580.439/639 Homework #5

Not to be handed in; use these problems to study for the midterm. Solutions posted Oct 13.

Problem 1

Consider the following pair of differential equations:

$$\frac{dx}{dt} = -\frac{1}{\varepsilon}\left(x^3 - 3x + y\right) \quad \text{and} \quad \frac{dy}{dt} = g(x,y)$$

Assume that ε<<1, so that the first equation is fast compared to the second, i.e. $dx/dt \gg dy/dt$. For the two functions $g(x,y)$ listed below, draw the (x,y) phase space, including the nullclines, and equilibrium points. Tell whether the equilibrium points are stable; use the fact that ε<<1 to simplify the equations. Tell whether it can be proven that the system does or does not have a limit cycle. Draw approximate trajectories in phase space for the system starting at the initial values listed; in drawing these, make use of the assumption that ε<<1 discussed above.

a) $g(x,y) = -(y - 3x - 4.5)$ initial values = (-1, -0.5) and (0, -1)

b) $g(x,y) = -(y - 5x)$ initial values = (-1, -0.5) and (0, -1)

Problem 2 (Izhikevich, p55 ff)

Consider a membrane model containing a leakage channel G_L and a sodium channel modeled as $G_{Na}m_\infty(V)$ where

$$m_\infty(V) = \frac{1}{1 + e^{(V_1 - V)/V_2}}$$

where G_L=19 mS, G_{Na}=74 mS, E_L=-67 mV, E_{Na}=60 mV, V_1=1.5 mV, V_2=16 mV, and C=10 μFd. Note that this system has only one state variable. There is an external current I_{ext} applied to the model.

a) Write the differential equation(s) needed to model this system.

b) One-dimensional systems are usually analyzed in a phase plane with membrane potential V on the abscissa and the time derivative of membrane potential dV/dt on the ordinate. Make such a plot for this system. Compute the equilibrium points of this system (with I_{ext}=0)?

c) Classify the equilibrium points as stable or unstable and explain why. Remember that in this 1 dimensional system, the trajectories are all lines along the x-axis of the phase plane.

d) As I_{ext} increases, a bifurcation occurs. Sketch the plot of dV/dt versus V at the bifurcation. What is the approximate value of I_{ext} at the bifurcation? What kind of bifurcation in 2-state-variable systems does this one resemble? What happens to membrane potential when I_{ext} is driven above the bifurcation value?

Problem 3

By passing a brief current pulse into the MLE (Morris-Lecar Equations) through I_{ext}, it is possible to change the membrane potential V without changing the potassium activation w. This corresponds to the biological experiment of injecting a current pulse into a cell, changing its membrane potential, without changing channel gating. The model (or cell) is then allowed to evolve according to the relevant differential equations. In this problem, you will draw arrows to show the effects of such a current injection in the phase plane, where the current injection is designed to start or stop a limit cycle. The phase planes are taken directly from Rinzel and Ermentrout's chapter.

In each of the examples on the next page, a phase plane is shown with isoclines, equilibrium points, stable (SPO) and unstable (UPO) limit cycles, etc. In each case show where, along the trajectories in the phase plane, a depolarizing voltage step could be applied to accomplish the goal stated at left. Your answer has three parts:

(1) Draw an arrow on the phase plane to show the trajectory corresponding to the voltage step. The arrow should show the (approximately) smallest voltage change possible. The arrow should start on the trajectory specified and end at a point that would accomplish the goal specified.

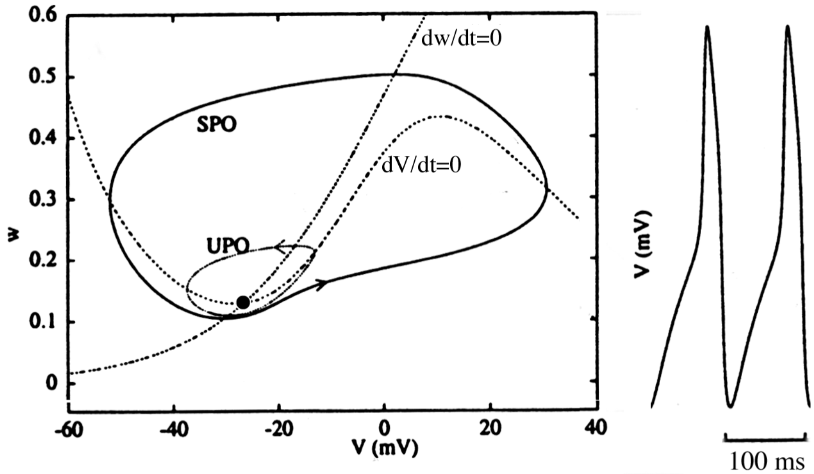
(2) Write a sentence describing what is necessary to achieve the goal (e.g. "cross the stable manifold").

(3) On the voltage vs. time plots at right, show the trajectory that results from the voltage step. The voltage plots show a short segment of the voltage-time plot preceding the voltage step (e.g. a limit cycle or a constant voltage at the equilibrium point) and do not necessarily end at the time point at which the voltage step must be applied. Your plot should start somewhere along the voltage plot and should correspond to the arrow in the phase plane and the trajectory that will follow it.

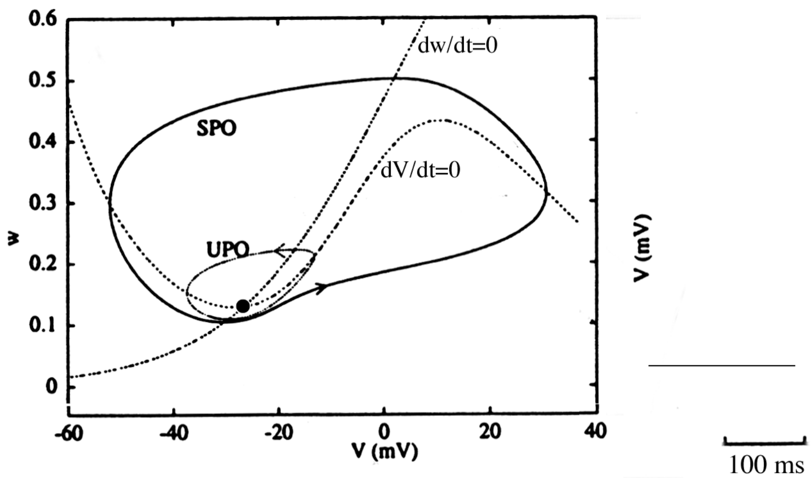
These phase planes are from the MLE and were used as examples in class, in the R&E chapter, and in the course notes.

You can draw your answers to Problem 3 on this and the next page or redraw the phase planes. Equilibrium points are identified as: S – stable; N – saddle node; U – unstable.

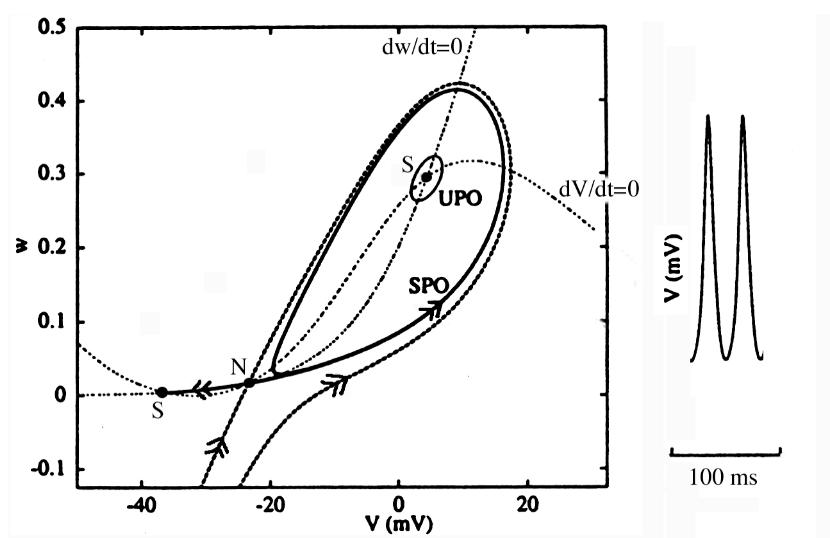
Part a) Stop the limit cycle, returning the system to its stable equilibrium point (black dot)



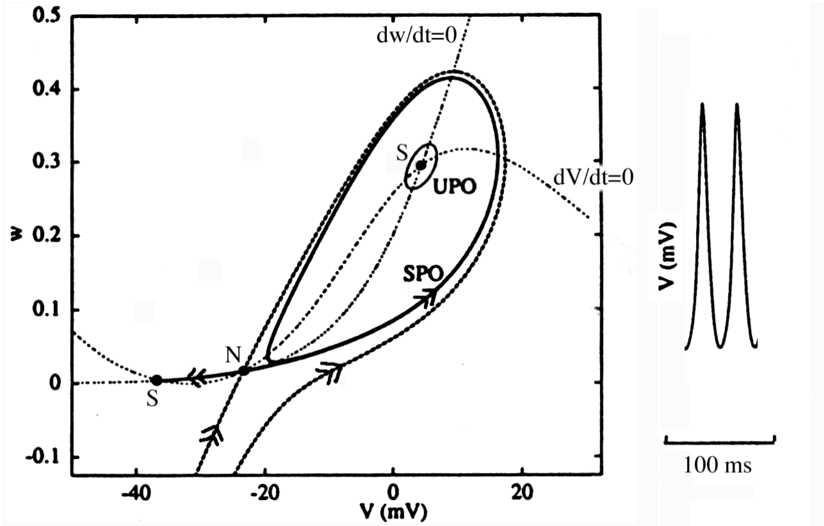
Part b) Initiate the limit cycle beginning with the system at its stable equilibrium point (black dot).



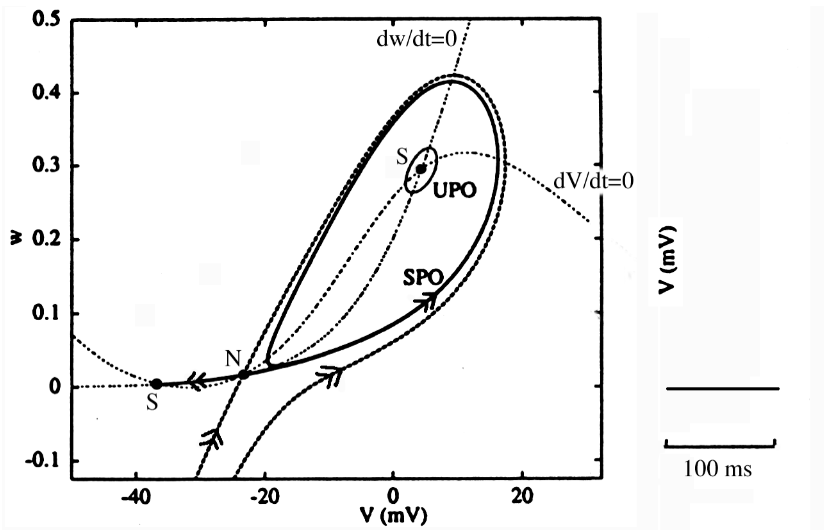
Part c) Stop the limit cycle (SPO) and transfer the system to the resting potential (S, near –35 mV).



Part d) Stop the limit cycle (SPO) and transfer the system to the upper equilibrium point (S, near +5 mV).



Part e) Beginning at the resting potential (S near –35 mV), transfer the system to the limit cycle (SPO).



Part f) Is it possible to transfer the system between the two stable equilibrium points with a single depolarizing voltage pulse? If so, show how on the phase plane below. If not, tell why not. Consider both directions, from rest near –35 mV to S near +5 mV and vice-versa.