligned}$$



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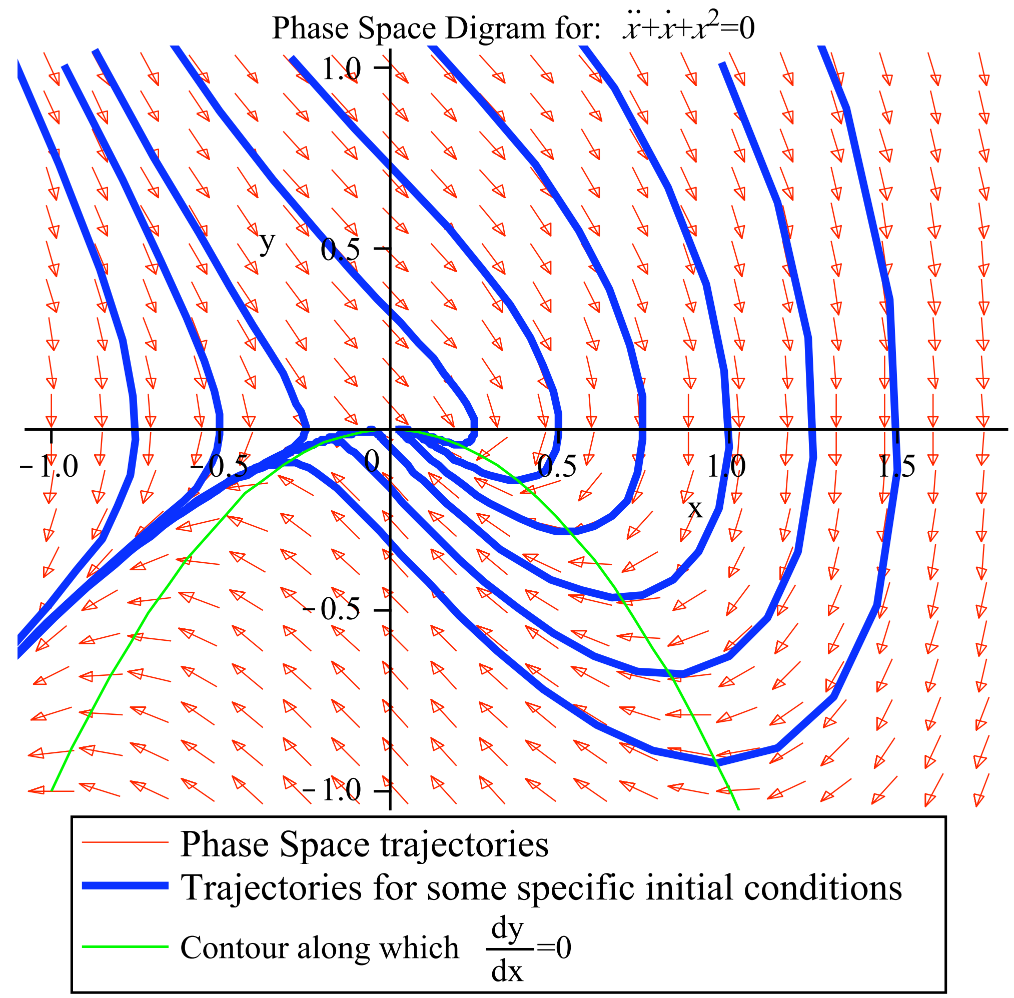

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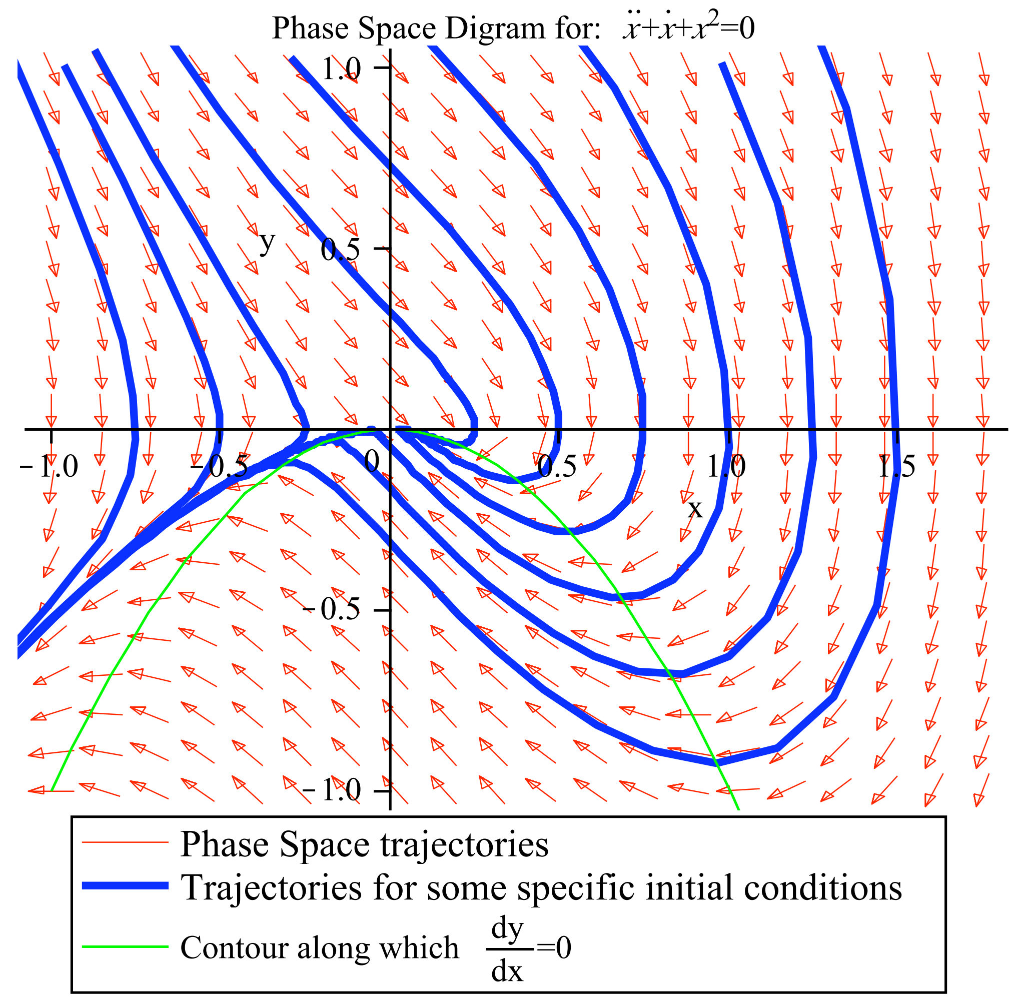

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we deduce:

- Crossing the x axis (y=0) the slope is always infinite.
- Crossing the y axis (x=0) the slope is always -1.
- Crossing the contour $y = -x^2$, the slope is always zero (for $x \neq 0$).
- For y<0, outside (inside) the contour $y = -x^2$, the derivative is positive (negative) and the trajectories are monotonically increasing (decreasing).

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- Thus, to find the critical exponent we have to glue the two solutions.

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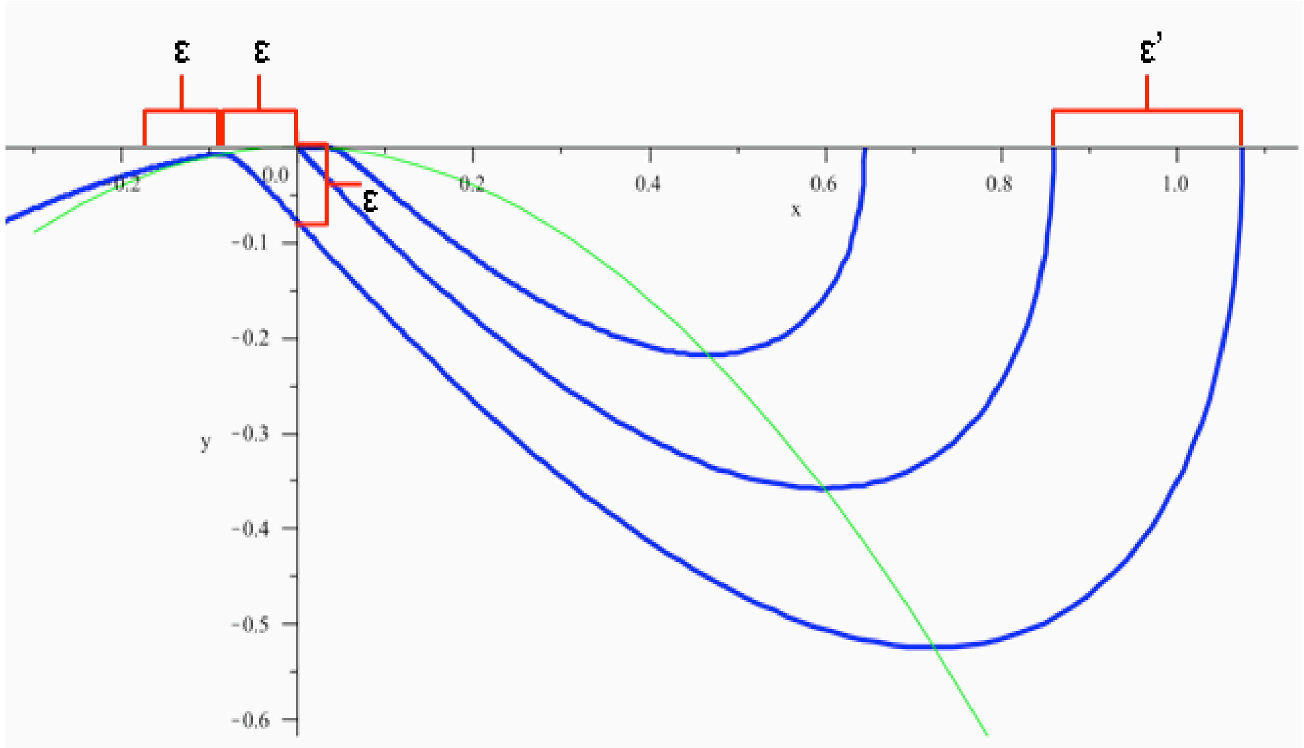
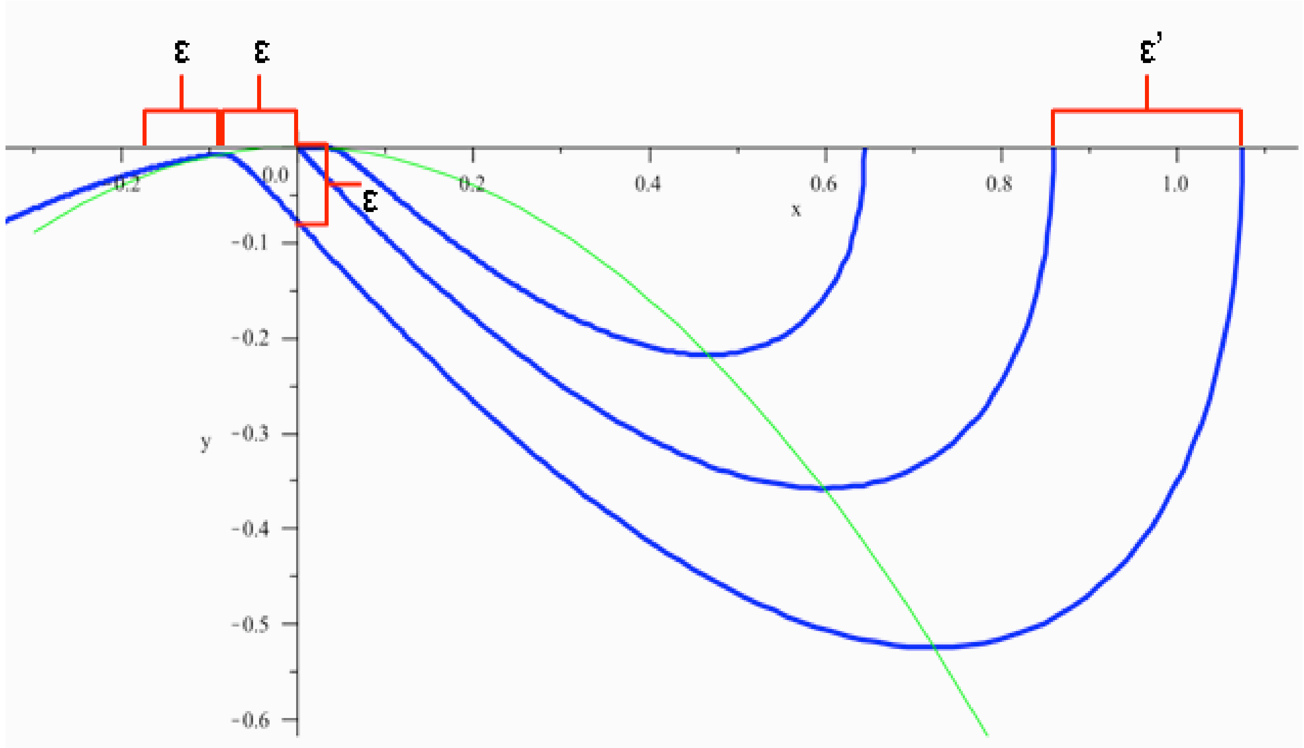


Gluing the Solutions

- We start with approximation (I):

$$x = ce^{-t} - \epsilon \quad , \quad y = -ce^{-t}$$



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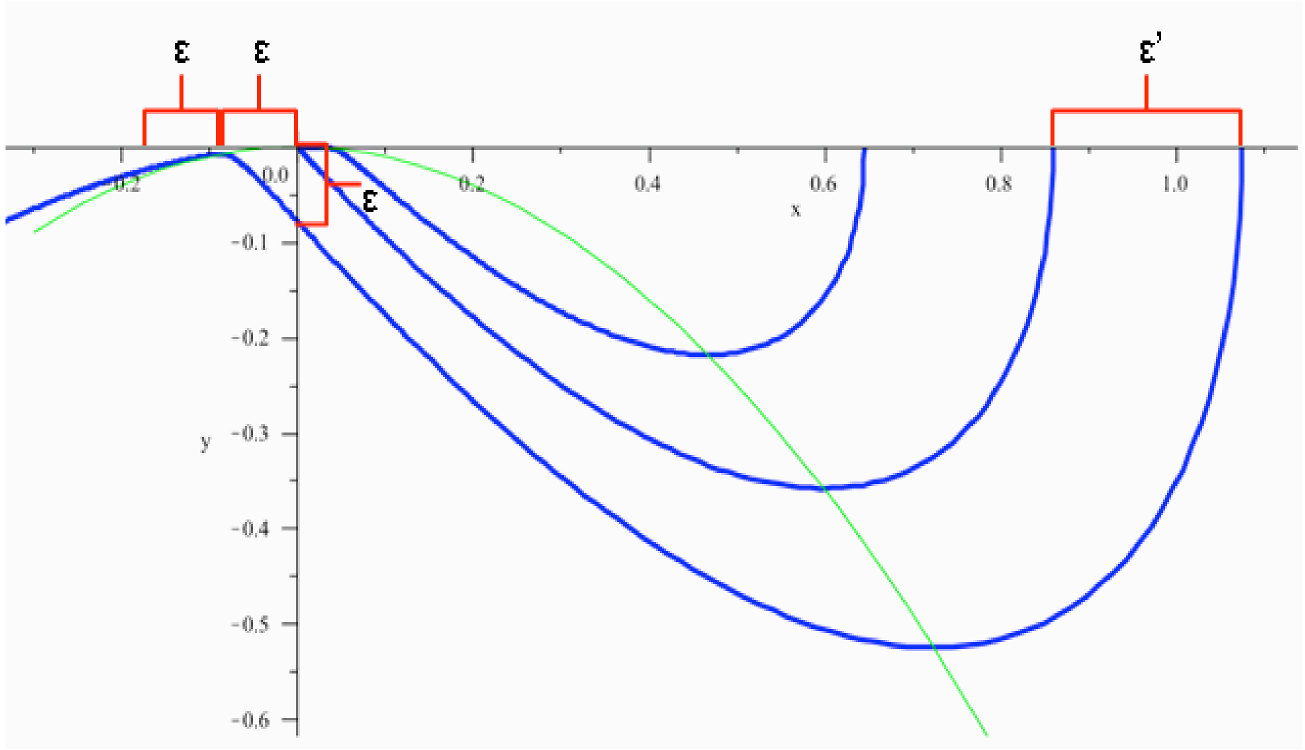

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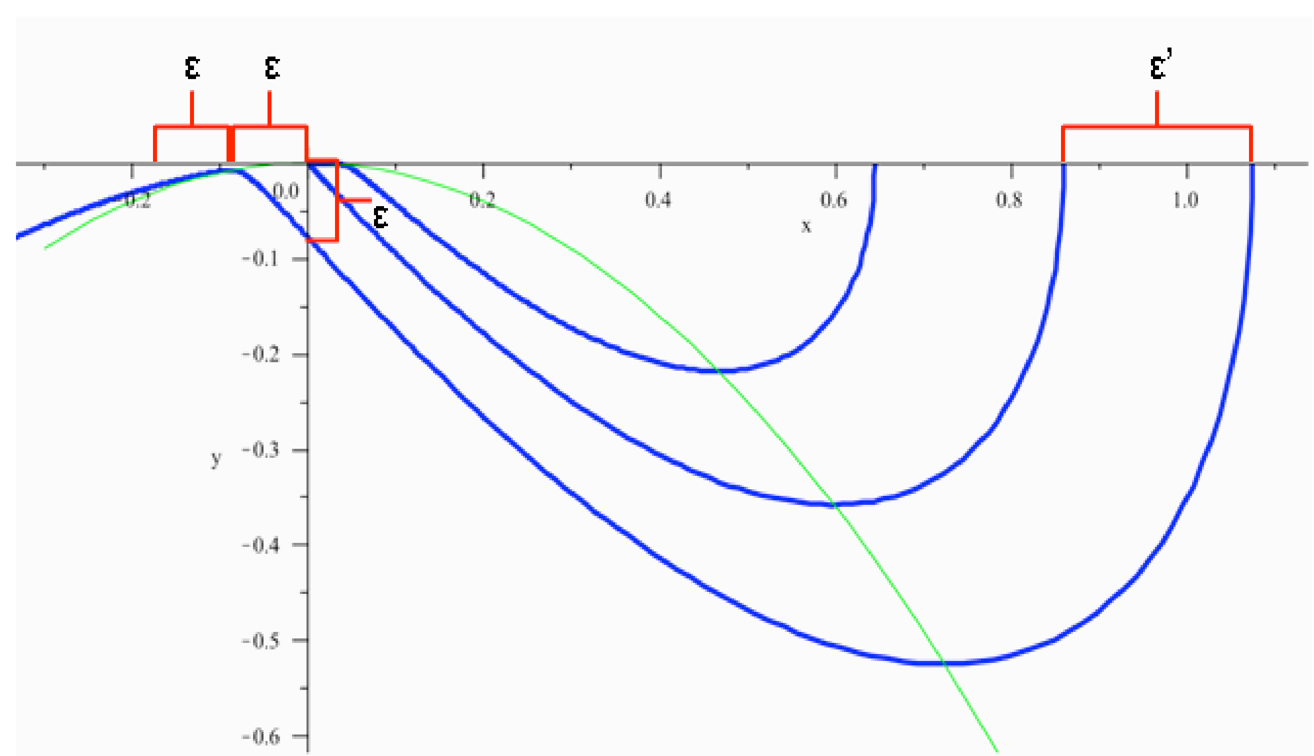

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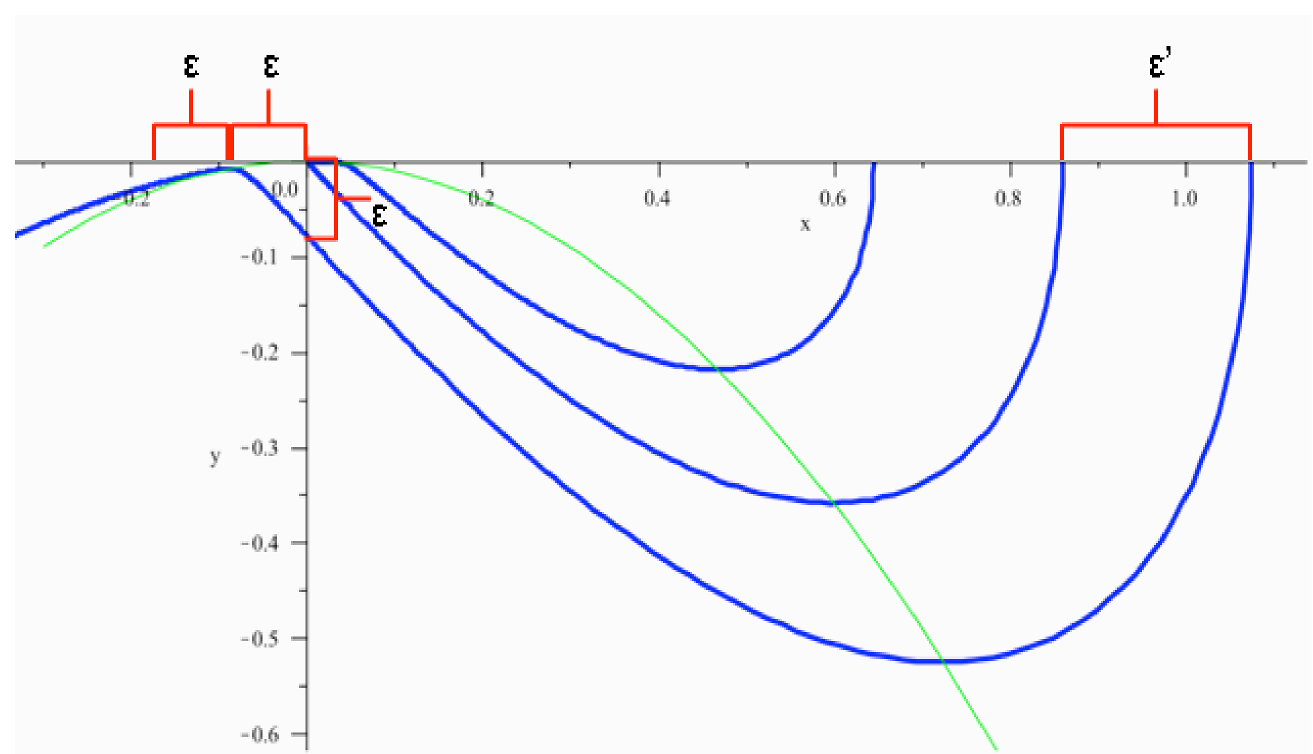
- Thus the first approximation is satisfactory up to $x \sim -\epsilon$, which, is in the vicinity of the curve along which approximation (II) is valid. However, as we saw above, the solution trajectory crosses this curve parallel to the y axis and therefore must deviate from it.



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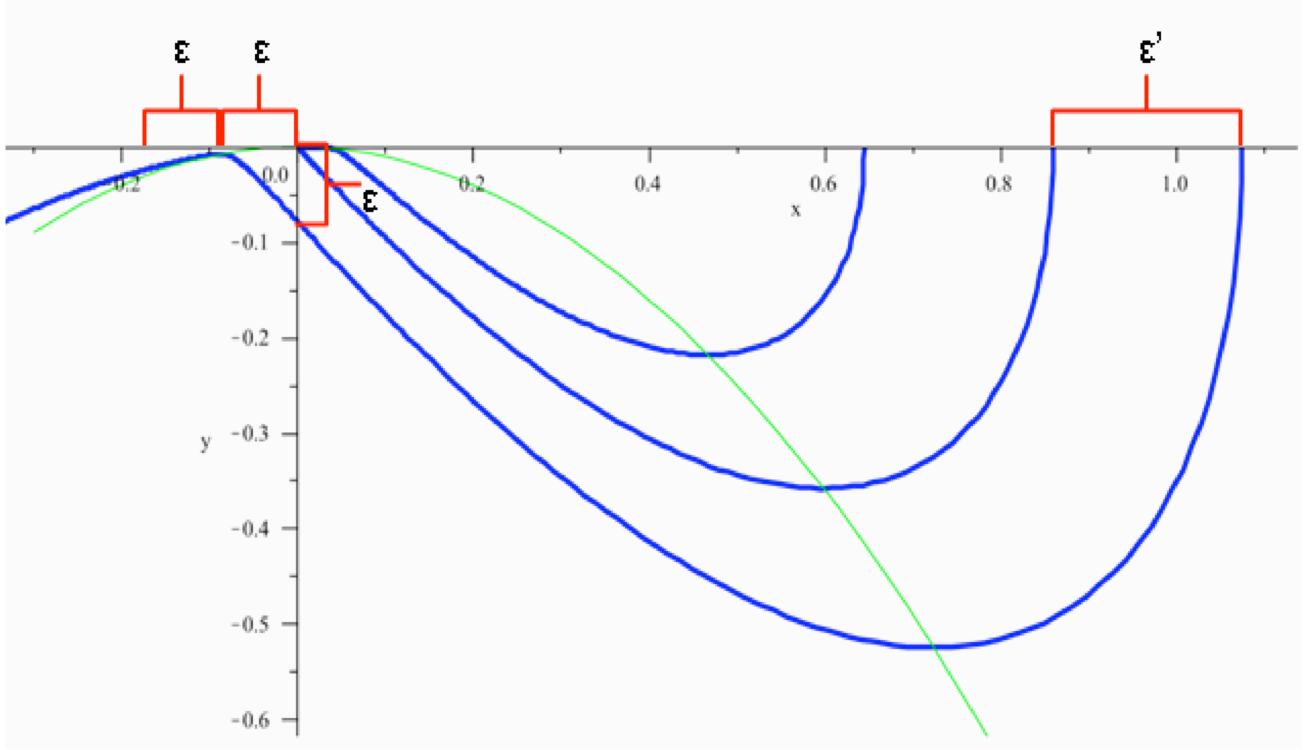

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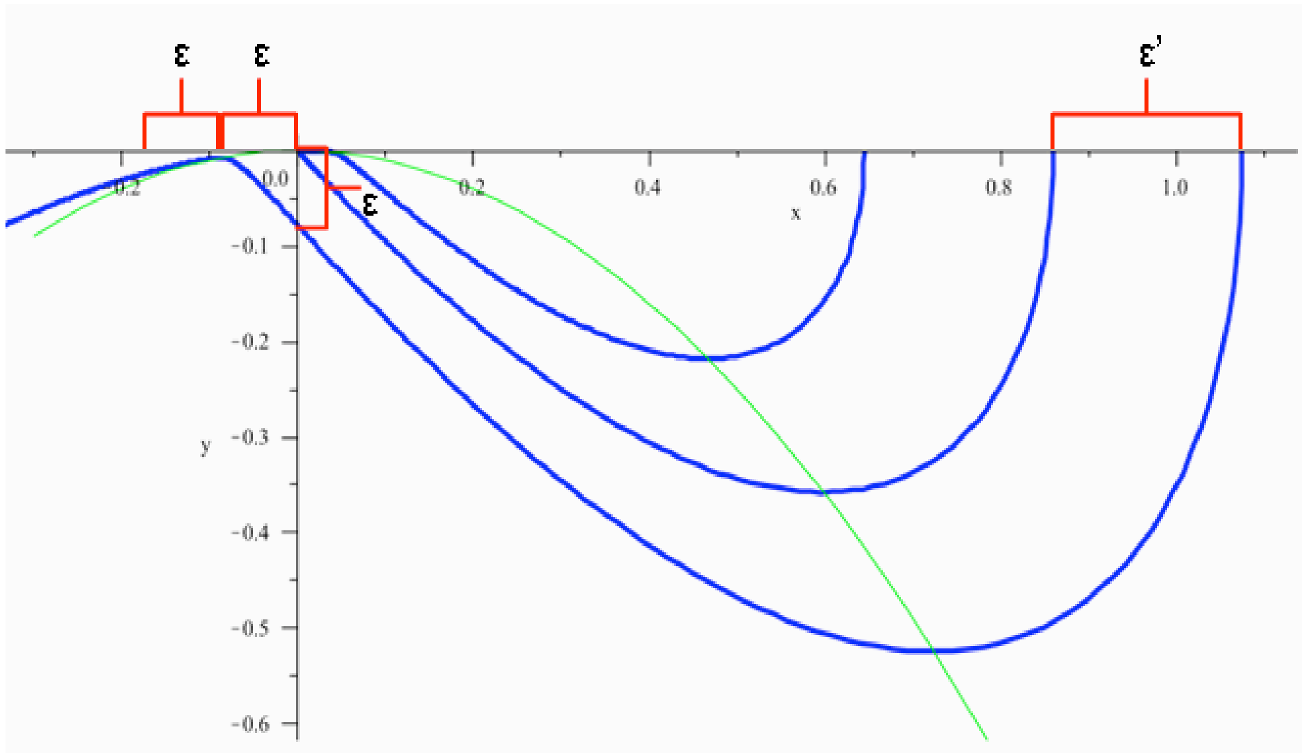

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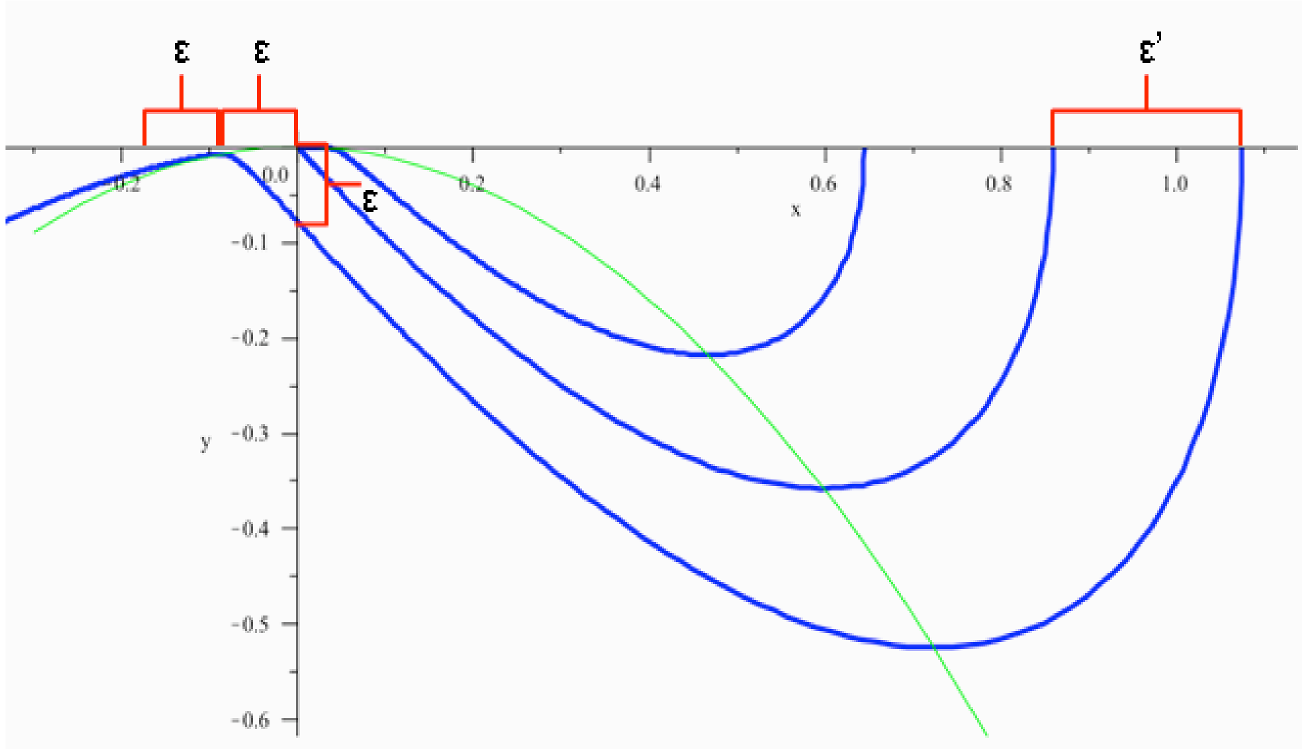

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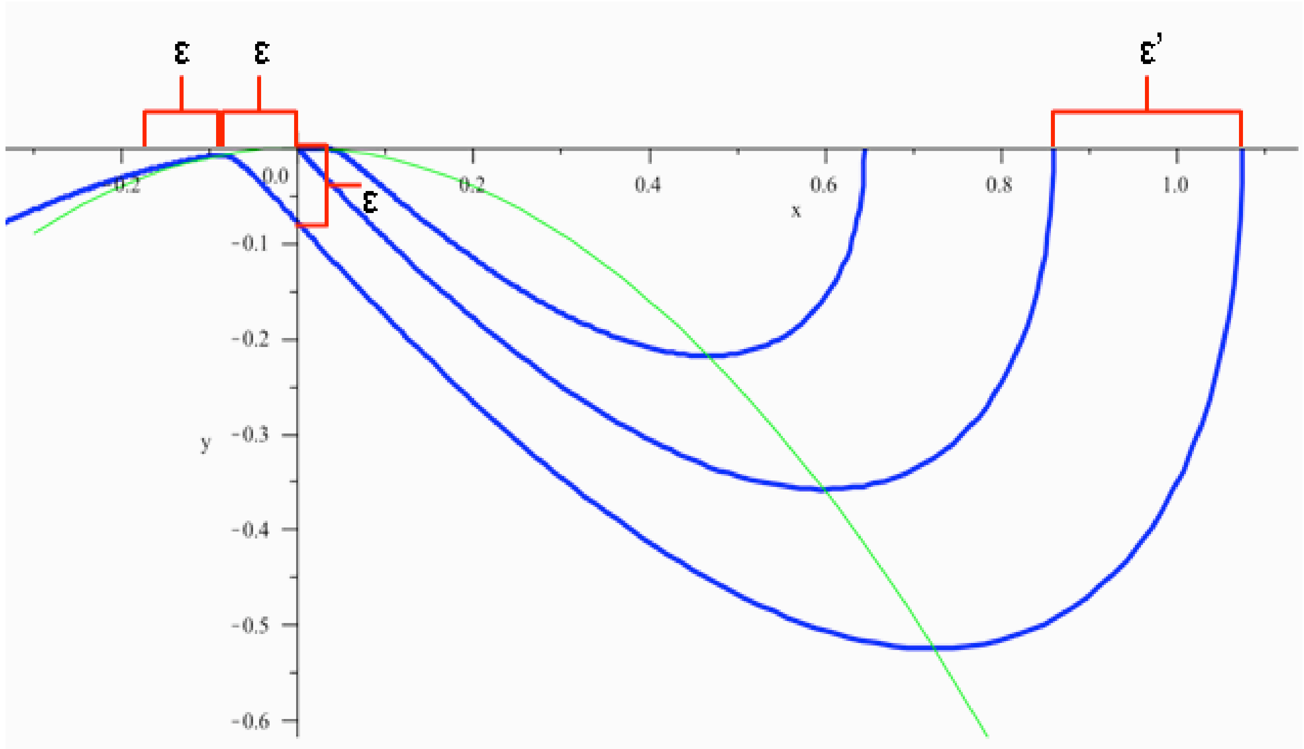

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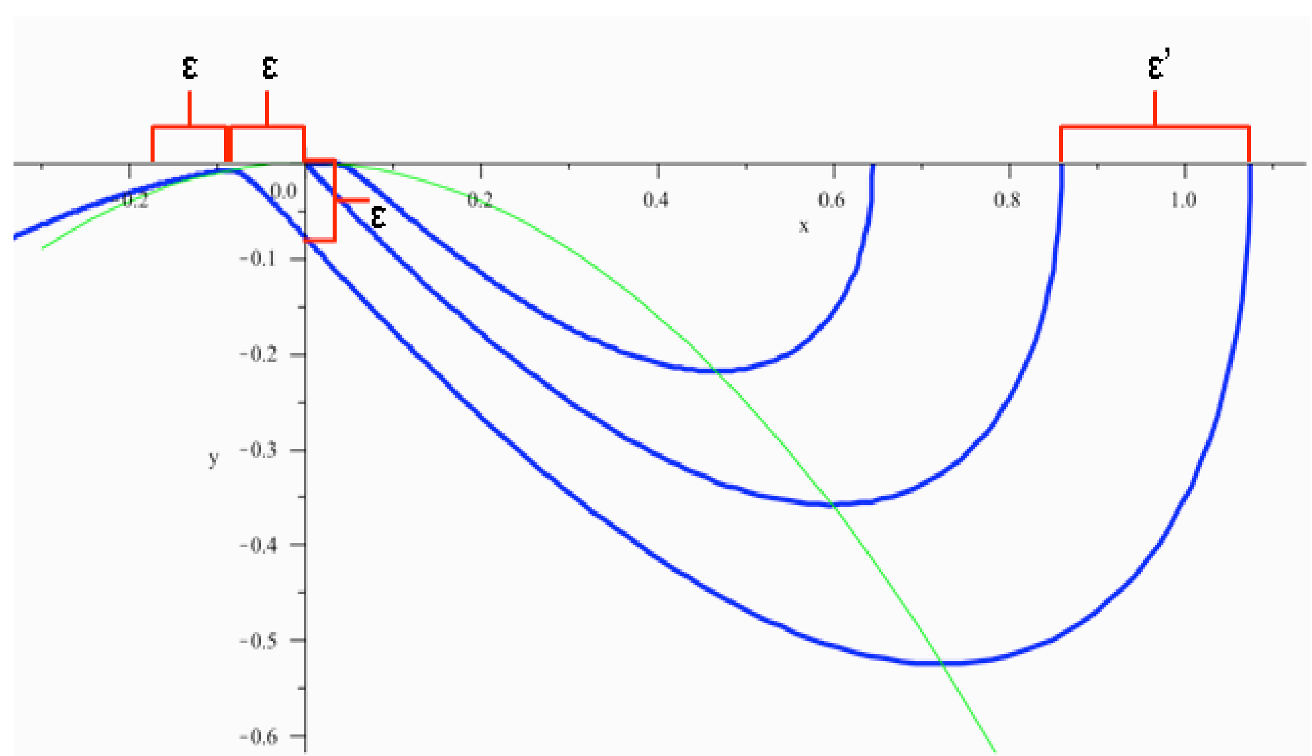

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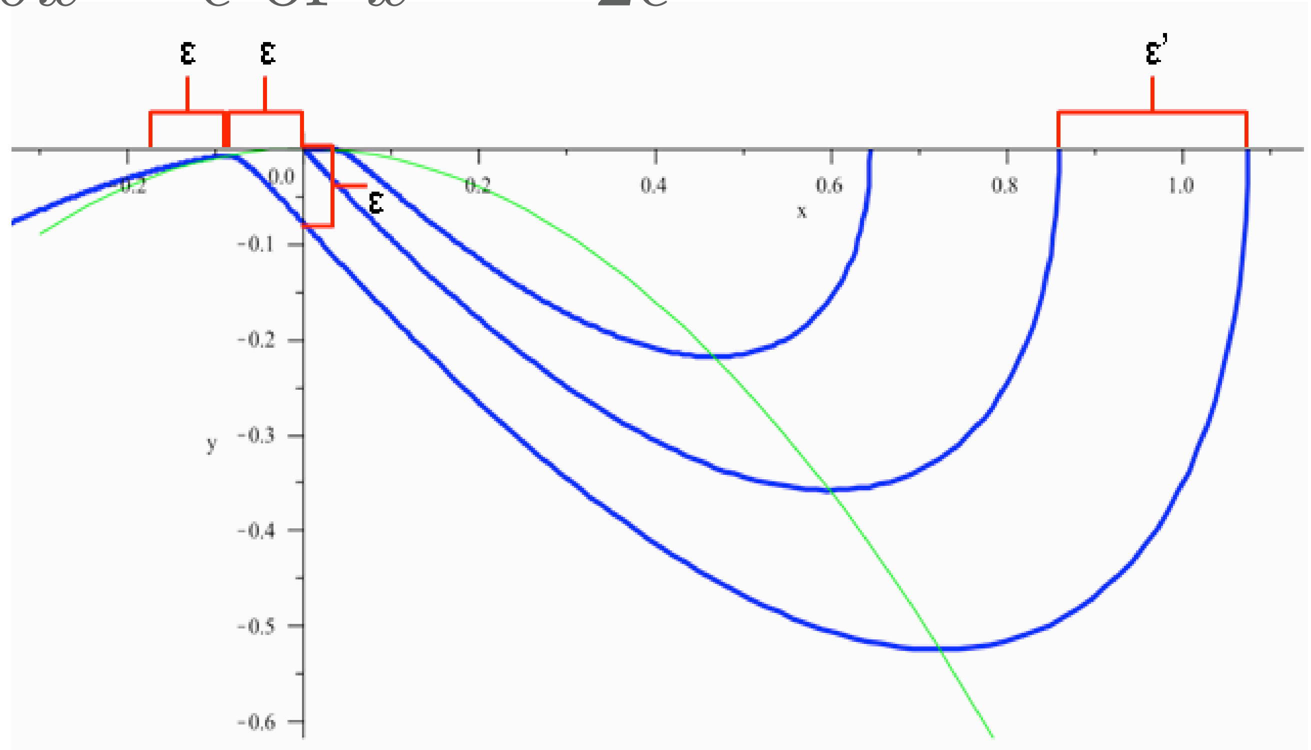

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- The remaining missing link is the relation between ϵ' and ϵ. Since the equations are regular in this region the relation between them is linear:

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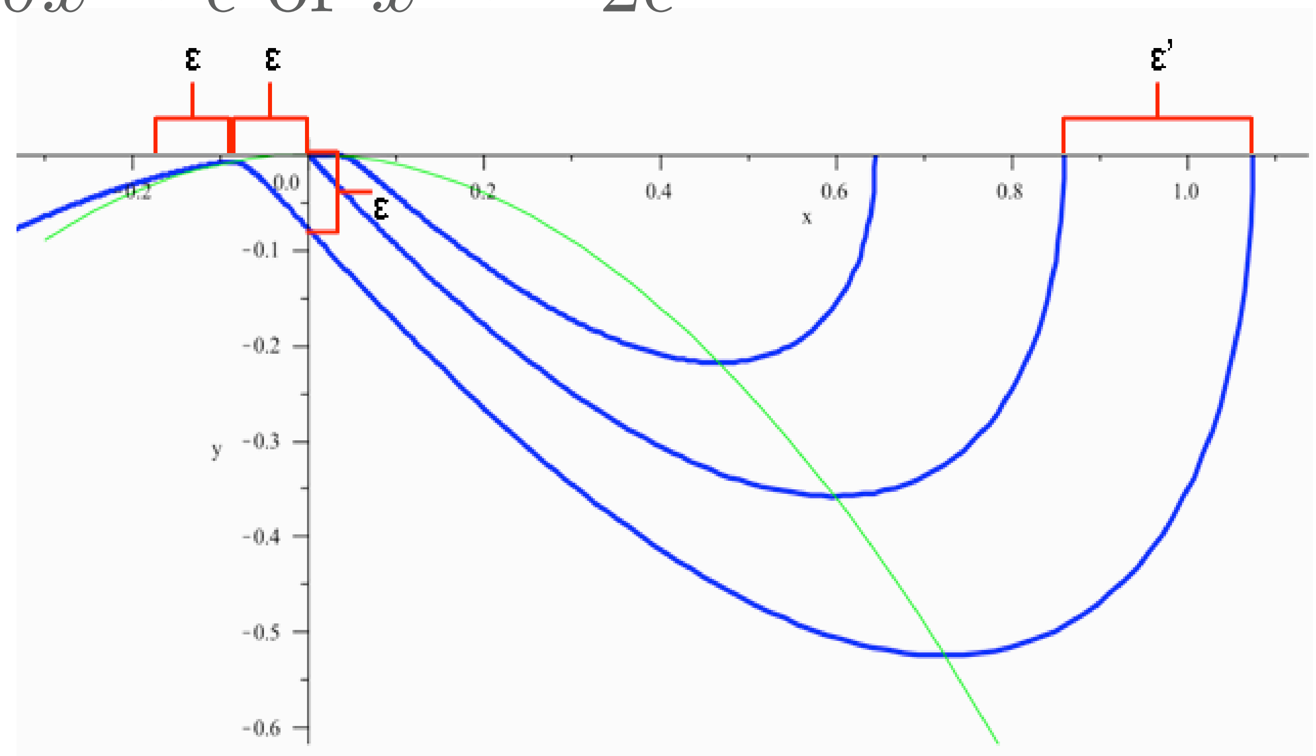

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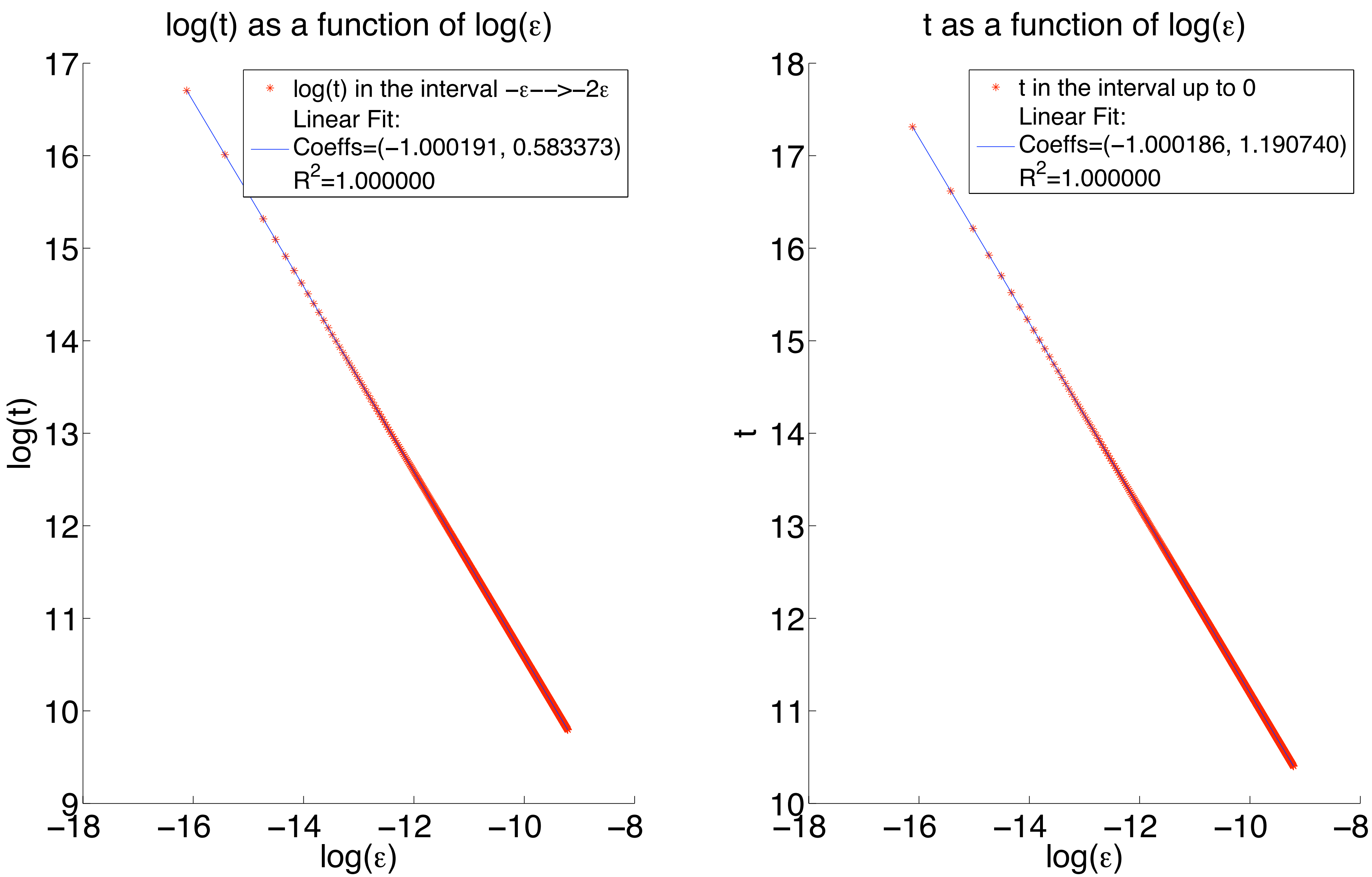

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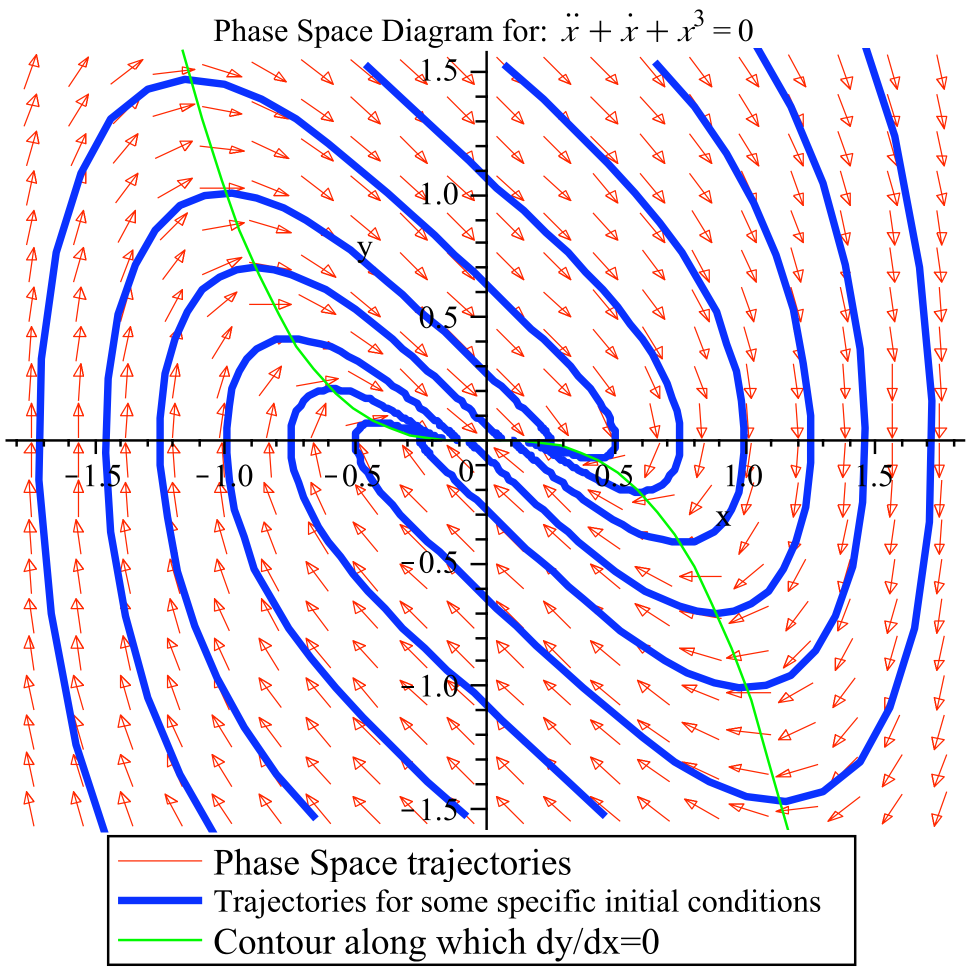

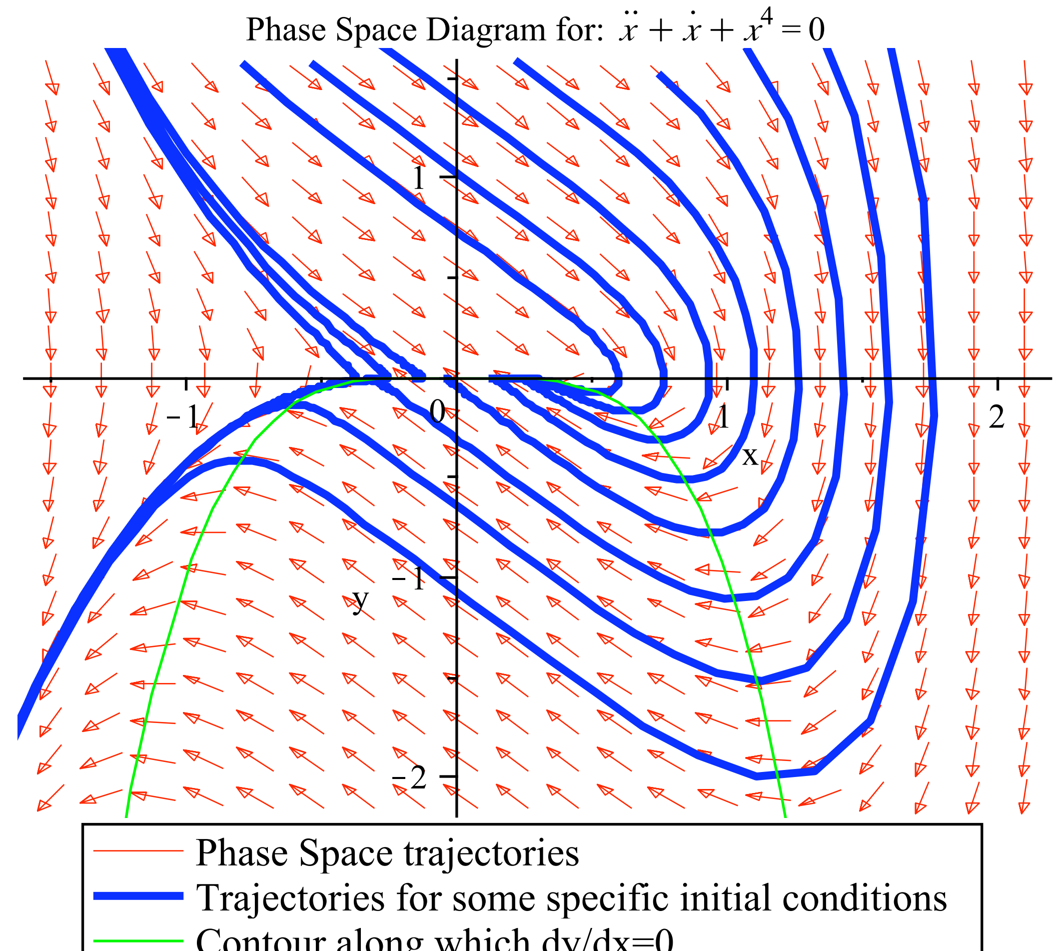

Outline

- The inflationary paradigm - reminder
- Large field and small field inflation
- Inflection point inflation (IPI)
- Phase transition (PT) in IPI
- Generalizations of the PT phenomenon
- Analytic discussion
- **Somewhat different PT in a stringy IPI model**

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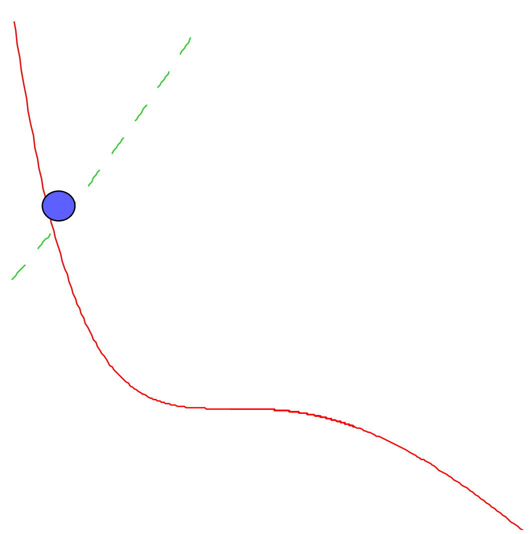

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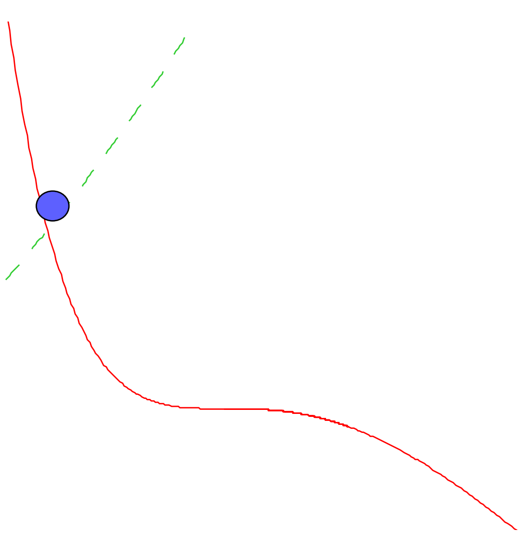

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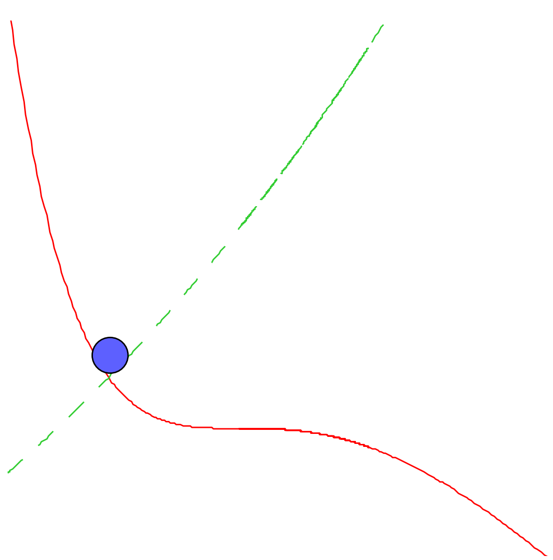

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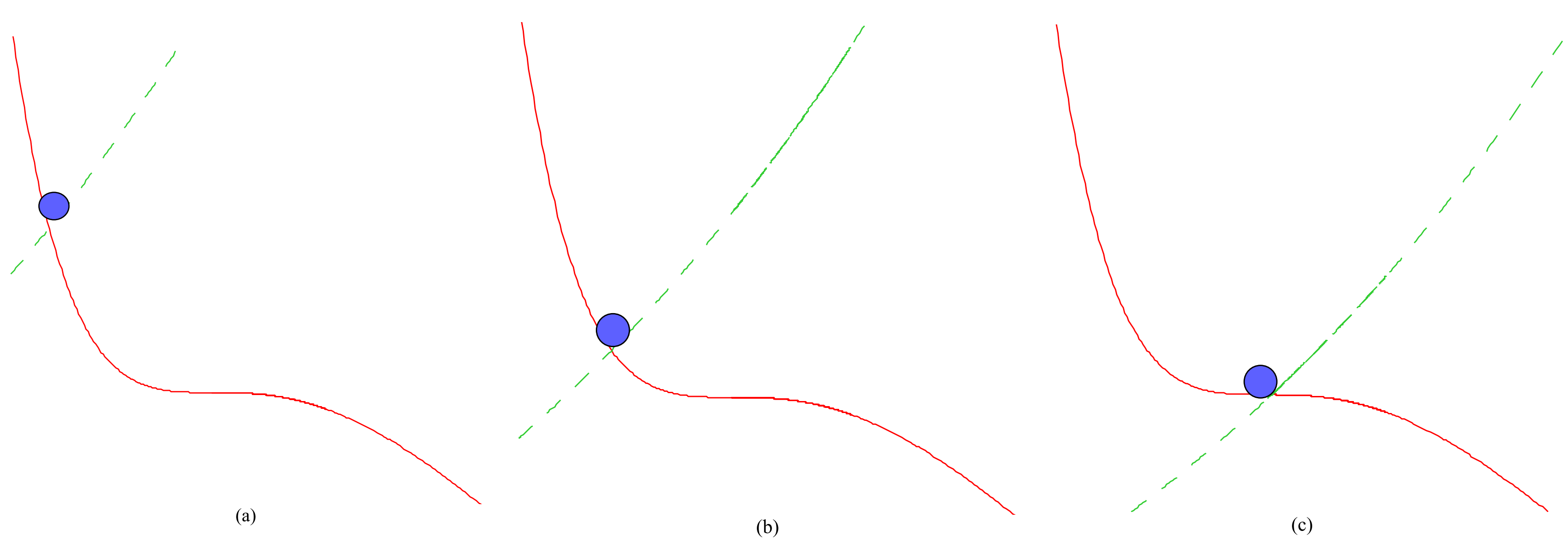


Time-Dependent Potential

- We focus on the non-perturbative states given by (0+p)-branes (branes that wrap p cycles in M and are point-like objects in the four dimensional uncompactified space-time). Their mass in the Einstein frame is:

$$m_{0+p} = T_{0+p} L^p \left(\frac{L_0}{L}\right)^{d/2}$$

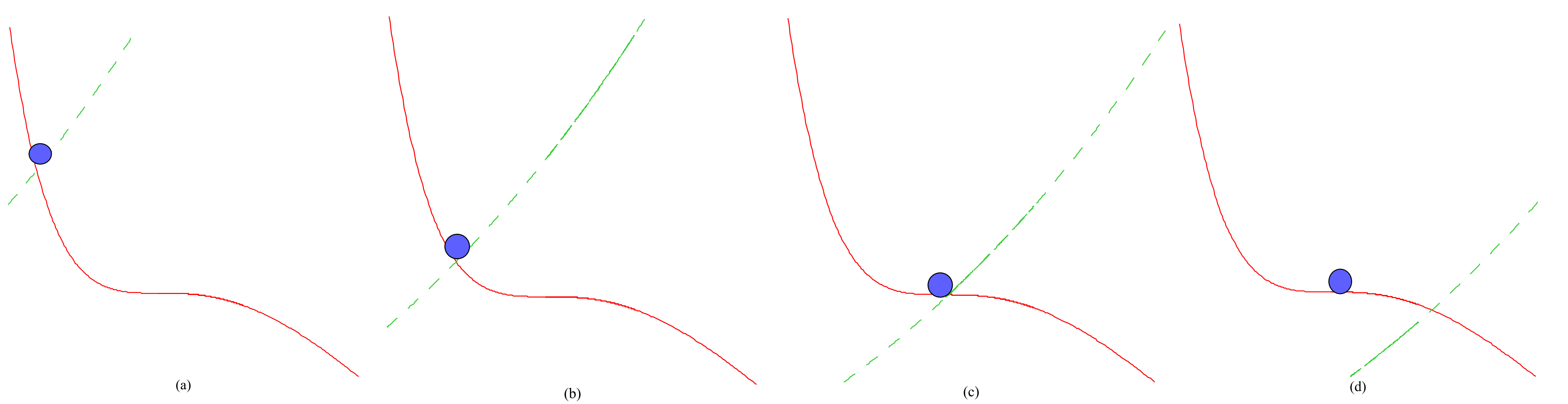
This leads to:

$$V_{0+p} = n_{0+p} T_{0+p} L_0^3 L^{p-3}$$

Which is a potential term which blow up as $L \longrightarrow \infty$ and turns into the term:

$$V_{dynamic} = n(t) \exp(3\alpha\phi)$$

- This is a natural solution to the overshoot problem, as can be seen below:



Phase Transition

Phase Transition

- Indeed, for different values of the inflection point, we find an initial density that is big enough to slow down the inflaton so that inflation occurs.

$L_{inflection} \backslash n_0$	$1.5e-05$	0.000315	0.00081	0.00541	0.099035
2	0.46215	0.46474	0.46909	0.51401	51130.0048
5	0.42172	0.45187	0.51436	41.6388	∞
8	0.42516	0.89218	44.8478	∞	∞
10	0.43725	53.3611	∞	∞	∞
20	384.1207	∞	∞	∞	∞

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- In addition, we also see scaling behavior as the critical n_c is approached, again with an integer critical exponent of -1.

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  - In the small field region, N depends on the initial condition and in order to obtain a large N the initial condition generally has to be fine tuned.
  - In the large field region the efficiency of the attractor mechanism is maximized and all non-singular initial conditions yield the same N.

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  - In the large field region the efficiency of the attractor mechanism is maximized and all non-singular initial conditions yield the same N.

- The surprising aspects of our findings are:
  (I) The transition between the two regions happens at *finite* values of the parameters that define the potential.
  (II) In some cases critical behavior takes place at the transition between the two regions.