The Johns Hopkins University
Department of Electrical and Computer Engineering

505.460 — Introduction to Linear Systems

12 December 1993

Final Exam.

Name: _____

Question	Maximum	Marks
1.	15	
2.	15	
3.	10	
4.	10	
5.	10	
6.	10	
7.	10	
8.	10	
9.	10	
10.	10	
Total	80	

Some useful mathematical facts:

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta); \quad \sin(\alpha+\beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$

$$\cos(\alpha) = \frac{1}{2j}\left[e^{j\alpha} - e^{-j\alpha}\right]$$

$$\int x\sin x\,dx = \sin x - x\cos x + C; \qquad \int x\cos x\,dx = \cos x + x\sin x + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C; \qquad \int xe^{ax}\,dx = \frac{1}{a^2}(ax-1)e^{ax} + C$$

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, \quad |a| < 1. \qquad \sum_{k=0}^{N} a^k = \frac{1-a^{N+1}}{1-a}$$

$$a^n = e^{n\ln a}$$

$$\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}; \qquad \sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2}$$

1. Explain why sampling of bandlimited signals must be done at a rate higher than twice the highest frequency of the signal.

2. Single-side-band Modulation (SSB) and Quadrature Multiplexing (QM) are two ways of achieving the same goal. Explain what this goal is, how each of the two methods achieves it, and what the advantages and disadvantages of each method are.

3. The input to a linear time invariant filter with frequency response $H(\omega) = \frac{1}{1+j\omega}$ is given by $x(t) = e^{-t}u(t)$. Compute the output $y(t)$.

4. For the complex function

$$X(\omega) = \frac{2 + j\omega}{3 + j\omega};$$

compute the magnitude and phase.

5. Consider a general system whose output $y(t)$ is related to the input $x(t)$ by the following expression

$$y(t) = \frac{dx(t)}{dt}.$$

Which of the following properties does the system have?

Mermoryless? ___

___

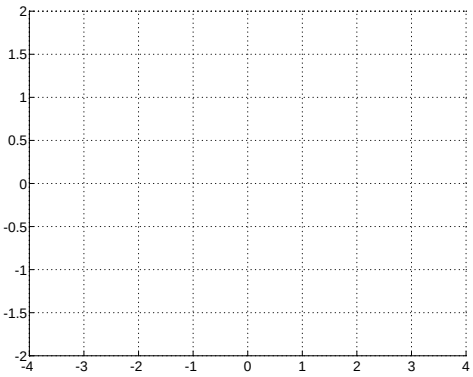
Linear? ___

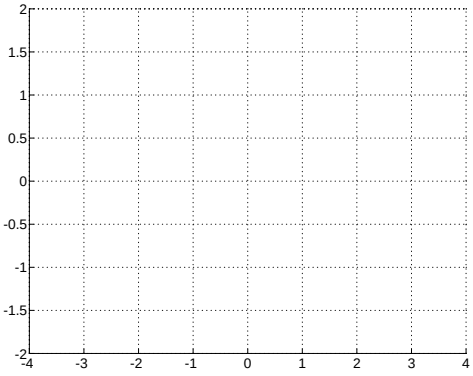
___

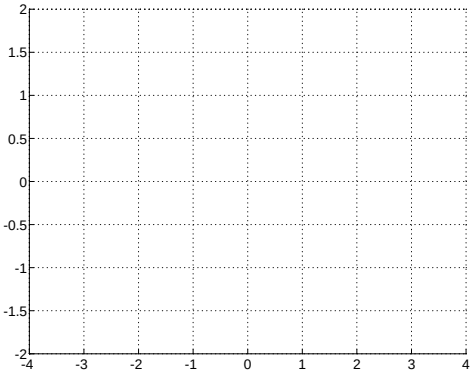
___

[You will get two marks for each correct answer, lose one mark for each incorrect answer.]

6. Determine and sketch the even and odd parts of the signal depicted below:







7. Determine the Fourier series representation of the following signal:

$$x(t) = \cos(4t) + \sin(8t).$$

8. The following is the Fourier transform of a continuous-time signal. Determine the signal from its transform.

$$X(\omega) = \frac{2\sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}.$$

9. Let $x[n]$ be periodic with period N and Fourier Series representation

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}.$$

Assume that N is even. Is

$$x[n] - x[n - \frac{N}{2}]$$

periodic? If so, calculate the discrete-time Fourier series representation.

10. Compute the discrete-time Fourier transform of the signal

$$x[n] = \delta[4 - 2n].$$