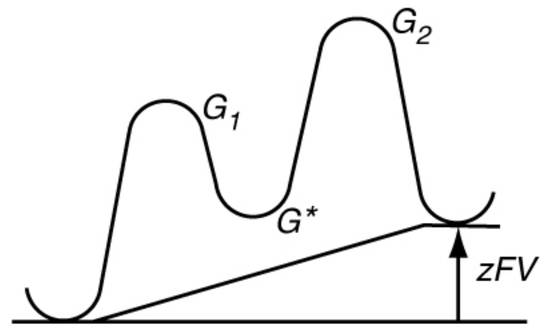
580.439/639 Midterm Exam, 2013

1 hour, closed book, answer all questions. 11 points for all questions, except Problem 3 part b), 5 points.

Problem 1

Consider the barrier model at right. There is a single binding site in the channel with energy G^* and two energy peaks G_1 and G_2. Use this model to explain the idea of steady state and to distinguish it from equilibrium. Write equations to express these conditions in terms of the parameters of the model. Tell which of the variables given in the model contribute to the steady state and which to the equilibrium. You should assume the flux is zero for equilibrium and you may add parameters if needed.



Problem 2

In a certain neuron in the cochlear nucleus, there are three ion channels that are active in the vicinity of resting potential: an ungated leak channel with conductance G_{leak}; a voltage-gated potassium channel with conductance $G_K n$; and a voltage-gated H channel, with conductance $G_H h$. n and h are HH gating variables like the variables with the same names in the HH model. Note that the potassium channel has only an activation gate (assumed to be n not n^4 for simplicity) and the H channel has only an inactivation gate.

The parameters of these channels are given below:

$$n_\infty(V) = \frac{1}{1+e^{-(V+60)/k}} \qquad\qquad h_\infty(V) = \frac{1}{1+e^{(V+60)/k}}$$

$$E_K = -90 \quad E_H = -30 \quad \text{and} \quad E_{leak} = -40$$

$$G_K = G_H >> G_{leak}$$

These neurons have a very fast membrane time constant (~1 ms). Presumably the reason for the two channels that gate in the vicinity of the resting potential is to produce this fast time constant.

Part a) Write the differential equations to model this neuron in the vicinity of rest, i.e. when the contribution of other channels can be ignored.

Part b) Write the equation that must be solved to find the resting potential $\bar{V}$.

Part c) Argue that the resting potential is -60 mV with the values given for the parameters above and with I_{ext}=0. It may help to make a graph of the currents. Hint: the numbers have been chosen to make this easy.

The value of -60 mV is exact if G_{leak}=0. If G_{leak} is small but not zero, is the resting potential larger than or smaller than -60 mV?

Part d) For a first-order linear differential equation like the following for a parallel RC circuit:

$$C\frac{dV}{dt} = I_{ext} - \frac{V}{R}$$

the time constant of the exponential solutions is $\tau = RC$. By analogy, write an equation for the time constant of the model of Part a) at the resting potential. Of course, in this case the time constant may change as channel gating changes the conductances of the channels, but the time constant is still a rough indicator of the timing of responses to injected current. Write your result in terms of the parameters in the definition above, don't insert the numerical values of $h_\infty(\bar{V})$ and $n_\infty(\bar{V})$ determined in Part c).

Part e) Show that the time constant determined in part d) actually does not change with membrane potential in the vicinity of the resting potential (i.e. where this model is valid). Of course, this will require inserting the numerical values.

Problem 3

Consider the following system of differential equations:

$$\frac{dx}{dt} = ax - y - x(x^2 + y^2)$$
$$\frac{dy}{dt} = x + ay - y(x^2 + y^2)$$

Part a) Inspection of the equations shows that there is an equilibrium point at the origin. Linearize the equations at this equilibrium point and characterize it as to stability. On the basis of your eigenvalues, argue that there is a bifurcation (with respect to changing the parameter a) when $a=0$. What kind of bifurcation is it?

Part b) An idea of how this system works can be gotten by changing to radial coordinates using the transformation

$$r = \sqrt{x^2 + y^2} \quad and \quad \Theta = tan^{-1}(y / x)$$

With this transformation, r is the distance of a point in the phase plane from the origin and Θ is the angle, relative to the x axis, of a vector to the point. Show that, with this transformation, the differential equations above can be written in a simpler form.

$$\frac{dr}{dt} = ar - r^3 \quad and \quad \frac{d\Theta}{dt} = 1$$

Working out these equations can be hairy if you chose the wrong method, so **DON'T DO IT UNTIL YOU'VE FINISHED THE REST OF THE EXAM**.

Part c) Argue from the radial form of the differential equations that for $a>0$, the system has a limit cycle encircling the origin at radius $a^{1/2}$. (Hint: remember $r \geq 0$ in the radial model.)

Part d) Argue from the results in parts a) through c) that if $a<0$, the origin is a stable equilibrium point and all trajectories in the phase plane lead to the equilibrium point at the origin.