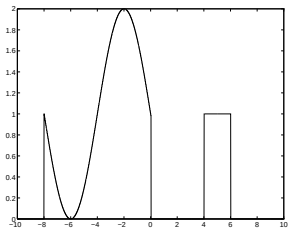
The Johns Hopkins University
Department of Electrical and Computer Engineering

505.460 — Introduction to Linear Systems — Spring 1996

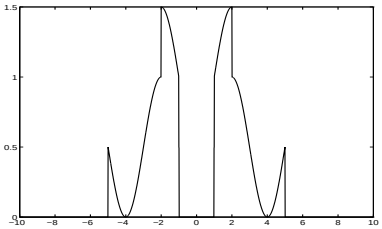
Quiz No. 1.

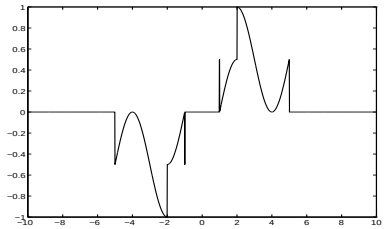
1.

t	-8	-6	-4	-2	0	2	4	6
$1-\frac{t}{2}$	5	4	3	2	1	0	-1	-2
$2x(1-\frac{t}{2})$	$0 \to 2$	0	2	4	$2 \to 0$	0	$0 \to 2$	$2 \to 0$



2. Find and plot the even and odd parts of the signal.





3. $u(t+2) - u(t+1) + \left[1 - \cos(\frac{\pi t}{2})\right]\left[u(t-1) - u(t-5)\right]$

4. (a) Memoryless? No. $y(t)$ depends on $x(t+1)$.
   (b) Causal? No. $y(t)$ depends on $x(t+1)$.
   (c) Linear? Yes.

$$\begin{aligned}\frac{1}{2}\int_{t-1}^{t+1}\left(a_1x_1(\tau)+a_2x_2(\tau)\right)d\tau &= a_1\frac{1}{2}\int_{t-1}^{t+1}x_1(\tau)d\tau + a_2\frac{1}{2}\int_{t-1}^{t+1}x_2(\tau)d\tau \\ &= a_1y_1(t)+a_2y_2(t)\end{aligned}$$

   (d) Time-invariant? Yes. Let $x_1(t) = x(t-T)$.

$$\begin{aligned}\frac{1}{2}\int_{t-1}^{t+1}x_1(\tau)d\tau = \frac{1}{2}\int_{t-1}^{t+1}x(\tau-T)d\tau &= \frac{1}{2}\int_{t-T-1}^{t-T+1}x(\mu)d\mu \qquad (\mu=\tau-T) \\ &= y(t-T)\end{aligned}$$

   (e) Stable? Yes. If $|x(t)| \le M$ $|y(t)| \le \frac{1}{2}\int_{t-1}^{t+1}|x(\tau)|d\tau \le M\frac{1}{2}\int_{t-1}^{t+1}d\tau = M$

5. Note that $x_2(t) = 2x_1(t-1) + x_1(t+2)$ and so, due to the LTI properties of the system:

$$\begin{aligned}y_2(t) &= 2y_1(t-1)+y_1(t+2) \\ &= 2(e^{-2(t-1)}-e^{-3(t-1)})u(t-1) + (e^{-2(t+2)}-e^{-3(t+2)})u(t+2)\end{aligned}$$