Characteristics of Long-run Return and Risk: A Unified Performance Metric

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Abstract It is well documented in the literature that long-run asset prices do not follow a random walk, thus their returns are not independent and identically distributed (i.i.d.) over time. But how can this notion—long-run returns and volatilities being horizon dependent—be incorporated into formal pricing models? In this paper, we develop a unified risk-adjusted return metric that is applicable to both private assets and public securities. Since such metric is based on a pair of empirically determined return and risk characteristic lines that depict the horizon impact on return and volatility, our performance metric is rooted in empirical evidence rather than assumptions. The results suggest that long-run asset performance cannot be adequately measured by single-period return and volatility. Rather, prudent long-run investment decisions must include careful consideration of the anticipated holding period and a proper understanding of the long-run return and risk characteristics.

The purpose of this paper is to develop a generalized investment performance metric to facilitate performance comparison and portfolio diversification across private and public asset markets. More specifically, we propose a unified risk-adjusted return measure that is based on a unified model of long-run buy-and-hold investment process that is valid for both private real estate and public financial assets.

Ironically, it can be said that the performance measures have always been “unified,” in the sense that real estate investment analysis has traditionally relied on the pricing models and theories that are developed for financial assets. That is, investment return and risk have long been measured in the same way for both private and public assets, despite the many obvious differences between the two types of assets and their respective markets. This kind of practice is akin to the proverbial apples to oranges comparison, and it has resulted in some peculiar findings when real estate performance is compared to stocks. For example, one such finding is the so-called “real estate risk premium puzzle,” which refers to the widely-documented observation that real estate exhibits comparable returns but much lower risk (volatility) relative to stocks and bonds. Another is often
known as the “real estate allocation puzzle,” which refers to the large gap between academic findings and institutional realities with regard to the optimal weight of private real estate in mixed-asset portfolios.

The nonsensical findings resulted from treating real estate the same as stocks has motivated researchers to develop alternative performance metrics and pricing models that are unique for real estate. Lin and Vandell (2007), Lin and Liu (2008), and the series of works by Cheng, Lin, and Liu (2008, 2010, 2013, among others) have demonstrated that the finding of extremely low risk of real estate is a myth caused by the failure of a conventional risk metric (standard deviation of returns) to capture the unique risks involved in real estate, namely the liquidity risk and the horizon dependence [as opposed to independent and identically distributed (i.i.d.)] of property returns.¹ By explicitly quantifying and integrating illiquidity into the traditional risk metric, these researchers propose distinctly different performance metrics for real estate, which provide plausible solutions to the long-standing real estate risk premium puzzles, and have led to alternative portfolio theory that extends the classical modern portfolio theory (Cheng, Lin, and Liu, 2013).

The distinctly different performance metrics for real estate, however, implies that the private asset and financial securities are more than two separate asset classes standing side-by-side; they are effectively in two different worlds: the i.i.d. world versus the non-i.i.d. world, so to speak. The i.i.d. world is the classical finance paradigm of an efficient market, where security asset returns being i.i.d. is a critical assumption that enables classical works by Sharpe (1964), Lintner (1965), Black (1972), Merton (1973), Ross (1976), and Breeden (1979), among others, develop single and multi-beta asset pricing models, which imply that the expected return on a security is a linear function of factor risk premiums and their associated betas. The non-i.i.d. world is the framework of those afore-mentioned studies on private assets that do not utilize the i.i.d. assumption.

However, there are good reasons to believe that the two worlds need not be separated by the i.i.d. assumption, especially when long-run performance is examined within a buy-and-hold framework. For example, Fama and French (1992, 1993, and others) have shown that “the relation between market beta and average return is flat, even when beta is the only explanatory variable.” Instead, Fama and French find that empirically determined variables such as size and book-to-market ratio can better explain the cross-section variation of stock returns. Barber and Lyon (1997), Kothari and Warner (1997), and numerous others show that the long-run behaviors of common stocks are quite different from their short-run performances. In other words, if it can be argued that short-run stock returns are reasonably i.i.d., the long-run stock performance very likely violates such assumption.

Such a notion prompts us to seek performance metrics without the i.i.d. assumption. That is, to bring the financial assets out of the surmised i.i.d. world to join real estate in a non-i.i.d. world, where a simple buy-and-hold investment
strategy model applies to both assets, and a unified performance metrics can be developed. This is a new process in the literature because, while it is significant that the earlier works took real estate into a non-i.i.d. world while leaving the financial assets in the classical finance i.i.d. world, with each world having its own performance metrics, a fair comparison between stocks and real estate actually requires the performance metrics to be the same. We accomplish this task in this study. In doing so, we also improve some aspects of the earlier works. For example, the performance metrics proposed by earlier studies focus only on the risk, not the returns. They demonstrate that risks are non-i.i.d but assume returns are still i.i.d. We empirically demonstrate that both long-run risk and return are horizon dependent, and so should be the risk-adjusted returns.

We approach the task in these steps: First, we empirically examine the long-run behavior of various financial and real assets. These examinations produce what we called characteristic lines of return and risk (CL-return and CL-risk, respectively) for each asset. Based on the characteristic lines, we formulate alternative models of holding-period return and volatility that do not rely on the classical i.i.d. assumption. Second, we examine a simple buy-and-hold investment strategy, and discuss the roles of investment horizon and illiquidity in formulating the ex ante expected return and risks. We then present the development of a unified performance metric in terms of risk-adjusted returns. We also demonstrate the application and computation of the new performance metric with real world data. We close with concluding remarks.

**Impact of Holding Period on Long-run Asset Performances**

In classical asset pricing models, it is generally assumed that, in an efficient market, asset returns are i.i.d. Under this assumption, the return and risk of holding an asset for one period is the same as the per-period risk of holding the asset over multiple periods. Investment horizon, as well as the timing of investment, is therefore irrelevant. Consequently, the same inferences should be obtained regardless of the time intervals of the data being analyzed. However, empirical research has well documented that investment horizons do matter and that long-run asset performance tends to be quite different from their short-run behaviors. Levy (1972) was perhaps the first to examine the horizon impact on investment risk. The author documented that the CAPM betas obtained using data of different time intervals are significantly different. Poterba and Summers (1986) and Lo and MacKinlay (1988) document that asset prices do not follow random walks and that portfolios of different sizes tend to have different correlation structures. Keim and Stambaugh (1986) and Campbell (1991) show that stock prices exhibit various degrees of predictability, as opposed to a random walk. Chou, Hsu, and Zhou (2000) examine the cross-sectional variations of stock returns over different time intervals and find the CAPM beta to be insignificant in explaining monthly stock returns but is reasonably effective in explaining returns over longer intervals (semiannual or annual).
If the random walk assumption describes the price behavior in an efficient market, the inefficient real estate market cannot, and should not exhibit such characteristics. Indeed, there is ample evidence documenting that real estate returns do not satisfy the i.i.d. assumption. For example, Case and Shiller (1989) test the efficiency of the housing market in the United States and find that housing returns are serially correlated and partially predictable. Englund, Gordon, and Quigley (1999) analyze a large sample of Swedish housing prices and reject the random walk hypothesis in favor of a first-order serial correlation. Gao, Lin, and Na (2009) use two large panel datasets and find that house prices often exhibit serial correlation and mean reversion. Leung (2007) constructs an equilibrium model and finds that the autocorrelations and cross-asset correlations of the equilibrium prices and return of assets (housing and equity) will be non-zero. In the commercial real estate market, Young and Graff (1995) find that the NCREIF property index returns are neither normal nor stable over time. More recently, Cheng, Lin, Liu, and Zhang (2011) use the BDS test developed by Brock, Dechert, and Scheinkman (1987) to conduct a direct test for the null hypothesis of i.i.d. in time series data. They apply the BDS test to a wide variety of asset classes, such as common stocks, REITs, commercial real estate, and the residential market. Their results confirm that real estate is not i.i.d., and reveal that the real estate market exhibits are more divergent from the i.i.d. condition than that of stock markets.2

Given this body of research, there is a strong reason to believe that asset return and risks are horizon-dependent, regardless whether they are real estate or securities. In this section, we empirically examine the horizon impact on asset return and risks.

We select several major asset classes to investigate including common stock indices such as the Dow Jones Industrial, S&P 500, and NASDAQ, as well as the NCREIF national property index and the OFHEO housing index.3 The choice of these indices is intended to be representative, rather than exhaustive. The purpose here is to establish a reasonable empirical foundation for the theoretical development in the next section. The indices begin at different times and are reported for different time intervals; some are available daily, others only quarterly. For easy calculation and comparison, we chose quarterly indices for all the above asset classes. All the indices are obtained from public sources. The time span is 1977:Q4–2007:Q2.4

To examine the holding period impact on asset return and risk, we use the quarterly index data to simulate a buy-and-hold strategy where the holding period ranges from 1 to 36 quarters. Taking an index as the original sample, the simulation goes in the following steps. Starting with a certain holding period, say one year, we randomly pick any quarter to “buy” the portfolio at the index level, then we “sell” at the index level at the end of that holding period (i.e., four quarters later), and compute the “return” of this investment. By repeating this process and randomly “buying” the market at any time for 5,000 times through re-sampling, we obtain 5,000 simulated annual returns. We then compute the mean
and standard deviations for these simulated returns, which is the average return and risk corresponding to a four-quarter holding period. Denoting the average return and risk with \( \mu \) and \( \sigma \) respectively, we repeat the whole process to obtain \( \mu_{\tau} \) and \( \sigma_{\tau} \) for every holding-period, \( \tau = 1, 2, 3, \ldots, 36 \) quarters.\(^5\) For easy examination, we standardize the returns and standard deviations by scaling them in such a way so that the return and standard deviation for holding both for one quarter is 1.00. The “scaled” return and standard deviations are then plotted in Exhibit 1. Furthermore, given the “scaled” standard deviation for a single quarter holding period being 1, we also plot the multi-period return and risk increasing along the path of i.i.d. condition. \( \mu_{\tau}/\mu_1 = \tau \) and \( \sigma_{\tau}/\sigma_1 = \sqrt{\tau} \), where \( \mu_1 \) and \( \sigma_1 \) are the return and risk of single-quarter holding and \( \tau \) is the number of quarters of the holding periods.) The above process is then repeated for all the indices selected, and the results are displayed in Exhibits 2–5.

Exhibits 1–5 display the standardized return and risk for the major asset classes. For lack of a better term, we tentatively call these lines the characteristic lines of return or risk (CL-return and CL-risk, respectively). Each CL in Exhibits 1–5 depicts the relation between an asset’s return (risk) and the investment horizon (or holding periods). Each point (except the point \((1,1)\)) on a CL represents the ratio
between the return (risk) of multi-quarter versus single-quarter investment. These lines show how investment horizon correlates to the buy-and-hold return and risk of investing in each asset class. It should be noted that these lines do not indicate the magnitudes of return and risk. They only indicate how return and risk change over time relative to other asset markets.

It is immediately clear from Exhibits 1–5 that the long-run outcomes of a simple buy-and-hold investment strategy appear quite different from what we would expect if asset prices follow the random walk. The CLs in every exhibit clearly drift away from the i.i.d. path as the holding period increases, and all CLs lie above the i.i.d. paths. It is also interesting to notice that, for security assets (Exhibits 1–3) the CL-return increases faster than CL-risk, while for private real estate (Exhibits 4 and 5), the opposite seems true.

The observation that CL-risk lies above the i.i.d. path suggests that periodic risk (i.e., annualized) becomes higher as the holding period increases. This is in contrast to the findings of several recent studies such as MacKinnon and Zaman (2009), Pagliari (2011), and Rehring (2012), who suggest that the variance of real
estate returns declines or “decays” in the long run. The cause of the difference may be in the diverse tactics used to approach the question. Whereas the above three studies all rely on return-generating models based on the VAR approach developed in Campbell and Viceira (2005, p. 37), their examinations, as Campbell and Viceira acknowledge, “are not directly looking at the long-horizon properties of returns but at the long-horizon properties of returns imputed from (their) first-order VAR.” In contrast, we observe the historical data as is, and directly compute multi-period return series from quarterly real estate indices without using any return-generating models. A similar finding is reported by Collett, Lizieri, and Ward (2003, p. 207), who directly observe U.K. commercial data and reach the similar conclusion that “even allowing for appraisal smoothing, private real estate looks less attractive once more realistic and longer time horizons are considered.”

The simple conclusion from these observations is that long-run asset performances clearly do not follow i.i.d. paths. While the reason behind such return and volatility characteristics is definitely worthy of further research, we ask a more practical question: In light of these observations, what is a reasonable alternative to the i.i.d. assumption to describe asset return and volatility over time?
We propose an empirical approach similar to that in Cheng, Lin, and Liu (2013), which is to use a linear model to approximate the CLs in Exhibits 1–5. We recognize that some of the CLs are not linear throughout the 36-quarter holding period. But within a certain holding period, linear approximation appears reasonable. For the financial asset classes (Dow, NASDAQ, S&P500, and NAREIT), both CL-return and CL-risk appear rather linear up to at least 16 quarters (4 years), which is longer than the typical holding period of most common stocks. For real estate (NCREIF and OFHEO), the linear patterns extend throughout the entire 36 quarters, perhaps because the typical holding period of real estate is much longer. Obviously, the reasonable holding period during which the CL-return and CL-risk remain linear is an empirical matter that should be examined on a case-by-case basis. Once such a linear range is determined, a simple linear model can be used to estimate the slope of the CL-return and CL-risk for each asset class.

We begin with the CL-return. Let $r$ be the holding period and given the line passes point $(1, 1)$, the CL return can be modeled as:
where \( \theta \) is the slope of the CL-return and \( \varepsilon \) is an error term. To save notations, we omit the subscript of \( \mu_i \) and simply use \( \mu \) to denote the asset’s single-quarter average return, thus (1) can then be written as:

\[
\frac{\mu_r}{\mu} = 1 + (\tau - 1)\theta + \varepsilon, \tag{1}
\]

It is easy to see that if \( \theta = 1 \), \( u_r = \tau u + \varepsilon \), and the average periodic expected return is \( \mu_r/\tau = u \), which is the i.i.d. condition where the expected periodic return is independent of the holding period.
Similarly, we can model the CL-risk with the following linear model:

\[
\frac{\sigma_{\tau}}{\sigma_1} = 1 + \beta(\tau - 1) + \eta. \quad (3)
\]

where \( \beta \) is the slope of the CL-risk and \( \eta \) is an error term. We simplify the notation and rewrite (3) as:

\[
\sigma_{\tau} = \sigma + \beta(\tau - 1)\sigma + \eta. \quad (4)
\]

In (2) and (4), the magnitude of \( \theta \) and \( \beta \) indicates the speed to which an asset’s return and risk increases with the holding period, or the extent to which the asset’s returns deviate from the i.i.d. condition. These linear models provide the alternative to the i.i.d. assumption. For comparison, we use the following model:

\[
\begin{align*}
\mu_{\tau} &= \mu + \theta(\tau - 1)\mu \\
\sigma_{\tau} &= \sigma + \beta(\tau - 1)\sigma
\end{align*}
\quad (5)
\]

to replace the i.i.d. assumption:

\[
\begin{align*}
\mu_{\tau} &= \tau \mu \\
\sigma_{\tau} &= \sqrt{\tau\sigma} \text{ or } \sigma_{\tau}^2 = \tau \sigma^2.
\end{align*}
\quad (6)
\]

The slope of the CLs, \( \theta \) and \( \beta \), can be estimated with the following regressions:

\[
\begin{align*}
\mu_{\tau} - \mu &= \theta(\tau - 1)\mu + \epsilon \\
\text{and} \\
\sigma_{\tau} - \sigma &= \beta(\tau - 1)\sigma + \eta.
\end{align*}
\quad (7)
\]

Note that both regressions have zero intercepts.

Using quarterly time series of our selected major indices over 1977:Q4–2007:Q2, \( \theta \) and \( \beta \) for these assets are estimated as shown in Exhibit 6. Note that for financial assets, the linear model is only fitted to the range of 0–16 quarters in Exhibits
A Model of a Buy-and-Hold Investment Process

We have established an alternative to the traditional i.i.d. assumption (equation 5) for describing asset return distributions over time. We next use a simple model to describe the buy-and-hold investment process. While the buy-and-hold strategy for financial assets is straightforward, it is perhaps less so for real estate. For easy discussion, we describe a typical investment process in Exhibit 7. Suppose that an investor acquires an asset at time 0 and holds it for a certain time and then puts it up for sale. Because real estate cannot be sold immediately and requires some selling time [the so-called time-on-market (TOM)], the investor must anticipate such time and start listing the property before the end of his anticipated holding period. If his planned holding period is \( \tau = t + E[TOM] \), he needs to list the property at time \( t \). Note that at time \( t \), the anticipated \( TOM \) is a random variable as he does not know when the property will be sold or at what price. In other words, the real estate investor faces two ex ante uncertainties: the uncertainty of

1–3. For real estate, the estimates use the entire 36 quarters. The \( t \)-test for \( \theta \) is for the null hypothesis of \( \theta = 1 \), which is what i.i.d. assumption would imply. The \( t \)-test for \( \beta \) is for the null hypothesis of \( \beta = 1 \), which is the “alternative assumption” on risk structure proposed by Lin and Liu (2008). The results suggest that the slopes of both CLs are significantly different from 1. As for a test for \( \beta \) against the i.i.d. assumption, it has been done by Cheng, Lin, Liu, and Zhang (2011), who use a more sophisticated BDS test specifically for i.i.d. conformation, and their results strongly reject the i.i.d. hypothesis.

Note: The table reports the regression coefficient estimates for \( \theta \) and \( \beta \) based on equation (7). The high adjusted \( R^2 \)'s of all regressions suggest a strong linear pattern for all CL-returns and CL risks. \( T \)-statistics tests whether the coefficients are different from 1, which for \( \theta \) is a test against the i.i.d. assumption, and for \( \beta \) is a test against the “alternative assumption” in Lin and Liu (2008).
the TOM, as well as the uncertainty of the price. Therefore, the ex ante return of the real estate investment, $\tilde{r}_{t+\text{TOM}}$, can be considered as a function of the two stochastic variables: the random variable TOM and the random sale price (or return) upon a successful sale ($\tilde{r}_{t+\text{TOM}}/\text{TOM}$). In compassion, since the financial assets can be sold immediately, the investor’s marketing period is zero (i.e., $\text{TOM} = 0$). In other words, an investor holding both financial assets and real estate will list real estate for sale at time $t$ and continue to hold financial assets to sell whenever the real estate is sold at $t + \text{TOM}$.

**Liquidity Risk of Real Estate**

The process in Exhibit 7 highlights another complexity of investing in real estate—the illiquidity risk as indicated by the random TOM required for selling the asset. Proper estimates of the real estate risk, however, require more than incorporating the horizon impact into the performance metric. Private assets like real estate cannot be easily bought and sold at any time an investor desires. Since the potential loss of welfare due to such an inability to trade out of a position when needed can be significant, illiquidity risk must be properly accounted for by rational investors. Furthermore, illiquidity risk not only can be significant, it is also a systematic risk that cannot be diversified away in a portfolio. Generally speaking, the expected TOM is a function of market conditions, and is not under the full control of the seller. In hot markets, all properties are sold rather quickly, while in cold markets the average TOM will be substantially longer. A seller may be able to influence the selling time with listing strategies subject to their financial constraints, but cannot fully control the average TOM under a given market condition. A quick sale typically results in a significant price discount from the property’s fair value. In other words, liquidity risk is priced by the market. Conventional risk measures that ignore liquidity risk fail to account for this component of systematic risk.
A Generalized Performance Metric

We now approach the central objective of this study—the development of a unified investment performance metric that is valid for both financial and real estate assets. In addition to the empirical-based assumption in (5), we assume real estate illiquidity is measured by the random TOM, which is distributed at \( t_{TOM}, \sigma^2_{TOM} \). Note that we do not specify a particular distribution here as long as the mean and standard deviation of TOM can be estimated. Naturally, for a financial asset, illiquidity risk does not apply, so \( t_{TOM} = \sigma^2_{TOM} = 0 \).

\( TOM \) and \( \tilde{r}_{t+TOM} | TOM \) can be regarded as two stochastic variables. By applying the conditional variance formula for any two stochastic variables, the ex ante variance of real estate returns can be computed as:

\[
\text{Var}^{\text{ex-ante}}(\tilde{r}_{t+TOM}) = \text{Var}(E[\tilde{r}_{t+TOM} | TOM]) + E[\text{Var}(\tilde{r}_{t+TOM} | TOM)].
\] (8)

From (5), we have:

\[
E[\tilde{r}_{t+TOM} | TOM] = (t + TOM)\theta u + (1 - \theta)u
\] (9)

and

\[
\text{Var}(\tilde{r}_{t+TOM} | TOM) = [(t + TOM)\beta \sigma + \sigma(1 - \beta)]^2.
\] (10)

We can thus rewrite equation (8) as follows:

\[
\text{Var}^{\text{ex-ante}}(\tilde{r}_{t+TOM}) = \text{Var}([t + TOM]\theta u) + E([(t + TOM)\beta \sigma + \sigma(1 - \beta)]^2).
\] (11)

Simplifying equation (11) yields:

\[
\text{Var}^{\text{ex-ante}}(\tilde{r}_{t+TOM}) = (t + t_{TOM})^2\beta^2\sigma^2 + (\theta^2u^2 + \beta^2\sigma^2)\sigma^2_{TOM} + 2(t + t_{TOM})\sigma\beta(1 - \beta) + \sigma^2(1 - \beta)^2.
\] (12)
Therefore, the average periodic ex ante risk for the expected holding period until
sale \((t + t_{\text{TOM}})\) can be expressed as:

\[
\sigma^{\text{ex-ante}} = \sqrt{(t + t_{\text{TOM}})\beta^2\sigma^2 + 2\sigma^2\beta(1 - \beta) + \frac{(u^2\theta^2 + \beta^2\sigma^2)\sigma_{TOM}^2 + \sigma^2(1 - \beta)^2}{(t + t_{\text{TOM}})}}. \tag{13}
\]

For the financial asset, \(t_{\text{TOM}} = \sigma_{TOM}^2 = 0\). Equation (13) thus becomes:

\[
\sigma^{\text{ex-ante}} = \sqrt{t\beta^2\sigma^2 + 2\sigma^2\beta(1 - \beta) + \frac{\sigma^2(1 - \beta)^2}{t}}. \tag{13}'
\]

For the ex ante expected return, we first calculate the expected ex post return, conditional on a successful sale \((TOM)\), and then use the law of iterated expectations:

\[
E^{\text{ex-ante}}[\tilde{r}_{t+TOM}] = E[E[\tilde{r}_{t+TOM}|TOM]] \\
= E[(t + TOM)\theta u + (1 - \theta)u] \\
= (t + t_{\text{TOM}})\theta u + (1 - \theta)u. \tag{14}
\]

Hence, the average periodic ex ante return is:

\[
u^{\text{ex-ante}} = \theta u + \frac{(1 - \theta)u}{t + t_{\text{TOM}}}. \tag{15}
\]

Similarly, for the financial asset, equation (15) becomes:

\[
u^{\text{ex-ante}} = \theta u + \frac{(1 - \theta)u}{t}. \tag{15}'
\]
From equations (13) and (15), we can obtain a unified reward-to-risk ratio (the Shape ratio) for the financial and real estate assets as:

$$S = \frac{\theta u + (1 - \theta)u(t + t_{TOM}) - r_f}{(t + t_{TOM})\beta^2\sigma^2 + 2\sigma^2\beta(1 - \beta) + (u^2\theta^2 + \beta^2\sigma^2)\sigma^2_{TOM} + \sigma^2(1 - \beta)^2}.$$

(16)

When this ratio is applied to financial assets, we insert $t_{TOM} = \sigma^2_{TOM} = 0$, and apply the respective $\theta$ and $\beta$ for that particular asset. Note that $r_f$ is risk-free rate.

Equation (16) suggests that the risk-adjusted return of an asset is determined by many factors, such as investor’s investment horizon, asset illiquidity ($t_{TOM}$, $\sigma^2_{TOM}$), single-period return distribution ($u$, $\sigma^2$), and the slopes of the asset’s CL-return and CL-risk ($\theta$ and $\beta$). Equation (16) shows that, as expected, the real estate illiquidity [both the uncertainty of TOM ($\sigma^2_{TOM}$) and the expected TOM ($t_{TOM}$)] lead to higher ex ante risk. In other words, both the first and second moments of TOM ($t_{TOM}$ and $\sigma^2_{TOM}$) play an important role in the overall real estate risk. This is different from Lippman and McCall (1986), who suggest that only the first moment (expected TOM) is a sufficient measure for illiquidity.

Note that equation (16), or the Sharpe ratio in general, has the total risk at the denominator. In contemporary financial theory, it is generally differentiated into a systematic and unsystematic component, and only systematic risk is priced. However, this should not diminish the usefulness of equation (16) for the same reason that Sharpe ratio cannot be deemed useless. Obviously, in the thinly-traded private market, since unsystematic risk cannot be easily and inexpensively diversified away by investing in a tradable “market portfolio,” all investors hold a limited number of assets that also are indivisible and unique. This effectively implies that portfolios that contain real estate are subject to both systematic and unsystematic risks. As Levy (1978) and Merton (1987) show, amid such market unsystematic risk can affect assets’ equilibrium price, the total risk is actually more relevant than its systematic component.

It is perhaps also necessary to note that, despite its appearance, equation (16) is still confined within the traditional mean-variance framework, in which it implicitly assumes that the underlying asset return distributions (financial and real estate) are reasonably normal and have finite moments. This is debatable to some extent, particularly for real estate, in light of the findings of Young and Graff (1995), Young, Lee, and Devaney (2006), and Young (2008), among others, that real estate return distributions seem to exhibit “fat tails” and asymmetry. The fat
tail issue implies that risk measurements derived based on sample volatility measurements, such as the traditional variance or the one in equation (16), tend to underestimate the true investment risk for real estate because they underestimate the impact of an extreme result. The asymmetry issue implies that risk measures based on higher moments, such as skewness and kurtosis, may be more desirable and consistent with the perception of a typical risk-averse investor who is mainly concerned with the “downside risk” of investment. In fact, there is also a long stream of research in the finance literature that seeks alternative risk metrics and portfolio models beyond the traditional mean-variance framework. One of the most studied asymmetric risk measures is perhaps the so-called semivariance, or more broadly known as the lower partial moment (LPM) of an asset return distribution. Markowitz (1959), Mao (1970), Bowa and Lindenberg (1977), and Fishburn (1977) were among the first to suggest semivariance as an alternative risk measure to standard deviation. On the other hand, Liu, Hartzell, and Grissom (1992), Vines, Hsieh, and Hatem (1994), and Cheng (2005) have examined skewness and kurtosis as alternative risk metrics for commercial real estate. While this line of research has yielded important insights that point to promising directions for future research, it is beyond the scope of the current study.

**Empirical Demonstrations**

In this section, we demonstrate the application of our unified performance metric using the same data that produced Exhibits 1–5. In order to estimate equation (16), we need to know the following parameter values:

1. The quarterly return and volatility of each asset class \((u \text{ and } \sigma)\). These numbers are obtained directly from the sample data series.

2. The slopes of the CL-return and CL-risk of each asset class \((\theta \text{ and } \beta)\), such as those displayed in Exhibit 6. For financial asset, we subdivided the samples into two periods because we suspect the \(\theta\) and \(\beta\) may be sensitive to market cycles and conditions. Accordingly, we re-estimate \(\theta\) and \(\beta\) for each period for the demonstration below. For real estate, however, since we have to assume a much longer holding period, it is not feasible to conduct a two-period analysis. This choice is solely due to data limitation.

3. The risk-free rate is \(r_f\), which we obtained from Ken French’s website. For the period analyzed, the average quarterly risk-free rate is 0.5%.

For financial assets, since \(t_{TOM} = \sigma_{TOM}^2 = 0\), equation (16) is simplified as:

\[
S = \frac{\theta u + (1 - \theta)u t - r_f}{\sqrt{\beta^2 \sigma^2 + 2 \sigma^2 \beta (1 - \beta) + \frac{\sigma^2 (1 - \beta)^2}{t}}}.
\] (17)
To examine the impact of holding-period on different assets’ Sharpe ratios, we simulate a range of holding periods from 1 to 10 quarters. Exhibit 8 displays the results for the three financial asset classes.

In Exhibit 8, when the holding period is one quarter, the estimated Sharpe ratios are the same as traditional single-period Sharpe ratio. But now we can see that the ratios do vary across different holding periods, and the pattern is not linear. Also, the Sharpe ratios appear very different for the two periods, suggesting the CLs are sensitive to sample period. For the 1977–1997 period, the results seem to suggest there is an optimal holding period for each asset where they exhibit the highest Sharpe ratios when holding periods are 4–6 quarters. In addition, the rank order of Sharpe ratios for the three assets vary as holding period changes. For example, in Panel A of Exhibit 8, the rank order of the three assets under 1 quarter is exactly reversed when the holding period is 10 quarters. This is new evidence that long-run asset performances are different from their short-run behaviors. A final observation is a general pattern that the long-run Sharpe ratios tend to be lower than those of short-runs. This contrasts the conventional view that stocks are safer in the longer terms, but is consistent with the finding of Bodie (1995) that stock performance declines as holding period increases.

Next, we present the simulation results for the two real estate indices: NCREIF and OFHEO. In addition to parameters discussed before, the real estate calculation requires information about the distributions of time-on-market ($t_{TOM}$ and $\sigma_{TOM}^2$). Since $TOM$ is a random variable ex ante, we simulate a range of possible TOM to reflect various ex ante market conditions. With regard to $t_{TOM}$ and $\sigma_{TOM}^2$, we follow the assumptions by Cheng, Lin, and Liu (2010) that ex ante TOM follows a negative exponential distribution. This form of distribution has a convenient mathematical property: the variance is equal to the square of its mean, i.e., $\sigma_{TOM}^2 = t_{TOM}^2$. Thus equation (16) can be re-written as:

$$S = \frac{\theta u + (1-\theta)u/(t + t_{TOM}) - r_f}{\sqrt{(t + t_{TOM})\beta^2\sigma^2 + 2\sigma^2\beta(1 - \beta)}}$$

$$+ \frac{(u^2\theta^2 + \beta^2\sigma^2)t_{TOM}^2 + \sigma^2(1 - \beta)^2}{(t + t_{TOM})}.$$  

(18)

With regard to the holding period, because real estate is typically held for multiple years, we simulate a range of typical holding periods from 1 to 9 years. The results are in Exhibit 9.

Despite the obvious differences between commercial and residential real estate, the Sharpe ratios displayed in Exhibit 9. Clearly, the ratios consistently decline as the holding period increases, although neither asset class exhibits the kind of
### Exhibit 8 | Sharpe Ratios of Financial Assets across Holding Periods

<table>
<thead>
<tr>
<th>Quarterly Return</th>
<th>Quarterly Volatility</th>
<th>Holding Period (quarters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Dow Ind.</td>
<td>3.04%</td>
<td>6.06%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>3.20%</td>
<td>7.09%</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>3.89%</td>
<td>8.73%</td>
</tr>
<tr>
<td>Dow Ind.</td>
<td>1.39%</td>
<td>5.03%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>1.19%</td>
<td>8.43%</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>1.86%</td>
<td>12.41%</td>
</tr>
</tbody>
</table>
optimal holding period for financial assets shown in Exhibit 8. However, this point should be interpreted with caution because these calculations do not take the real estate transaction cost (typically large) into account. As Cheng, Lin, and Liu (2010) show, once transaction cost is properly accounted for, there appears to be an optimal holding period for real estate as well. Another observation is that for a given holding period, the Sharpe ratios decline as the TOM increases. That is, higher illiquidity increases the overall risk and reduces the Sharpe ratios. Finally, in case of the potential fat tail issues with the asset return distributions, the sample volatility may underestimate the true return volatility, causing the ratios in Exhibit 9 to be overestimated.

Conclusion

In this study, we explore the possibility for a generalized investment performance metric that is applicable to both real estate and financial assets, in order to facilitate direct performance comparison and portfolio diversification of mixed-asset portfolios. The central result of this study is the development of a unified risk-
adjust return measure (equation (16)) in which private and public assets are measured in the same way, differing only in the model’s parameters.

Through empirical examination of the long-run return and volatility of several major asset classes, we find the common ground of stocks and real estate. First, both assets’ return and volatility deviate significantly from the i.i.d. path, making it unnecessary to use the assumption. Second, both assets’ return and volatility appear to increase linearly as the holding period increases up to a certain point. This finding enables us to use a set of linear models to describe the holding period return and volatility for all assets, differing only in the slopes of their return and risk characteristics ($\theta$ and $\beta$). Unlike the earlier studies that use similar linear models for real estate but still assumes i.i.d. for financial assets, we use the linear model to replace the i.i.d. assumption for all assets and for both return and volatility. This is a critical base for a unified performance metric. The individuality of each asset determines its own $\theta$ and $\beta$, which can be conveniently estimated using empirical data.

The $\theta$ and $\beta$ are a pair of concepts that adds new insights into the long-run asset performance research. Their significant deviation from the i.i.d. world suggests that long-run asset performance cannot be adequately measured by single-period return and volatility. Rather, prudent long-run investment decision must give careful consideration to the anticipated holding period and proper understanding of the long-run return and risk characteristics.

**Endnotes**

1 Financial assets are subject to liquidity risk as well. But as Cheng, Lin, and Liu (2013) pointed out, the illiquidity of stocks are negligible compared to that of real estate. By the standard of real estate, all financial assets are liquid.

2 There are many other papers in this line of research including Young (2008) and Young, Lee, and Devaney (2006).

3 The OFHEO indices are renamed as FHFA indices because the office was merged into the Federal Housing Financing Agency in late 2008 during the height of the financial crisis. We retain the OFHEO name simply for its familiarity among researchers.

4 We intentionally exclude the recent crisis period starting later than 2007.

5 Some readers may have concerns with our calculation by using overlapping periods. Initially, we wanted to use non-overlapping periods. But three issues quickly arose: First, given the length of real estate indices, it was impossible to obtain meaningful results for long holding periods. For example, for 36-quarter (9 years) holding periods, we only have three non-overlapping periods, which were inadequate. Second, non-overlapping causes loss of information, as a lot of data points are not used, which effectively means that we would use only a small part of the sample instead of the whole sample for the calculation, especially for longer holding periods. Third, perhaps the most serious problem was that non-overlapping introduces seasonality bias into the calculation. For example, if the index happens to begin at the first quarter, a 4-quarter holding-period returns series with non-overlapping data would not use index points in all other quarters, which implies that investors can only buy and sell in the first quarter of every year (or
every other year if the holding-period is 8 quarters, and so on). In addition, we tried an alternative method for verification. For the 4-quarter holding period, for instance, we subdivided the whole series by the four quarters so that each subgroup only contains data from the same quarter of every year. We first calculated the mean and variance for each subgroup, then averaged them across the subgroups. The results based on the two methods turned out to be very close. The means are identical and the variances are very close under any holding period scenario.

6 For detailed descriptions of their computation process, see Cheng, He, Lin, and Liu (2015).

References


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