The Johns Hopkins University
Department of Electrical and Computer Engineering

505.460 — Introduction to Linear Systems

Spring 1995

Final Exam

Name: ____

Instructions: Answer three (3) out of the 5 questions in each of parts A and B. Clearly mark below which ones you are attempting by circling the number in the table. All questions are of equal value. The answers should be written in the loose sheets provided. Start each question in a new page. Time Alloted: 2.5 hours.

Marks

Question	Maximum	Marks	Question	Maximum	Marks
A1.	10		B1.	10	
A2.	10		B2.	10	
A3.	10		B3.	10	
A4.	10		B4.	10	
A5.	10		B5.	10	
Total A	30		Total B	30	

Some useful mathematical facts:

$$\cos^2(\alpha) + \sin^2(\alpha) = 1$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha + \beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$

$$\cos(\alpha) = \frac{1}{2}\left[e^{j\alpha} + e^{-j\alpha}\right]$$

$$\sin(\alpha) = \frac{1}{2j}\left[e^{j\alpha} - e^{-j\alpha}\right]$$

$$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}, \qquad \text{for } |a| < 1$$

A1. (a) Sketch the magnitude $|H_h(\Omega)|$ of a discrete-time high-pass filter (make sure that you cover at least $\Omega \in [-4\pi, 4\pi]$).

(b) Briefly describe how the impulse response for this filter can be obtained from that of a low-pass filter $h_l[n]$.

A2. (a) Explain what is meant by coherent modulation of a signal.

(b) What is the advantage/disadvantage of this type of scheme?

(c) Give an example of such a modulation scheme.

A3. The output to discrete-time filter is given by the recursion:

$$y[n] = \frac{1}{3}\Big(x[n+1] + x[n] + x[n-1]\Big)$$

(a) For discrete-time filters, what do FIR and IIR stand for? What type is this filter?

(b) Find the transfer function

$$H(\Omega) := \frac{Y(\Omega)}{X(\Omega)}$$

(c) If $x[n] = \delta[n]$, what will the output of this system be?

A4. Recall that, according to Parseval's relation, the energy of a signal is given by:

$$E_X = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

(a) If $|X(\omega)|^2 = \frac{1}{1+\omega^2}$, and the signal is fed into an ideal low pass filter with cutoff frequency W write down an expression for the energy of the output E_Y.

(b) What frequency must W be so that the energy of the output is at least 99% of the energy of the input? **Hint:**

$$\int \frac{1}{1+s^2} ds = \tan^{-1} s + C$$

A5. What is meant by the word *aliasing*? Make sure that you explain where it comes from in enough detail so that I know that you understand it. How can it be prevented in applications? (give two ways)

B1. Let $x(t)$ be a real, even, periodic function with period T.

(a) Show that the coefficients in the Fourier series representation satisfy $a_k = a_{-k}$.

(b) Show that

$$x(t) = a_0 + \sum_{k=1}^{\infty} 2a_k \cos(2\pi kt/T)$$

B2. Let $f(t)$ and $g(t)$ be two periodic functions with period T. Let

$$h(t) := \frac{1}{T}\int_{-T/2}^{T/2} f(\tau)g(t-\tau)d\tau$$

(a) Show that $h(t)$ is also periodic with period T.

(b) If the Fourier series representations of $f(t)$ and $g(t)$ are

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T} \quad \text{and} \quad g(t) = \sum_{l=-\infty}^{\infty} b_l e^{j2\pi lt/T}$$

show that

$$h(t) = \sum_{m=-\infty}^{\infty} c_m e^{j2\pi kt/T}$$

where $c_m = a_m b_m$.
Hint: recall that

$$\frac{1}{T}\int_0^T e^{j2\pi k/T} dt = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases}$$

B3. Compute the discrete-time Fourier transform of the signal

$$x[n] = \begin{cases} 1 & k > 0 \\ 0 & k = 0 \\ -1 & k < 0 \end{cases}$$

B4. Consider a system whose input-output relation can be written as:

$$y(t) = \int_{-\infty}^{\infty} \delta(\tau)u(\tau+1)x(1-\tau)d\tau$$

Which of the following properties does this system have? Make sure to justify your answer. (50% of the marks will be awarded based on your justification)

(a) Linearity?

(b) Stability?

(c) Time-invariance?

(d) Causality?

What is the output to $x(t) = \cos(2\pi t)$?

B5. Let $x(t) = e^{-2(t-1)}u(t-1)$ be the input to a system with transform $H(\omega) = \frac{1}{3+j\omega}$. Show that the output

$$y(t) = \left(e^{-2(t-1)} - e^{-3(t-1)}\right)u(t-1)$$