580.439/639 Final Exam 2008

3 hours, closed book except for 2 sheets of paper, answer all problems. 20 points for each question plus 20 points for your name.

Problem 1

Part a) What is the difference between metabotropic and ionotropic synapses? In which class are NMDA receptors? How do NMDA receptors differ from other receptors in their class?

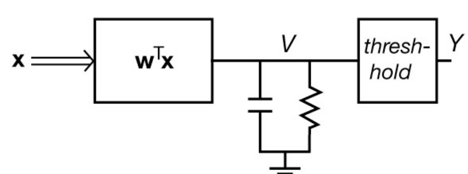
Part b) In class, a model for thalamocortical neurons was discussed in which the neurons showed two modes of spiking, a burst mode and an encoding mode. These were accompanied by different resting membrane potentials (~-65 and ~-80 mV, respectively). How might inhibitory neurotransmission differ in the two modes? Explain why.

Part c) Describe some factors that determine the amplitude of the EPSP produced in the soma by a synapse somewhere on the dendritic tree. Tell how each factor affects the amplitude.

Part d) Explain what is meant by spike-timing-dependent plasticity and show a graph that illustrates the spike-timing part. Be careful in labeling the x-axis of your graph; indicate clearly what are the two events whose occurrence times are important in the timing. Explain whether the graph you draw is for Hebbian (synapses are strengthened to promote spiking when the synapse fires) or anti-Hebbian (synapses are weakened to oppose spiking when the synapse fires) plasticity.

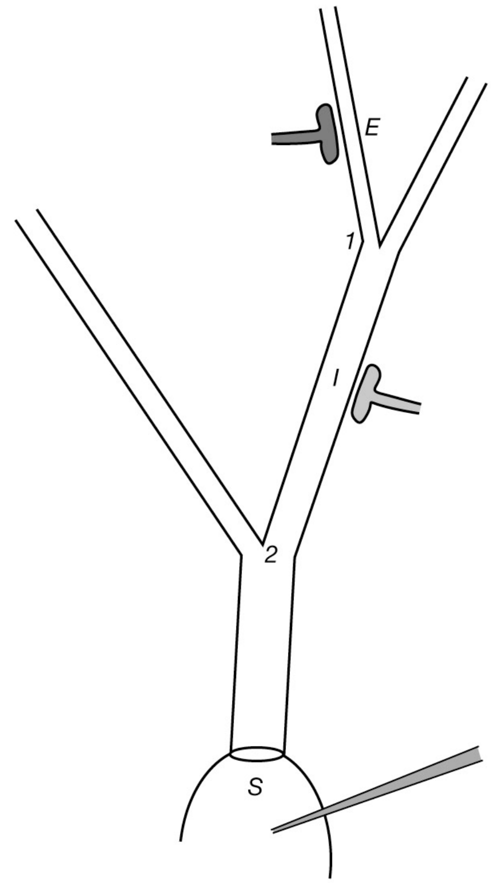
Part e) Suppose a cell has a homeostatic plasticity that strengthens or weakens synapses globally to keep the cell's average firing rate in a reasonable range. Suppose it also has an anti-Hebbian plasticity that acts at individual synapses (i.e. the only synapse that is weakened is the one that meets the anti-Hebbian coincidence rule). Explain how this could result in uniformity of synaptic strength measured by EPSP size in the soma.

Part f) The usual neural network neuron model is shown below. Current is injected in the soma by the weighted sum of the synaptic inputs $\mathbf{w}^T\mathbf{x}$; the soma is modeled by a parallel capacitance and linear resting conductance as shown. Some sort of output nonlinearity, like a squashing function, can be used to model neural nonlinearities. Sometimes it is interesting to study the effects of various voltage-gated potassium conductances to simulate refractoriness, accommodation (a rise in threshold in response to sustained excitatory input), etc. In this case the nonlinearity is a threshold circuit that produces action potentials. Redraw the circuit to incorporate a potassium channel in the place where it would function like that in a real neuron and write the differential equation for such a model.



Problem 2

The dendritic tree sketched at right has an excitatory synapse at point E and an inhibitory synapse at point I. Various other points on the tree are marked for convenience.



Part a) In class, a formula useful for deriving the transfer impedance from a synapse (say E) to the soma was derived, a chain rule for transfer impedances. If points i, j, k are in-line on a direct path to the soma, then

$$K_{ik} = \frac{K_{ij} K_{jk}}{K_{jj}} \ .$$

Write a similar rule for computing K_{ik} that uses only K_{ij}. (Hint consider voltage gains).

Part b) Show how to use the modified rule to compute K_{ES}, the transfer impedance from the excitatory synapse to the soma in the figure at right. Explain what you are doing at the branch points.

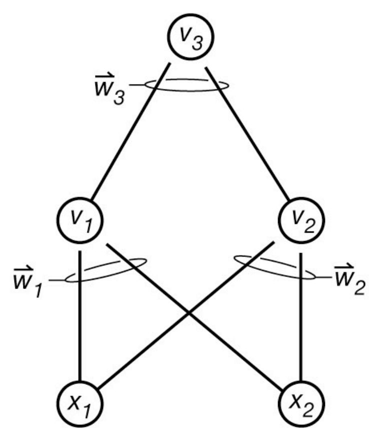
Part c) Suppose the inhibitory synapse at I is a shunting synapse, meaning that it has its main effect by shunting excitatory currents rather than by hyperpolarizing the membrane. Show that activation of a synaptic conductance at I reduces K_{ES}. Assume the D.C. steady state for this problem.

Part d) In part c), the inhibitory synapse was on-path from the excitatory synapse to the soma. Repeat the calculation for an inhibitory synapse distal to E. Do you think the inhibitory effect is smaller or larger (this may be hard to prove in limited time)?

Problem 3

Part a) Consider a linear perceptron $V=\mathbf{w}^T\mathbf{x}$, with weight vector $\mathbf{w}$ and input vector $\mathbf{x}$. There is no bias term. What input vector $\mathbf{x}$ gives the maximum output value V, under the constraint that $|\mathbf{x}| = 1$ and $\mathbf{w}$ is fixed? Hint: remember that $\mathbf{a}^T\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\phi_{ab}$, where $\phi_{\mathbf{ab}}$ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

Part b) Does it change your answer to Part a) if the perceptron is nonlinear, $V = S\left[\mathbf{w}^T\mathbf{x}\right]$, for $S[\ ]$ a continuous strictly monotone increasing function (like $1/(1+\exp(x))$)



Part c) Now consider a two-layer network consisting of three perceptrons as at right. Suppose the elements are linear. Show that the result of part a) is still true with a modified weight vector. Work out the modified weight vector.

Part d) Now suppose the two layer network is made of three perceptrons in which

$$V = S\left[\mathbf{w}^{\mathrm{T}}\mathbf{x}\right]$$

for a squashing function like that used in part b). Assume that there is a maximum for some input vector **x** with the constraint $|\mathbf{x}| = 1$ and write an equation that must be satisfied at the maximum. This will be hairy and you won't be able to get very far, so don't worry about solving for **x** explicitly.