The Johns Hopkins University
Department of Electrical and Computer Engineering

505.460 — Introduction to Linear Systems — Fall 2004

Final exam

Name: _____

Instructions

1. Answer **any six of the seven questions.**
2. All questions are of equal value.
3. If you answer more than six questions, I will grade the first six that I see.
4. You have 2.5 hours.

Allowable aids

1. Table 3.1 (page 206) & Table 3.2 (page 221)
2. Table 4.1 (page 328) & Table 4.2 (page 329)
3. Table 5.1 (page 391) & Table 5.2 (page 392)
4. One standard size, double sided formula sheet.

1. Let $x(t)$ be the complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

with frequency (in radians) ω_0.

(a) What is the fundamental period?

(b) Suppose that the discrete-time signal $z[n]$ is obtained by uniform sampling of $x(t)$ with sampling interval T_s; that is,

$$z[n] = x(nT_s) = e^{j\omega_0 nT_s}$$

Find the condition on the value T_s so that $z[n]$ is periodic.

(c) What is its period?

2. Assume that the input

$$x(t) = u(t)$$

is applied to the system with impulse response

$$h(t) = e^{-2(t-1)}u(t-1)$$

Find the output $y(t) = (h \star x)(t)$.

3. Determine the Fourier series representation of the signal

$$x(t) = \cos\left(2t + \frac{\pi}{4}\right)$$

4. Suppose that input

$$x(t) = e^{-2t}u(t)$$

is applied to a system with impulse response $h(t)$. The output is observed to be

$$y(t) = e^{-3t}u(t)$$

(a) Find the Fourier Transform of the input; i.e. find $X(j\omega)$.

(b) Find the Fourier Transform of the output; i.e. find $Y(j\omega)$.

(c) Use parts (a) and (b) to determine the Fourier transform of the system $h(t)$; i.e. find $H(j\omega)$.

5. An ideal *band-stop* filter is one specified by

$$|H(j\omega)| = \begin{cases} 0 & \Omega_L < |\omega| < \Omega_h \\ 1 & \text{othewise} \end{cases}$$

   (a) Sketch the magnitude frequency response of this filter, for $|\omega| < 500\,\text{Hz}$ if $\Omega_L = 20\,\text{Hz}$ and $\Omega_H = 100\,\text{Hz}$

   (b) Suppose that you are trying to build this filter, but all you have are ideal high- and low-pass filters. Explain how you could go about it.

6. The signals

$$x(t) = 2\sin(20\pi t)$$

and

$$y(t) = \sin(20\pi t)$$

are to be sampled and stored digitally.

(a) How fast much each be sampled so that perfect reconstruction is possible?

(b) How does your answer change if $z(t) = x(t)y(t)$ is to be sampled?

(c) How does your answer change if $z(t) = (x \star y)(t)$ (i.e. the convolution) is to be sampled?

7. In the double-side band modulation scheme:

$$y(t) = x(t)\cos(2\pi f_m t)$$

with demodulation:

$$z(t) = y(t)\cos(2\pi f_d t)$$

and $w(t) = (h \star z)(t)$. What is the relationship between f_m, f_d and the bandwidth of $x(t)$? Also, what type of system is $h(t)$ if we are going to have $w(t) = x(t)$?