580.439/639 Midterm Examination, 2001

Answer all questions. Closed book, except for one 8.5"x11" sheet of paper.

Problem 1

Consider a cell with the ion concentrations inside and outside given in the table at right.

Ion	Inside	Outside
Sodium	15	140
Potassium	135	5
Calcium	10^{-4}	2
Chloride	22	135

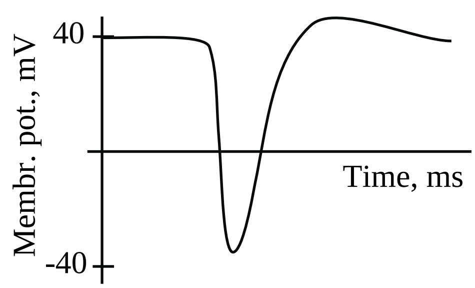
7 pts **Part a)** Draw an electrical circuit for ion flow through this membrane, assuming all ions are permeant. Label the batteries.

7 pts **Part b)** What are the minimum and maximum theoretical values of membrane potential in this cell, assuming that there is no source of external current to affect the membrane potential?

7 pts **Part c)** Suppose that there is a channel in the membrane that admits both potassium and sodium, with a *conductivity* for potassium that is 5 times larger than that of sodium. At what membrane potential is the current through this channel expected to be zero, ignoring the effects of gating? That is, what is the reversal potential for this channel? It may help to draw a circuit model of this channel consisting of two parallel branches, one for each ion. (Note: if you insist, you can solve this assuming the *permeability* is 5 times larger for potassium, just state your assumptions.)

6 pts **Part d)** Suppose that the chloride conductivity of this membrane is ordinarily very small and that the resting potential is –60 mV. What will be the qualitative effect on the membrane potential (depolarization or hyperpolarization) of opening chloride channels? Which direction will chloride ions move through the membrane?

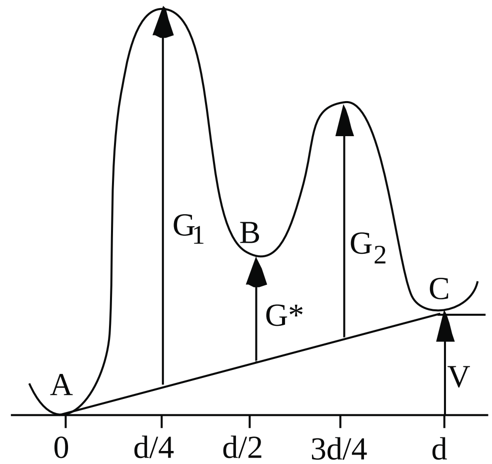
7 pts **Part e)** Consider a different cell from a space invader with the same concentration gradients as in the table above. When a culture of tissue from the creature is analyzed, it is found that the resting membrane potentials of its cells are near +40 mV and action potentials are negative, as sketched at right. Explain how such potentials could come about, assuming the basic mechanisms of ion channels and gating are similar to those with which we are familiar. There are several possibilities; which is most likely if it is determined experimentally that changing the calcium and potassium concentrations in the extracellular space has no effect on the membrane potential, but changing the sodium and chloride concentrations does?



5 pts **Part f)** When an action potential occurs in the cell of part e), which way does charge flow through the cell membrane, into or out of the cell? What ion or ions carry this charge and in which directions do the ions flow?

Problem 2

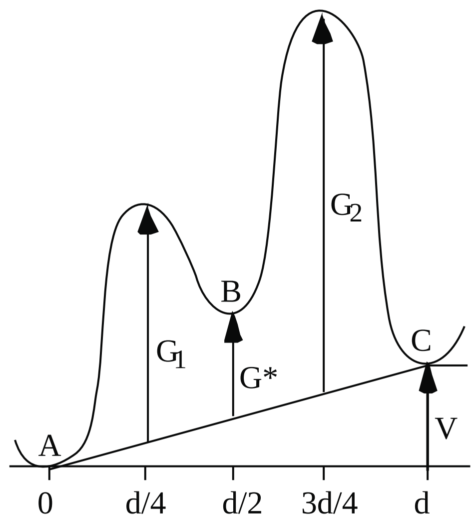
Consider the two barrier models sketched in the figure below. The barrier heights are given as G_1, G^*, and G_2 and the transmembrane electrical potential is V. The peaks and valleys in the barrier diagram are spaced evenly through the membrane, as indicated on the abscissa; d is the thickness of the membrane. The sites A and C are solution on either side of the membrane (outside and inside, respectively), and B is bound to a site in the channel. Parts a, b, and c refer to either diagram.



7 pts **Part a)** Write equations that express the condition under which there is zero net flux of ion between sites A and B. Repeat for sites B and C. You should be able to write equations for the zero-flux concentration B in terms of given concentrations A and/or C, the total concentration of channel X, and the membrane potential V. These equations provide a condition of *equilibrium* between A and B or between B and C.

6 pts **Part b)** Is it possible for the ion to be at equilibrium at all three sites A, B, and C? Under what conditions?

6 pts **Part c)** Suppose the system is at steady state and suppose that A and B are at equilibrium, as defined in part a) above. Must B and C necessarily be at equilibrium also? Explain why or give a counterexample.



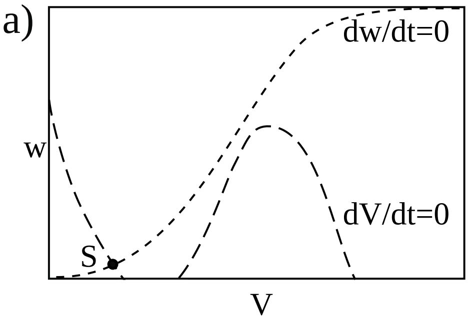
7 pts **Part d)** Consider the difference between the two barrier systems shown. In class, it was claimed that these two would rectify in different directions. Explain what is meant by this (e.g. show a rectifying plot with labeled axes). Show that these two systems rectify differently by working out the fluxes for the two cases. To simplify the problem, assume that independence holds (low concentrations) and that the larger barrier dominates the flux, e.g. that $\exp(-G_1/RT) << \exp(-G_2/RT)$ for the top barrier and vice-versa for the bottom one.

Problem 3

Consider the phase planes drawn below. The equations in each case are a Hodgkin-Huxley system like the MLE. Nullclines and equilibrium points are shown; equilibrium points are labeled as to type (S stable, U unstable, or D saddle). Sometimes the manifolds of a saddle are given in part. For each case, state whether the system does or does not have a limit cycle, based on the information given, and tell why. If a limit cycle exists, sketch its approximate location. You may assume that an outer boundary meeting the conditions of the Poincaré-Bendixson theorem exists in each case.

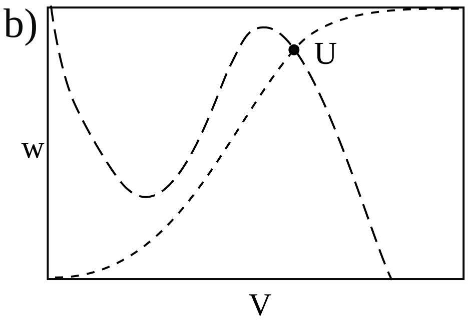
7 pts

a)



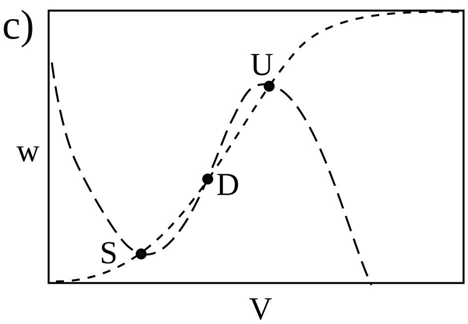
7 pts

b)



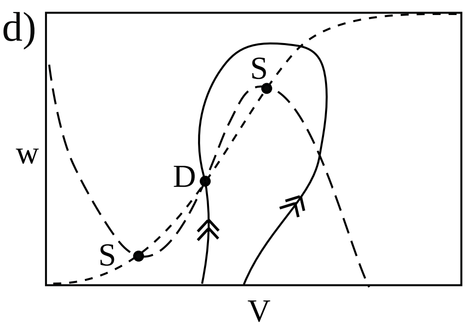
7 pts

c)



7 pts

d)



7 pts

e)