580.439/639 Midterm Exam 2002

1.5 hours. Answer all questions; closed book except for 1 sheet of paper.

Problem 1

Neurotransmitter receptor channels that are activated by GABA generally admit mainly chloride ions. Because E_{Cl} is negative to the resting potential, these channels are inhibitory. However, these channels are not perfectly selective for Cl^- in that they carry a mixed current of Cl^- and HCO_3^- (bicarbonate, abbreviated B^-) ions.

10 pts **Part a)** Assuming *independence*, i.e. that ions flow through the channels independently of one another (or that the concentration of ion is low enough that most channels are unoccupied), write an expression for the current I_{GABA} through a population of these channels as a function of the membrane potential V, the concentrations of the ions in solution (C_{out}, C_{in}, B_{out}, and B_{in} for chloride outside and inside, bicarbonate outside and inside, respectively), and the mobilities u_{Cl} and u_B. (Hint: think GHK).

10 pts **Part b)** Sketch the current-voltage relationship for this channel, based on the model of part a), for the following parameters: C_{out} = 150 mM, C_{in} = 3.2 mM, B_{out} = 5 mM, B_{in} = 2.8 mM, and $u_{Cl} = u_B$ (it will be sufficient to show a sketch based on the asymptotes for the individual currents). Does the current differ much from the result for a purely chloride channel (consider both negative and positive potentials here)?

5 pts **Part c)** Write an expression for the membrane potential at which the current through the channel goes to zero.

10 pts **Part d)** Often the voltage at which the current through a channel is zero is the equilibrium potential. Is that true in this case? If not, tell why and tell which way chloride and bicarbonate **ions** are moving at the zero-current potential.

Problem 2

10 pts **Part a)** Set up a model for flux through the channel of Problem 1 at concentrations too high for independence. Assume a two-barrier channel with a single site that can be occupied by either chloride or bicarbonate (but not both). That is, use the same energy diagram as was considered in detail in lecture. Set up the equations in terms of the rate constants and do not bother to express them in terms of energies. Assume that the permeation is in steady state. The answer to this is the equations only, **don't bother to solve them**. You should end up with five unknowns and five equations.

10 pts **Part b)** What does the fact that the permeabilities of the channel for Cl^- and HCO_3^- are approximately equal (at low concentrations) tell you about barrier heights for permeation of the two ions? What does it tell you about the energy of binding to the site in the channel?

Problem 3

The Van-der-Pol oscillator (VDP) is described by the following non-linear differential equation:

$$\frac{d^2y}{dt^2} \square \square(1 \square y^2)\frac{dy}{dt} + y = 0 \quad \text{for } \square > 0$$

5 pts **Part a)** Write the VDP equation in the form of a system of two differential equations (Hint: you've seen this before; try y_1=y and y_2= $\square^{-1}$ dy/dt).

15 pts **Part b)** Construct a phase plane for this system. Show the isoclines and equilibrium points. The isoclines should divide the phase plane into 6 regions. Draw an arrow or a pair of arrows in each one to show the direction of flow of trajectories in that region. Give equations for the isoclines and values for the equilibrium points. Indicate the direction of trajectories across the isoclines.

10 pts **Part c)** Analyze the stability of the approximate linear system at each equilibrium point. Give the Jacobian matrix and its eigenvalues at each equilibrium point.

10 pts **Part d)** Can you show that the system does or does not have a limit cycle? If so, give the argument. If not, tell what you can about possible limit cycles.

5 pts **Part e)** What would be the answer to part d) if you were told that the amplitude of the state variables are bounded by some functions $B_1(\square)$ and $B_2(\square)$, for $\square$>0?