580.439/639 Final Exam

Answer all questions. Closed book except for two sheets of paper. 17 points per part, plus 11 points for putting your name on all papers you hand in.

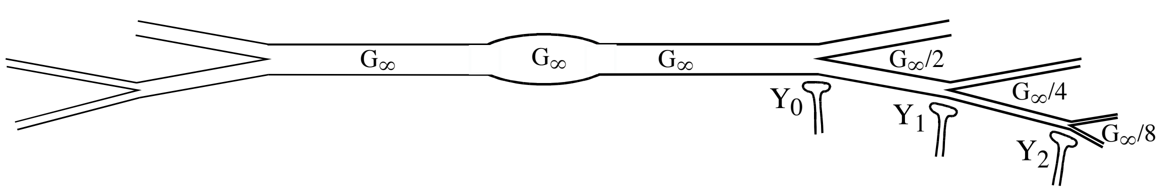
Problem 1

Part a) An ion may be at equilibrium across a membrane or it may be in a steady state of flux. Explain what these two conditions mean, with some equations. (Keep it simple, you don't need to discuss any particular mechanisms or models for ion movement through the membrane.)

Part b) Calcium can be released into the cytoplasm of a cell by two mechanisms: 1) opening of voltage-gated calcium channels or ligand-gated receptor channels in the plasma membrane; or 2) calcium-induced calcium release from intracellular stores, say endoplasmic reticulum or other membrane-bound organelle. It is straightforward to monitor the extent of calcium entry via pathway 1) using membrane-potential recording, but more difficult to monitor pathway 2). Explain why. Also explain how the second pathway might affect the membrane potential through physiological mechanisms.

Problem 2

Consider the bipolar cell sketched below. The dendritic branching is such that the equivalent cylinder theorem holds everywhere, including at the soma; that is, the soma is just part of the cable formed by the primary dendrites, all of which have the same properties. Only part of the tree is shown; assume that the branching is fully symmetric, as required by the equivalent cylinder theorem. Recently, it has been shown that the EPSP produced at the site of a synapse grows larger as the synapse moves further out on the dendritic tree. In the example below, the EPSP produced at the synapse furthest to the right (Y_2) would be the largest.



Part a) Explain qualitatively why this result should be so. Assume that the conductances of all the synapses are the same. (Remember, we're talking about the EPSP produced at the synaptic site, not the EPSP in the soma produced by that synapse.)

Part b) (Do this one last!) To analyze this situation, assume the synapses each inject a current I_0, so that the problem is linear. Then the EPSP at the synaptic site is given by $I_0 K_{ii}$, where K_{ii} is the input impedance of the dendritic tree at the site of the synapse. Work out values for K_{ii} at the three synapses shown above for the D.C. steady state (q=1). The synapses are assumed to sit right at the branch points, as drawn. Assume that each distal branch of the tree is terminated by its characteristic impedance $Y_L = qG_\infty/2^n$, where n is the number of branch points. This means that the distal branches appear to be infinitely long. The result will be messy. To simplify it further, assume that $\tanh(L) = 0.2$ for L the electrotonic distance between successive branch points.

Part c) How is the result changed if the synapse is assumed to change a conductance instead of injecting a current (assuming that only one synapse is activated at a time)?

Problem 3

Consider a semi-infinite cable, i.e. one running from x=0 to x=∞. The cable is driven at x=0 by injecting a step current $I_0u(t)$, as was done in the examples presented in class. This problem considers the total charge on the cable. (Note that x is distance in units like cm or μm, not dimensionless distance X).

Part a) Write the cable equation and boundary conditions appropriate for this situation. Assume the cable is at the resting potential at t=0 and assume that the membrane potential goes to rest (0) as $x \rightarrow \infty$. $I_0u(t)$ is the axial current of the cable at x=0, as usual.

Part b) Consider the total charge on the cable Q, defined as

$$Q(t) = c_m \int_0^\infty V(x,t)\,dx$$

where c_m is the capacitance of the cable membrane per unit length. By integrating the cable equation of a), show that the charge obeys the following equation:

$$\tau_m \frac{dQ}{dt} + Q = \tau_m I_0 u(t) \qquad (*)$$

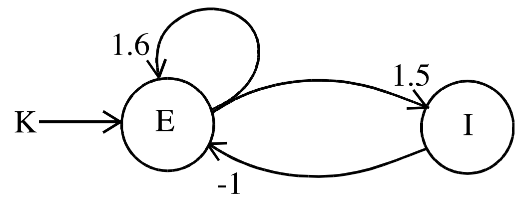
Part c) Equation (*) needs a boundary condition. What is it?

Part d) Solve Eqn. (*) with the boundary condition of part c) to write an expression for the development of total charge on the cylinder as a function of time.

Part e) The charge Q builds up to a steady (i.e. constant) value as $t \rightarrow \infty$, yet charge continues to flow into the cable at x=0 at rate I_0 coul/s. Explain where the charge is going. Show that your answer is consistent with the fact that the steady-state potential distribution on the cable $V(x, t \rightarrow \infty) = I_0 \exp(-x/\lambda)/G_\infty$.

Problem 4 (from Wilson, p. 121 ff)

Consider the two-neuron circuit shown at right. There is one excitatory neuron (E) and one inhibitory interneuron (I). They are interconnected as shown by the arrows, and the E neuron receives an input K. The interconnection weights are shown. The neurons are characterized by the usual Wilson-Cowan equations of the form



$$\tau_j \frac{dx_j}{dt} = -x_j + S(K_j + \sum_{\text{inputs } i} w_{ij} x_i) \qquad (**)$$

and $S(x)$ is the function $Mx^2/(\sigma^2 + x^2)$, with M=100 and σ=30. The time constants are 5 for the E neuron and 10 for the I neuron.

Part a) Write a set of differential equations of the form of (**) for this system, using E and I as the neural activation variables. Write equations for the nullclines, in the form $I_{null} = f(E_{null})$.

Part b) At top on the attached page are shown two phase planes. One is for K=0 and the other for K=20. Tell which is which and label the nullclines (i.e. tell which nullcline is for dE/dt=0 and which is for dI/dt=0).

Part c) Write equations for the terms in the Jacobian of this system, in terms of the variables in your answer to a). These can be expressed in terms of symbols, it is not necessary to substitute for the constants and calculate everything.

Part d) The Jacobians for the linearized systems near the equilibrium points in the two phase planes of part b) are given below. Characterize the equilibrium points.

$$J(E=0, I=0) = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.1 \end{bmatrix} \qquad J(E=12.7, I=28.9) = \begin{bmatrix} 0.42 & -0.39 \\ 0.32 & -0.1 \end{bmatrix}$$

Part e) Argue that the system for K=20 has a limit cycle. What can you say about the system for K=0? (Don't even think about Lyapunov.)

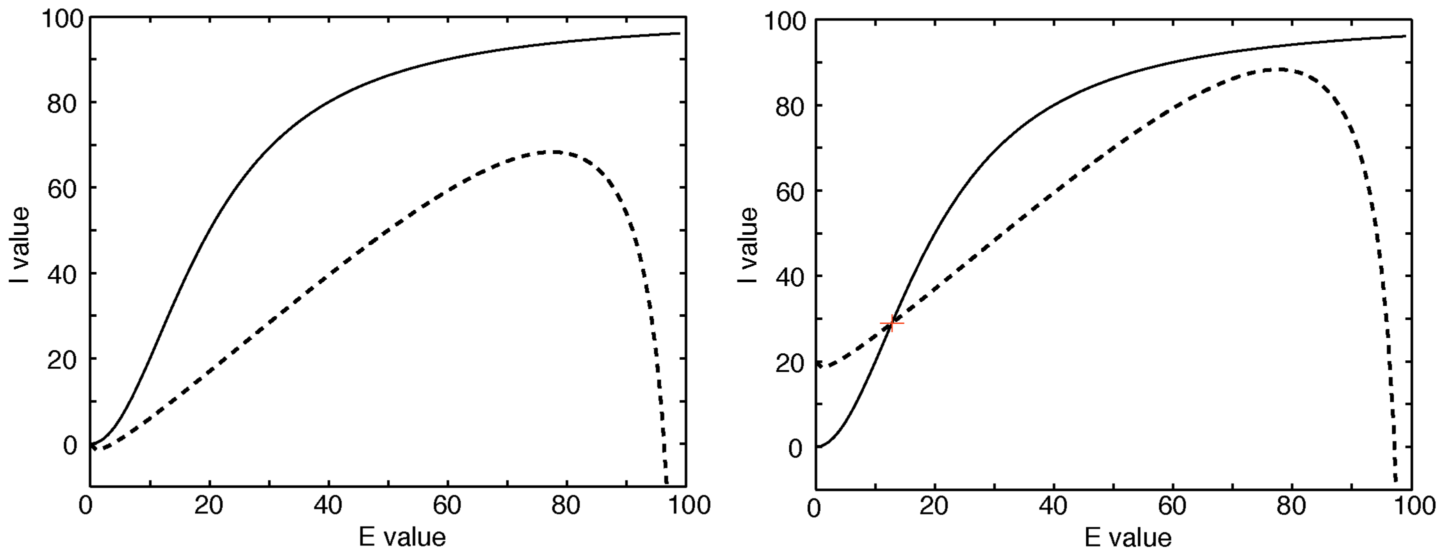
Part f) The phase plane for K=30 is shown at bottom on the attached page. The nullclines are drawn in the same line styles as in the plots at the top of the page. Arrows at each point in the plane show the direction of trajectories in that region. One particularly interesting full trajectory is shown, as the heavy dot-dash line, beginning near one equilibrium point and ending near another. There are now three equilibrium points, with the Jacobians below. Characterize these equilibrium points.

$$J(E = 18.0, I = 44.7) = \begin{bmatrix} 0.472 & -0.420 \\ 0.275 & -0.1 \end{bmatrix} \qquad J(E = 61.4, I = 90.4) = \begin{bmatrix} 0.201 & -0.250 \\ 0.028 & -0.1 \end{bmatrix}$$

$$J(E = 84.9, I = 94.7) = \begin{bmatrix} -0.0843 & -0.0723 \\ 0.0117 & -0.1 \end{bmatrix}$$

Part g) Does this system have a limit cycle? If so, draw a likely trajectory. If not, why not? You may have to guess a little bit about trajectories like the one drawn to come to a conclusion here.

Problem 4, Part b)



Problem 4, part f)