Homework 7

1. Show that for any three objects A, B, C in a cartesian closed category, there are isomorphisms:

   (a) $(A \times B)^C \cong A^C \times B^C$

   (b) $(A^B)^C \cong A^{B\times C}$

2. (a) Show that for any objects A, B in a cartesian closed category, there is a bijective correspondence between points of the exponential $1 \to B^A$ and arrows $A \to B$.

   (b) Is the category of monoids cartesian closed?

3. Consider the category of sets equipped with a distinguished subset, $(A, P \subseteq A)$, with maps $f : (A, P) \to (B, Q)$ being those functions $f : A \to B$ such that $a \in P$ iff $f(a) \in Q$. Show this category is cartesian closed by describing it as a category of pairs of sets.

4. (a) Show that in any cartesian closed poset with joins $p \vee q$, the following "distributive" law of intuitionistic propositional calculus holds:

   $$((p \vee q) \Rightarrow r) \Rightarrow ((p \Rightarrow r) \wedge (q \Rightarrow r))$$

   (b) Generalize the forgoing problem to an arbitrary category (not necessarily a poset), by showing that there is always an arrow of the corresponding form.

5. * Prove that in a CCC $\mathbf{C}$, exponentiation with a fixed base object C is a contravariant functor $C^{(-)} : \mathbf{C}^{\mathrm{op}} \to \mathbf{C}$, where $C^{(-)}(A) = C^A$.